INFLUENCE OF GPS DATA FIXED DURING THE FLIGHT FOR AERIAL SURVEY AND AERIAL TRIANGULATION

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ABSTRACT

Modern aerial surveying systems content GPS receivers. It allows to fix co-ordinates of projection centers (exposure stations) during the flight.

Theoretical model of mutual connection between the center of GPS receiving antenna and aerial camera in case of air carriers tilt is considered in the first part of the paper. As conclusions it is underlined that for accurate determination of co-ordinates of projection center the angles of aircraft tilt and angles of camera tilt have to be known. Several ways for solution of this task are proposed.

In the second part of paper the theoretical bases of analytical aerial triangulation in case of known co-ordinates of projection centers (aerial triangulation and terrestrial triangulation). Difference between already existing methods and proposed method is that in last case the control points are not needed at all.

At first sight, it is paradoxical conclusion because when linear elements of exterior orientation are known, angles of aerial image tilt can be arbitrary. For solution of this problem it is proposed to use photogrammetric connections which exist between all images of several flight strips.

In the third part of paper the results of testing of the aerial triangulation method using block of model images are shown. Investigations are implemented for three variants of control network

Objective accuracy evaluation has corroborated rightness of the theoretical aspects and allowed to recommend method for practical application.

1 INTRODUCTION

Fixation of space co-ordinates of the projection centers during the aerial camera carrier flight is very useful and perspective deed. In the paper it is shown, that in the case of fixation of such data with high accuracy, the block aerial triangulation is possible to build without control points. It can economise the cost and eliminate man works at the terrain, which could be hardly accessible, inaccessible, out-of-the-way or dangerous for people. Exactly in these cases the advantages of the photogrammetry as noncontact method of object investigation are seen.

2 GPS-DATA AND FIXATION OF CO-ORDINATES OF PROJECTION CENTERS

In the modern aerial surveying systems, among which the leaders are firms LEICA (Switzerland) and ZEISS (Germany), the co-ordinates of the projection centers are fixing during the flights with use of GPS-receivers. The simplest case is shown on the scheme (figure 1), when receiver A is installed at the same vertical line with the projection center S_1.

During the flight aerial camera is inclined on the angle $\psi,$ and the projection center is in the point S; its placement in chosen space system is determined by vector R_S:

$$R_S = R_A + r_S = R_A + M_\psi r_0 \qquad (1)$$

where r_0 - constant depended on installation of antenna A and aerial camera (projection center S_i). It is determined according to special methodologies.
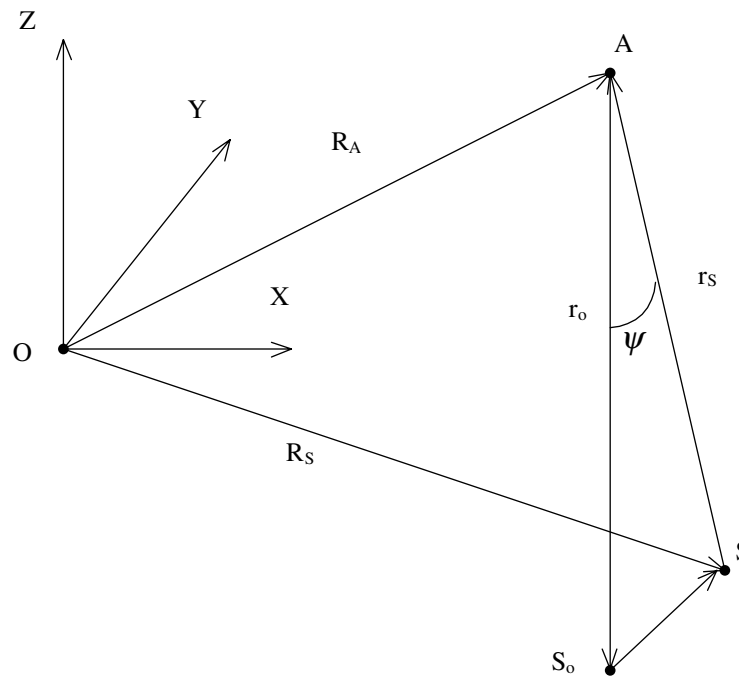
Fig. 1. Simple scheme of the fixation of projection center S placement.

Displacement of point S_1 to real position is created by angles of bank, tangage and drift of the carrier. Therefore matrix M_ψ should considered as matrix of direction cosines, which is calculated on three Eyler's angles. From the formula (1) it follows that position of point X could be determined only if the angles of the aerial carrier tilt are known. We can consider with some assumptions (although it does not correspond to realities and we will show it below) that tilt angles of aerial camera and aerial carrier are identical. For analytical block aerial triangulation these angles are determined by iterative way and with sufficient accuracy; it allows also to define more precise vector R_S.

