CLOSED-FORM SPACE RESECTION USING PHOTO SCALE VARIATION

by

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KEY WORDS: Algorithms, Close_Range, Convergent, Orientation, Transformation.

ABSTRACT

Space resection is the process by which the spatial location and orientation of a photograph are determined. However, because the observation equations are non-linear, approximations of parameters are necessary at the beginning of the adjustment. A closed solution means that there is no need for any initial values or estimates of the spatial position and the orientation of the photograph. A general solution for closed space resection is proposed and described. The solution is based on the analysis of the scale variations of the image distances between the control points. The proposed approach has been tested in close range photogrammetry project with highly oblique and convergent photography.

INTRODUCTION

The problem of space resection involves the determination of the spatial position and the orientation of a camera exposure station. In photogrammetric practice, the solution of this problem is most commonly arrived at through a least squares adjustment based on the collinearity condition. Since this is a nonlinear model, approximations of the exterior orientation parameters are necessary in order to initiate an iterative adjustment solution. To obviate the need for such initial estimates for the parameters, especially in the case of convergent or oblique photography which is often used in the case of close range photogrammetric applications, various efforts have been made in the past by photogrammetrists and mathematicians to develop a closed-form space resection solution. In addition, there has been considerable interest in such a solution in the field of robot vision since the early days of its application to the three-dimensional analysis (Sobel 1974).

Closed-form space resection can be achieved by using the Direct Linear Transformation (DLT) as was proposed by Abdel Aziz and Karara (1971). But a minimum of six object control points are needed. Other closed-form solutions introduced special additional constraints such as the assumption that the object plane is nearly parallel to the image plane (Rampal, 1979). Fischler and Bolles (1981) and Zeng and Wang (1992) both proposed a somewhat similar closed-form space resection solution that leads to a fourth degree polynomial in a variable that does not represent the spatial camera position nor its orientation. This leads to complexity of the solution resulting in multiple roots for the polynomial, two of which are imaginary.

Our approach for a closed-form resection solution is based on the well known fact that the scale in a perspective photograph is variable across the image plane. One explanation for this variation is the fact that during the imaging process in a frame camera, the third dimension (z image coordinate) in the image space is being forced to remain equal to the focal length of the camera at each image point. Conversely, a constant scale in the image space can be enforced, if the camera focal length is allowed to vary across the photo. If the scale between a set of observed image points, such as for some object space control points, is forced to remain unchanged, it will cause a move in the coordinates in the image space. This will result in a new set of image coordinates and a different focal length (z image coordinate) at each control point. The new set of coordinates may be viewed as a representation of a scaled and rotated three dimensional model of the object control points. Using a closed three dimensional transformation between the ground coordinates and the newly formed three dimensional coordinates will result in the camera spatial position and orientation parameters.

The collinearity equation which describes the geometrical relationship between the object point, camera spatial position and its exterior and interior orientation, and the image coordinates of the point, can be derived from a three dimensional conformal transformation. With this derivation the constant scale in the three dimensional transformation is eliminated and constant focal length is imposed to reflect the camera geometry. The resulting image is a three dimensional model with constant z (focal length) and a variable scale. With our proposed solution we regain the constant scale of the model by varying the image coordinates and the focal length of the points. One point needs to be fixed in its measured image coordinates and focal length to eliminate singularity problems.

INVARIENT PHOTO SCALE

The proposed concept of invariant photo scale is analyzed mathematically for the case of photo with three control points, which is the minimum control required for space resection.

By fixing the focal length at point 1 (f_1), the changed image coordinates $\hat{x}_i, \hat{y}_i$ (i=2,3) due to focal length perturbations at points 2 and 3 can be expressed as:

$$\hat{x}_i = \frac{x_i}{f_1} f_i \qquad i=2,3 \qquad (1)$$
$$\hat{y}_i = \frac{y_i}{f_1} f_i$$

The horizontal distances between the three points after these changes are given as:

$$d_{ij}^2 = \frac{1}{f_1^2}\left[\left(f_i x_i - f_j x_j\right)^2 + \left(f_i y_i - f_j y_j\right)^2\right] \qquad (2)$$

Where i=1,2 and j=i+1,3

The scale of the newly formed three dimensional image model can be expressed as:

$$scale = \left[\frac{d_{ij}^2 + (f_i - f_j)^2}{D_{ij}^2}\right]^{\frac{1}{2}} \qquad (3)$$

i=1,2 and j=i+1,3

where:

$$D_{ij} = \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2 + (Z_i - Z_j)^2}$$

and X,Y,Z are the object space coordinate in a right handed cartesian coordinate system.

