ACCURACY IMPROVEMENT IN COMPUTATIONAL CLOSE-RANGE PHOTOGRAMMETRY BY THE USE OF MULTI-CONTROL VARIABLE (MCV) MONTE CARLO METHOD

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ABSTRACT

Accuracy improvement by use of multi-control variate method (MCV) among other Monte Carlo methods is presented. The (MCV) method is particularly applicable in close-range photogrammetry as the accuracy of computed three dimensional coordinates of any detail point can be improved by the simultaneous comparison and consideration of a set of all available object space control point coordinates.

1. INTRODUCTION

Monte Carlo techniques are often used in many scientific and other disciplines and an important part of this effort is directed towards variance reduction in systems analyses. The applicability of some of these techniques to the normal case of terrestrial and close-range photogrammetry has been identified, developed and partly demonstrated in (Nagaraja,1991,92). Though at first sight it might seem that sampling procedures only apply in case of simulation studies, further reflection should indicate that conceptually and practically, it should be possible to incorporate these ideas in reduction of variance while dealing with practical data. However, this paper deals with a case study involving some data sets obtained by simulation only, yet closely conforming to actual data. It was not possible to use live or field data in this study.

The subject of variance reduction has received, outside photogrammetry, considerable attention and a number of methods have been developed. Hence, there are a few techniques that help to increase accuracy and hence efficiency of simulations, sometimes substantially, by producing less variable observations. Accordingly, there is a need to study such methods and possibilities.

Applications of simulation are not new in photogrammetry but new applications are still possible. Variance reduction techniques seek either increase in precision (decreased variance) for a fixed sample size or a decrease in sample size required to obtain a given degree of precision.
Several authors have cautioned us in using these techniques without first ensuring their applicability and effectiveness. If properly used, these techniques can provide tremendous increase in the efficiency of the model; however, if the intuition is faulty and the analyst does not use a reasonable design, the technique can also be quite unpredictable and perhaps actually increase variance for some techniques. Because of this characteristic reason, a systematic and thorough study of the selected method is both desirable and essential in adapting it for any specific application in photogrammetry.

The multi-control variables technique applies very well when there is a close repetition or a near equivalent to the process we are using in simulating that can be treated theoretically. Thus, in the normal case of close-range photogrammetry, just as also in aerial photogrammetry, we have an equivalence between computation of three dimensional coordinates of a non-control object space point and that of a given or known control object space point. We can then simulate the least accurate Y-coordinates (say, when it is known that the Y-coordinate axis and camera axis are parallel) of a selected object space point and that of the known control point simultaneously, using same random number streams in both computations. The difference in the known and computed coordinate of the control object space point is indeed an estimate of the correction that can be conveniently applied to the computed coordinate of the selected non-control object space point. This procedure will therefore cut out the variance due to common parameters, mainly the uncompensated systematic errors in the two processes, leaving only the component due to the error of the approximation in the variance. Obviously, this should be of a lower order of magnitude. In photogrammetry, as control point information is usually available and as its accuracy is generally higher, it can be used as a control variate. This same idea can be further extended to other known control points also, thus giving rise to the concept of a multi-control variable (MCV) technique. An attempt is made to present this extended application into the popular area of close-range photogrammetry in this paper.

2. THE MULTI-CONTROL VARIATE (MCV) METHOD

The idea of a single-control variable Monte Carlo technique for reduction of variation of observations, which thereby increases the precision of the estimate, can be readily extended to a multiple control variates technique. The basic computational concept to be used in this multiple case is explained in a nutshell in (Kobayashi, 1981). Following this concept, we proceed to define a new random variable Z,

$$Z(R) = Y(R) - b_i\,(X_i(R) - E[X_i]), \quad i=1,2,..,k.,$$

R: random number from the stream used. ...(2.1)

If Q denotes the covariance matrix of $X = [x_1, x_2, \ldots, X_k]$ & If C denotes the cross covariance vector between Y and X;

$$Q_{ij} = Cov[X_i, X_j], \quad i,j = 1,2,\ldots,k. \qquad \ldots(2.2)$$

$$\text{and } C = Cov[Y, X_i], \quad i = 1,2,\ldots,k. \qquad \ldots(2.3)$$

then the optimal value Bo for $B = [b_1, b_2, \ldots, b_k]$ is

$$B_o = C\,Q^{-1} \qquad \ldots(2.4)$$

which leads to

$$Var[Z] = Var[Y] - C\,Q^{-1}\,C^T = Var[Y](1 - R^2_{YX}) \qquad \ldots(2.5)$$

Where R_{YX} is the multiple correlation coefficient between Y and X. The square of the correlation coefficient is often called the coefficient of determination, as it represents the fraction of the total variation of Y explained by variation of X, Here, as E[Z]=E[Y], computed value of Z is used for Y. The idea behind the multiple-control variate variance reduction is similar to regression analysis (special case of analysis of covariance). However, in the regression analysis we usually wish to investigate the power of a set of predictive variables X in explaining the variation of a response variable Y, whereas in variance reduction by the multi-control variate method, we evaluate the additional reduction in the variance against the additional computation involved. We should bear in mind that it is possible to achieve any desired reduction of variance by using the mean of a sufficiently long simulation run, i.e., we could use the arithmetic mean in place of each observation. The MCV method has been successfully applied in studying the queueing system in industrial operations research. Referring to Graver, (Kobayashi, 1981) reports that multi-control variate method (three control variates only) cuts the variance to about 8% (that is by a factor of 12.5) of the initial value. It is interesting to note that the expected value of Z and Y would still be the same when the negative value before the summation in eq.2.1 is changed to a positive sign. This fact has been used to modify eq.2.1 as follows:

$$Z(R) = Y(R) - b_i(X_i(R) - E[X]) \text{ for } b_i<0 \qquad \ldots(2.6a)$$

$$Z(R) = Y(R) + b_i(X_i(R) - E[X]) \text{ for } b_i>0 \qquad \ldots(2.6b)$$

3. A SIMULATION EXPERIMENT BASED ON THE NORMAL CASE

In order to evaluate the potentialities of the MCV technique, it is necessary to set up a framework for the simulation study. This aspect is covered in this section. In preparing a data set, the true object space coordinates were assumed and the corresponding photo coordinates in the left and right photographs were calculated. The calculated photo coordinates conform to the 'normal case ' in terrestrial and close-range photogrammetry. Accordingly, the effect of the tilts and rotations is not included in the study and hence it results in a certain approximation. Obviously, the advantage gained is the simplicity of the model. Using three different representative object to camera distances, three data sets were set up.

An effort was made to create the data sets in such a way that they approach closely or imitate quite nearly the corresponding practical / live data. The true object space coordinates were perturbed using appropriate standard errors while assuming the usual normal distribution. All other input data were also modified and/or perturbed as necessary. The basic information used in simulating the data sets is shown in table 1. The MCV Y-coordinates, used in deriving the corrections to computed Y-coordinates of non-control points, were perturbed for incorporating the random errors that are characteristic of practical data.

The magnitude of the standard error used is tabulated in item 4. The 'Field Base' used in computing the photographic coordinates were perturbed using values of standard errors given against item 5 in the table. As regards photographic coordinates, three major sources of systematic errors: the error in the interior orientation parameters, the error due to film deformations, and error due to lens distortions, were considered. The corresponding mean values and standard errors are indicated in items 1,2,and 4. Using these, two values were simulated for each one of the parameters & the first one was used to incorporate the source of error, while the second one was used to counteract its effect. So, it is easy to see that the resulting photographic coordinates reflect the presence of uncompensated residual systematic error effects, which is so characteristic of a practical/ live data. The equations used in consideration of the radial and tangential lens distortions as recommended and used by (Merchant, 1972) and listed below. The data sets were used with 9 MCV control points, equally distributed, in all the three planes.

$$dx_r = x\,(K_1 r^2 + K_2 r^4 + K_3 r^6) \quad \ldots(3.1)$$

$$dy_r = y\,(K_1 r^2 + K_2 r^4 + K_3 r^6) \quad \ldots(3.2)$$

$$dx_t = [\,P_1 (r^2 + 2x^2) + 2P_2\,x\,y\,][\,1 + P_3 r^2 + \ldots] \quad \ldots(3.3)$$

$$dy_t = [\,2P_1\,x\,y + P_2 (r^2 + 2y^2)\,][\,1 + P_3 r^2 + \ldots] \quad \ldots(3.4)$$

where, the subscripts r and t refer to radial and tangential distortion effects respectively. Finally, the photocoordinates were perturbed for random measurement errors using indicated standard errors.

The data were analysed on an IBM compatible PC.

4. ANALYSES OF RESULTS

The results obtained by processing the various data sets discussed in section 2 are summarised and presented in table 2. As already stated, three data sets were simulated and analysed. For each data set, three cases 2a, 2b and 2c with different sample sizes of 36, 7 and 3 were chosen, the first case being a representation of the theoretical class, and the last one representative of the typical practical cases in industrial photogrammetry. The comparison between the entries in items 1 and 2 gives the improvement in quality which can be directly attributed to the MCV method. This improvement expressed in percent is averaged and shown against item 7. A comparison of the entries against items 3 and 4 indicates the improvement in bias. This again is expressed as percentage and then shown as entries against item 8, bias reduction. Finally, a comparison of entries against items 5 and 6 indicates the improvement in the standard error. This improvement is against calculated as a percentage and shown as entries again item 9. Thus, a summary of the improvements gained by the use of the MCV method can be easily read by looking at the entries against items 7,8 and 9. Accordingly, it is easy to see that improvements of the order of 60% to 70% are generally achieved. However, it is to be noted that 9 MCV control points distributed in the three planes, were used in achieving the stated improvement.

5. CONCLUSIONS

The conclusions are summarized as follows.