Approach (1) can be used for aerial survey of average and small scales, when the carrier flight is stable (almost horizontal), and angles of bank and tangage do not exceed 2-3°.

Quite another situation appears if the aerial survey is implemented in large scales, which are used, for example, for cadastral or engineering works. In this case the simplifications shown on fig. 1 are inadmissible. Let GPS-antenna be in point A_0 (fig. 2), when aerial carrier takes horizontal position and displaces to point A when aerial carrier is inclined. The carrier can be considered as solid with geometrical centers in point O_H and axis's X', Y', Z', which are parallel to corresponding axis's of some space system OXYZ. Aerial camera, as a rule, is placed on the gyro-stabilizing platform and it controls that optical axis does not deviate (or deviates only on the small angles) of vertical line. The symmetry center of gyro-stabilizing platform is in the point K and does not coincide with projection center S_0. When aerial carrier is inclined on the angle ψ the gyro-stabilizing platform will incline camera to another direction (what is not so important now), so the camera is really inclined on angle $\varphi (\varphi \neq \psi)$.

From the fig. 2. we have for unknown vector R_S:

$$R_S = R_A + r_{AS} \qquad (2)$$

Vector X can be found by two ways. First of them is following:
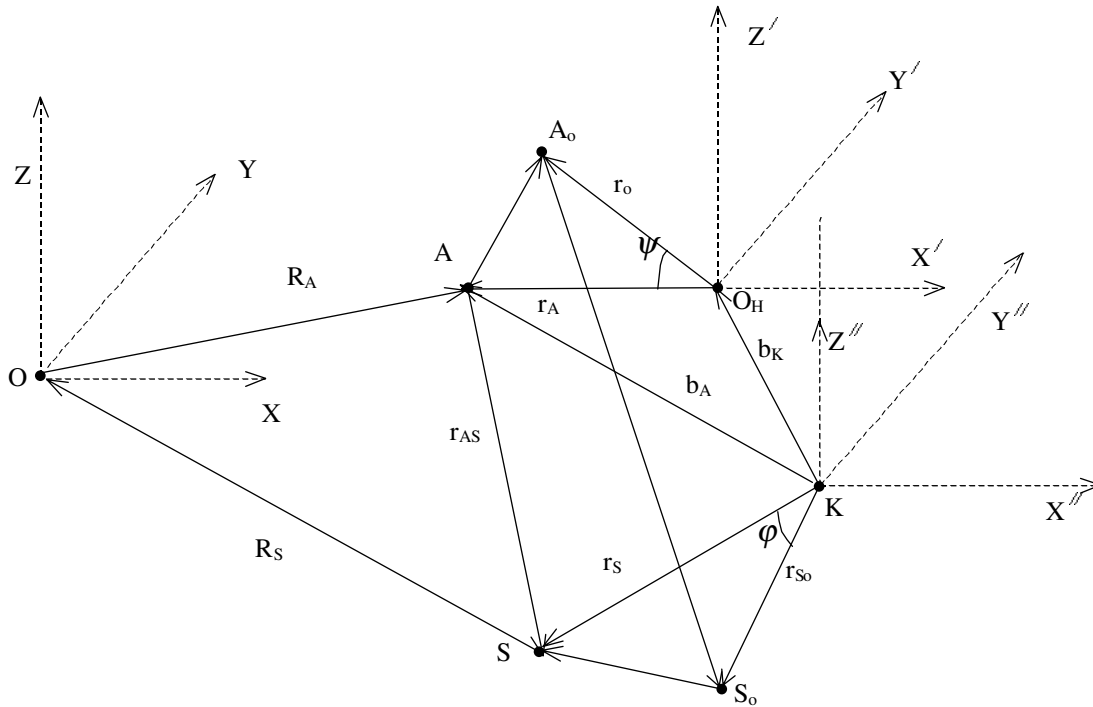
$$r_{AS} = b_k + M_\varphi r_{So} + M_\psi r_o \qquad (3)$$

Fig. 2. Scheme of the displacement of antenna and projection center when aerial carrier and camera are inclined.