Expanding Eq. (3) and eliminating the scale variable will result in equations;

$$f_2^2 + a_1 f_2 + b_1 f_3 + c_1 f_3^2 = \beta_1 \qquad (4)$$
$$d_1 f_2^2 + e_1 f_2 + g_1 f_2 f_3 + h_1 f_3^2 = \beta_2 \qquad (5)$$

where:

$$\alpha_1 = D_{12}^2 / D_{13}^2$$
$$\alpha_2 = D_{12}^2 / D_{23}^2$$
$$r_i^2 = x_i^2 + y_i^2 + f_1^2$$
$$a_0 = r_2^2 / f_1^2$$
$$a_1 = -2(x_1 x_2 + y_1 y_2 + f_1^2) f_1 / a_0$$
$$b_1 = 2\alpha_1(x_1 x_3 + y_1 y_3 + f_1^2) f_1 / a_0$$
$$c_1 = -\alpha_1 r_3^2 / r_2^2$$
$$\beta_1 = (\alpha_1 - 1) r_1^2 / a_0$$
$$d_1 = (1 - \alpha_1) r_2^2 / f_1^2$$
$$e_1 = -2(x_1 x_2 + y_1 y_2 + f_1^2) / f_1$$
$$g_1 = 2\alpha_2(x_2 x_3 + y_2 y_3 + f_1^2) / f_1^2$$
$$h_1 = -\alpha_2 r_3^2 / f_1^2$$
$$\beta_2 = -r_1^2$$

From Eq. (4), f_3 may be derived as:

$$f_3^2 = \frac{1}{c_1}\left[\beta_1 - f_2^2 - a_1 f_2 - b_1 f_3\right] \qquad (6)$$

Substituting Eq. (6) in Eq.(5), will result in,;

$$f_3 = \frac{\gamma_1 - \gamma_2 f_2^2 - \gamma_3 f_2}{g_1 f_2 - \gamma_4} \qquad (7)$$

where

$$\gamma_1 = \beta_2 - \frac{h_1}{c_1}\beta_1$$
$$\gamma_2 = d_1 - \frac{h_1}{c_1}$$
$$\gamma_3 = e_1 - \frac{a_1 h_1}{c_1}$$
$$\gamma_4 = \frac{b_1 h_1}{c_1}$$

Substituting Eq. (7) in Eq. (4) will result in the following fourth order polynomial equation:

$$n_4 f_2^4 + n_3 f_2^3 + n_2 f_2^2 + n_1 f_2 + n_0 = 0 \qquad (8)$$

where:

$$n_0 = -b_1 \gamma_1 \gamma_4 + c_1 \gamma_1^2 - \beta_1 \gamma_4^2$$

$$n_1 = a_1 \gamma_4^2 + b_1 \gamma_1 g_1 + b_1 \gamma_4 \gamma_3 - 2 c_1 \gamma_1 \gamma_3 + 2 \beta_1 \gamma_4 g_1$$

$$n_2 = \gamma_4^2 - 2 a_1 \gamma_4 g_1 - b_1 g_1 \gamma_3 + b_1 \gamma_4 \gamma_2 - 2 c_1 \gamma_1 \gamma_2 + c_1 \gamma_3^2 - \beta_1 g_1^2$$

$$n_3 = -2 g_1 \gamma_4 + g_1^2 a_1 - b_1 g_1 \gamma_2 + 2 c_1 \gamma_2 \gamma_3$$

$$n_4 = g_1^2 + c_1 \gamma_2^2$$

Interestingly, Fishler and Bolles (1981) and Zeng and Wang (1992) also arrived at a fourth order polynomial function of a variable, but the geometric representation of this variable is difficult to express. In Eq. (8), clearly the polynomial variable is the focal length at point 2.