1. The MCV technique is easily adaptable to the case of close-range photogrammetry.

2. The reductions in RMS, Bias and Variance are very substantial and hence worth the trouble.

3. For the practical realisation of the method, it is essential for anyone to be able to invert the Variance matrix Q.

TABLE 1: IMPORTANT MEAN VALUES AND STANDARD ERRORS USED IN THE SIMULATION STUDY (Unit : mm).

No Para.		Mean val.	Std.err.	No Para.		Mean val.	Std.err.
1 Geom	xo	0.0	0.05	5 Lens	k1	-0.72D-06	0.17D-07
(mm)	yo	0.0	0.07	Dist	k2	+0.67d-10	0.15D-11
	c	64.0	0.04		k3	-0.16D-14	0.39D-16
					p1	-0.57D-05	0.13D-06
					p2	-0.42D-05	0.14D-06
2 Film	ax	1.0	0.0001				
def.	ay	1.0	0.0001				
3 MCV	D1	9 points	0.5	5 Base	D1	1200.00	0.1
Yi-Va	D2	sym.plac	5.0	dist	D2	9600.00	0.4
riab.	D3	ed in XY	2.0	(B,mm)	D3	4800.00	0.4
(mm)		in obj.spa.					

Notes: Format size = 90x60mm.different scale factors ax,ay used in x & Y directions on left & right photos. Error free ideal coordinates are used for obj. spa. coordinates to enable evaluation of representative quality improvement that could be expected in practice. D1,D2,D3 refer to different data sets used in the investigations.
B:variable, f=64.0mm, Xo=Yo=Zo=0.0
S.E.: Sf=0.01 mm, Sx=Sy=Sx'=Sy'=0.005 mm

Table 2: COMPARISON OF ACCURACY IMPROVEMENT OBTAINED WITH USE OF NORMAL CASE MODEL AND DIFFERENT DATA SETS

No. Case&samp.size		2a:SS=36			2b:SS=7			2c:SS=3		
Particulars.		P1	P2	P3	P1	P2	P3	P1	P2	P3
1 RMS w/o use of	D1	2.7	3.3	4.3	3.5	4.6	6.0	4.7	6.1	7.9
MCV parameters	D2	20.	21.	22.	26.	28.	30.	35.	37.	40.
(mm)	D3	11.	14.	17.	15.	19.	24.	20.	25.	32.
2 RMS w/ use of	D1	1.5	1.3	1.9	1.7	1.4	2.3	1.9	1.5	2.4
MCV parameters	D2	19.	17.	16.	25.	23.	21.	32.	29.	26.
(mm)	D3	6.6	4.9	6.0	7.8	5.4	6.9	8.7	6.2	7.3
3 Bias w/o use of	D1	2.0	2.1	2.3	2.7	3.2	3.8	3.7	4.7	6.1
MCV parameters	D2	16.	16.	16.	21.	21.	22.	28.	29.	32.
(mm)	D3	8.5	8.8	9.5	11.	13.	16.	16.	19.	25.

(Continued)

No.	Case&samp.size Particulars.		2a:SS=36 P1	P2	P3	2b:SS=7 P1	P2	P3	2c:SS=3 P1	P2	P3
4	Bias w/ use of MCV parameters (mm)	D1	1.3	1.3	1.3	1.3	1.2	1.3	1.5	1.2	1.6
		D2	10.	10.	10.	14.	14.	13.	24.	22.	20.
		D3	4.8	4.7	4.7	5.5	4.8	4.5	6.4	5.0	5.2
5	S.E. w/o use of MCV parameters (mm)	D1	1.5	2.3	3.4	1.8	2.9	4.2	1.2	1.9	2.7
		D2	10.	12.	14.	13.	15.	17.	8.3	9.7	11.
		D3	6.6	9.4	14.	8.2	12.	17.	5.3	7.5	11.
6	S.E. w/ use of MCV parameters (mm)	D1	0.6	0.2	1.3	0.8	0.3	1.6	0.5	0.2	1.0
		D2	15.	13.	11.	18.	16.	14.	12.	10.	9.0
		D3	3.8	1.0	3.3	4.7	1.3	4.0	3.1	0.8	2.6
7	RMS reduction %	D1	Average:		53			62			69
		D2	Average:		18			19			23
		D3	Averaga:		19			63			71
8	Bias reduction	D1	Average:		38			60			71
		D2	Average:		36			37			26
		D3	Average:		46			63			74
9	SE. reduction	D1	Average		71			70			71
		D2	Average:		-08			-06			-06
		D3	Average:		73			73			73

Notes: D1,D2.D3 refer to data sets used in data processing. Object distances to front (P1), middle (P2) rear (P3) planes for the three data sets are as follows: D1: 3840, 4840, 5840 mm. D2:29000, 31250,33500 mm. D3: 16320, 19500, 23500 mm. Program names : MCV2A.FOR & MCV2A7.FOR. SS = sample size. Units:mm

6. RECOMMENDATIONS

The recommendations could be summarised as follows.

1. It is necessary to carry out further studies using the extended model that includes tilts and rotations.

2. A practical/live data analyses is required to further validate the workability of the method.

3. The computer processing of data needs to be optimized.

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