Magnitudes b_k, r_{So}, r_o have to be known from the concrete installation of antenna and aerial camera on the carrier. Another way is following:

$$r_{AS} + r_{SSo} + r_{SoAo} + r_{AoA} = 0$$

and as

$$r_{SSo} = r_{So} - r_S = r_{So} - M_\varphi r_{So} \qquad (4)$$

and

$$r_{AoA} = r_A - r_o = M_\psi r_o - r_o,$$

then

$$r_{AS} = -r_{So}(E - M_\varphi) + r_o(E - M_\psi) + r_{AoSo}$$

In these formulas E - singular matrix. Formula (4) is better then (3), as it is simpler to find vector r_{AoSo}, because points A_0, S_o exist physically while points O_H and K exist geometrically. Anyway, from the formulas (2)-(4) it is followed not comforted for practice conclusions:
- for finding of projection center we have to know the angles of inclination of aerial camera and the angles of inclination of aerial carrier, which are different.
-      for finding of projection centers we have to know the mutual position of antenna, carrier center, gyro-stabilizing platform center and also projection center when carrier and camera are horizontal.
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We will not consider here the second of mentioned above problems. It can be solved principally after implementation of the assembling works and special investigations. Talking about solving of the first problem the following ways can be proposed. For finding tilt angles of aerial carrier it is necessary to install three antennas at the points located on one straight line. Then centers of three antennas are fixing the plane and for finding the angles of it drift the formulas of analytical geometry can be used. For finding tilt angles of aerial camera it is necessary to use iterative process of block aerial triangulation. By this process tilt angles of aerial images will be defined more precisely as well as co-ordinates of point S. We have got positive results of this process modelling.

In certain cases some simplifications can be applied considering accuracy of gyro-stabilizing platform functioning. If to assume that accuracy of gyro-vertical simplification is $m\alpha \approx 5' - 8'$ and $KS_o \approx \dfrac{1}{2} f$ (f - focus distance of aerial camera), then error m_S of displacement of the projection center from the vertical is:

$$m_S = \frac{1}{2} f \frac{m_\alpha}{\rho'} \qquad (5)$$

When $f = 140$ mm, $m_S = 0,10 - 0,16$ mm. If boundary error of gyro-stabilisation is $25'$ then $m_S = 0,50$ mm.

Demonstrated results show that with application of good gyro-stabilisation we can assume that points S_o and S will be superposed, in other words, in formula (4) $r_{S_o} (E - M_\varphi) \approx 0$.

Tilt angles of aerial carrier in the real conditions of the flight can be up to several degrees. Displacement AA_o can be calculated this way:

$$AA_o = A_o O_H \frac{\alpha^o}{\rho^o} \qquad (6)$$

Where $A_o O_H$ - distance from aerial carrier center O_H to antenna. When $\alpha = 3°$ and distances are 1m, 2m, 3m, correspondingly there will be 0.05m, 0.10m, 0.15m. These simple calculations show, that antenna should be placed as close to the geometrical center of aerial carrier as possible. Nevertheless when aerial survey is implemented in large scale we can not neglect the factor of carrier tilt. Especially because that in this case the flights are implementing on small height where stability of atmosphere is not very good.

If we want to rich the accuracy of determination of co-ordinates of object points, for example, 5 centimetres, it is necessary to know tilt angles of aerial carrier. About one of the ways we have said above - it is installation of three GPS-antennas. Let note, that for reducing of influence of drift angle in the case of one GPS-antenna it should be located as close to axes $O_H Z'$ as possible (see fig. 2).

There is other method of determination of tilt angles of carrier. It is refusal of gyro-stabilizing platform and hardwired connection of camera with aerial carrier (but all means of vibration extinguishing for camera are kept). If to consider modern tendencies of digital processing of photo images, this way could be very realistic. Digital photogrammetrical station (DPS) does not impose any conditions on the tilt angles of images as photogrammetrical processing is implementing according to rigorous formulas. Even such task as creation of digital photoscheme (in western terminology photo mosaic) on the DPS is realising with high quality and in automatic regime. Tilt angles of carrier in this case are determined in iterative regime from block aerial triangulation with sufficient accuracy. According to these conditions the task of transfer from spatial co-ordinates of GPS-antenna to co-ordinates of project center is solving as well. Of course, refusal of gyro-stabilising platform will cause some contrarguments. But there is essential argument «for this» - this is reduction of prices of automating aerial surveying system approximately on 20-25%, and this is already considerable sum (cost of system is 300,000-400,000$).

Application of GPS-technology for aerial survey of large scale should be co-ordinated with time of exposition t and speed of aerial carries W. During the time of exposition the carrier will remove on the value

$$S = W \cdot t \qquad (7)$$

We should not to confuse this value with geometrical shift of imagery, because, as it is known, this shift in modern camera is compensated. Let calculate S – displacement of antenna during the exposition time. Assume $W = 160$km/hour, $t_1 = 1/100$, $t_2 = 1/1000$ (in second). There $S_1 = 44$ cm., $S_2 = 4$ cm.

If we are talking about high accurate works (for example, inventory making of urban land) that displacement $S_1 = 44$ cm. will make impossible to use GPS. On the contrary, displacement $S_2 = 4$ cm. will show the appropriateness of application of GPS. The technology of aerial survey connects with questions of choice of values t and W (sensitivity of film, illumination of land surface and other).