An iterative solution can be adopted to solve Eq. (8), by using the initial value of f_2 equal to f_1 . A closed form solution of Eq. (8) can be found in Dehu (1960). The solution will yield one to four real roots. In most cases this will lead to two imaginary and two real roots. If the solution leads to four real roots, the two real roots for f_2 which are in closest in value to f_1 (without changing sign) will be chosen. Such an approach can not be adopted if the variable in the fourth order polynomial as in Fishler and Bolles (1981) and Zeng and Wang (1992) does not represent an identifiable geometric entity. The availability of a fourth control point, will help to find the proper root from the two chosen real roots. Substituting the roots for f_2 in Eq. (7), will result in the determination of the corresponding roots for f_3 .

The perturbations of the image domain coordinates to enforce a constant scale, will result in a three dimensional image model. The control point coordinates in this model will be $(\hat{x}_i, \hat{y}_i, f_i)$ and i=1,2, and 3. $\hat{x}_i, \hat{y}_i$ are defined in Eq. (1). The relationship between the new image model and the object space is expressed by the following three dimensional conformal transformation:

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \frac{1}{s} M \begin{bmatrix} \hat{x}_i \\ \hat{y}_i \\ f_i \end{bmatrix} + \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} \qquad (9)$$

where:
X_o, Y_o, Z_o Camera Position in object Space.
M three dimensional orthogonal transformation matrix, such that M^T will represent the camera orientation matrix.

CLOSED FORM THREE DIMENSIONAL COORDINATE TRANSFORMATION

To obtain the exterior orientation parameters for the photograph, we need to solve Eq. (9) for X_o, Y_o, Z_o and omega, phi and kappa that defines the orientation matrix M^T. Although Eq. (9) is linear in X_o, Y_o, Z_o , it is non-linear in terms of the scale (s) and omega, phi and kappa rotational elements and require initial estimates. The initial value for the scale can be obtained from Eq. (3). The orthogonal orientation matrix M, can be computed using quaternions (Horn, 1987). This procedure can be summarized in the following steps:

1. Let the origin be the first point.
2. Take the line from the first to the second point to be the direction of the new x-axis.
3. Place the new y-axis perpendicular to the x-axis and assume a right handed coordinate system to define the direction for z-axis.
4. Let the coordinates of the three points in each system be expressed as a three dimensional vector:

   in image model : $v_{m,1}, v_{m,2}, v_{m,3}$

   in object space: $v_{o,1}, v_{o,2}, v_{o,3}$
5. Construct

   $$\overline{X}_m = v_{m,2} - v_{m,1}$$

   then

   $$\dot{X}_m = \frac{\overline{X}_m}{\left\| \overline{X}_m \right\|}$$

   is a unit vector in the direction of the new x-axis.

6. Now let

   $$\overline{Y}_m = (v_{m,3} - v_{m,1}) - [(v_{m,3} - v_{m,1}) \bullet \dot{X}_m] \dot{X}_m$$

   then

   $$\dot{Y}_i = \frac{\overline{Y}_m}{\left\| \overline{Y}_m \right\|}$$

   represent the direction of the new y-axis.
7. The $\overline{Z}$ axis is defined as:

   $$\overline{Z}_m = \overline{X}_m x \overline{Y}_m$$
8. Repeat steps 5 to 7 to the object space system to find $\overline{X}_o, \overline{Y}_o, \overline{Z}_o$.
9. The rotational elements to be computed are the one that performs the transformation:

$$\left|\overline{X}_m \quad \overline{Y}_m \quad \overline{Z}_m\right| \rightarrow \left|\overline{X}_o \quad \overline{Y}_o \quad \overline{Z}_o\right|$$

10. Adjoin the column vectors to form the following matrices:

$$M_m = \left|\overline{X}_m \quad \overline{Y}_m \quad \overline{Z}_m\right|$$

$$M_o = \left|\overline{X}_o \quad \overline{Y}_o \quad \overline{Z}_o\right|$$

11. Then

$$M = M_m M_o^T$$

The decomposition of the rotational matrix into the rotational elements can be performed by investigating corresponding trigono_ metric function values of that element as presented by Shih (1990).

RESULTS

To test this closed-form space resection algorithm, a program was written in C, and was implemented in a close range photogrammetry application software. Two image data sets were acquired using a 35 mm non-metric camera, with focal lengths of 11.000 mm and 51.142 mm. Each set used a different control field that contained a number of points. Table 1 and 2 shows the data for these sets and the results of the resection solution obtained by using the proposed newly developed algorithm and that obtained from the iterative collinearity solution. Using the new algorithm only three control points were used. This resulted in two possible resection solutions. The collinearity solution was then obtained by using the algorithm solution as an initial estimate . The collinearity solution that result in the lowest image residuals is adopted as the final solution.

CONCLUSIONS

The proposed new mathematical model has been tested and implemented in a newly developed software for close range photogrammetry applications. Most of he users of this software are not formally trained photogrammetrists, and consequently a closed-form space resection solution is a functional software requirement.

The minimum number of object control points required for the proposed solution are three. In general this will lead to two possible solutions. But in this new approach, the correct solution is achieved by using the proper focal length sign, eliminating the more tedious need for testing and searching for the correct spatial position and orientation elements.

Since the proposed approach is based on the scale variations of the image distances between the control points, it can be modified to work with machine coordinates instead of photo coordinates. This will provide an alternative approach to the use of DLT for processing non-metric imagery.

This approach also can be modified to process imagery taken with non-conventional with cameras, such as panoramic cameras and fish eye lenses.

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Dat Set No. 1 (Focal = 11.000 mm)			
Control Field			
Point No.	X	Y	Z
A1	12.968	10	19.44
A2	12.534	11.049	19.44
A3	11.484	11.484	19.44
A4	10.435	11.049	19.44
A5	10	10	19.44
A6	10.435	8.951	19.44
A7	11.484	8.516	19.44
B1	12.968	10	10
B2	12.534	11.049	10
B3	11.484	11.484	10

Photo No.	Closed Solution		Collinearity
1	Solution 1	Solution 2	Solution
X	42.425	50.532	42.484
Y	32.983	20.710	32.994
Z	43.537	-2.694	43.573
Omega	-34.121998	-153.515169	-34.133182
Phi	41.553542	65.458995	41.552162
Kappa	133.28245	-121.006813	133.264244

Photo No.	Closed Solution		Collinearity
5	Solution 1	Solution 2	Solution
X	-21.123	-16.072	-16.051
Y	3.982	-13.799	-13.761
Z	-10.248	43.226	43.175
Omega	161.96661	41.244502	41.237835
Phi	-51.14169	-33.186463	-33.184291
Kappa	74.448861	-31.898479	-31.888787

Photo No.	Closed Solution		Collinearity
9	Solution 1	Solution 2	Solution
X	47.951	46.882	48.564
Y	11.161	0.077	11.159
Z	7.704	22.626	7.559
Omega	156.905162	60.668651	156.896558
Phi	78.168973	64.234271	78.17292
Kappa	-70.334939	26.52283	-70.343181

TABLE 1

Dat Set No. 2 (Focal = 51.143 mm)			
Control Field			
Point No.	X	Y	Z
1	212.88	295.838	670.112
10	627.165	266.694	632.276
100	297.887	297.633	670.499
1001	1222.1	289.042	642.721
1000	6.791	289.482	675.543
160	317.973	84.622	374.536
109	627.016	246.874	635.654

Photo No.	Closed Solution		Collinearity
1	Solution 1	Solution 2	Solution
X	2026.539	-905.673	-905.689
Y	-1715.83	-43.362	-43.345
Z	1338.82	2050.393	2050.393
Omega	66.039076	6.762263	6.76591
Phi	28.263847	-48.083883	-48.082952
Kappa	13.314491	7.031545	7.037074

Photo No.	Closed Solution		Collinearity
2	Solution 1	Solution 2	Solution
X	508.685	230.171	508.687
Y	-27.807	-1341.854	-27.811
Z	2518.133	-71.984	2518.132
Omega	5.927573	110.130703	5.927163
Phi	-3.342656	-11.848267	-3.342869
Kappa	-3.097286	-4.247518	-3.09521

Photo No.	Closed Solution		Collinearity
3	Solution 1	Solution 2	Solution
X	1852.056	-261.635	1852.0851
Y	-33.891	-1263.801	-33.8971
Z	2238.223	-93.421	2238.2361
Omega	8.161803	112.522266	8.161595
Phi	34.391304	-27.463022	34.393644
Kappa	-0.911878	-4.862724	-0.912485

TABLE 2