So application of GPS-technology for fixation of co-ordinates of projection centers demands a deep understanding of aerial surveying process and connected with it photogrammetrical processing of images.

3 AERIAL TRIANGULATION AND GPS-DATA

In the practice of photogrammetrical works there are two cases when co-ordinates of the projection centers are known (fixed with errors which can be considered negligible small). First of them concerns terrestrial survey when co-ordinates of projection centers are known from geodetical works. Definitely saying, in that case the co-ordinates of phototheodolite station are known and co-ordinates of the projection centers are determined when angular elements of external orientation, constant r of camera and horizon of instrument i are known (fig. 3) .
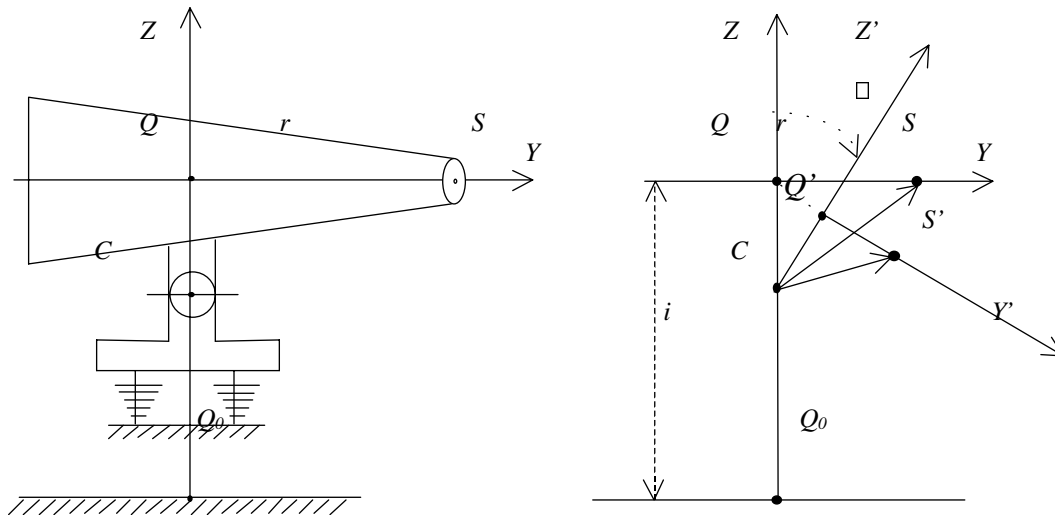

Fig. 3. - Holonomic constraint between points of photo station and projection center when phototheodolite is inclined.

Here Q_0 – photo station center, C – center of rotation (tilt) of phototheodolite,
Q – point of intersection of the main optical axes of camera with axes Z, Q'- shifted point Q when phototheodolite has tilt on ω; S and S' – position of projection center accordingly when $\omega = 0$ and $\omega \neq 0$; r – camera constant, i – horizon of instrument.

As we can see from fig. 3, projection center S and point Q are situated on the main optical axes of the camera; therefore there is no need to determine the space co-ordinates of the projection centers, it will be enough to find the co-ordinates of the point Q. According to these reasoning modified equations of collinearity, where point Q figures, but not the center of projection are formulated:

$$x = f \frac{b_1\left(X^G - X_Q^G\right) + a_1\left(Y^G - Y_Q^G\right) + c_1\left(Z^G - Z_Q^G\right)}{b_2\left(X^G - X_Q^G\right) + a_2\left(Y^G - Y_Q^G\right) + c_2\left(Z^G - Z_Q^G\right) - r} ;$$

$$z = f \frac{b_3\left(X^G - X_Q^G\right) + a_3\left(Y^G - Y_Q^G\right) + c_3\left(Z^G - Z_Q^G\right)}{b_2\left(X^G - X_Q^G\right) + a_2\left(Y^G - Y_Q^G\right) + c_2\left(Z^G - Z_Q^G\right) - r} ,$$

(8)

where a_i, b_i, c_i - directional cosines, calculated on angles α, ω, κ , at that

$$\alpha = A + \varphi \quad - 90°$$

(9)

(here A – direction angle of photographing basis, φ - angle of slant);
X^G, Y^G, Z^G - spatial co-ordinates of the object points;
X_Q^G, Y_Q^G, Z_Q^G - spatial co-ordinates of point Q;
x, z – plane rectangular co-ordinates of image point.

On aerial triangulation constructions instead of equations (8) linearised equations are used:

$$A \cdot \delta_Q + B \cdot \delta_\psi + C \cdot \delta_T + l_x = \vartheta_x;$$

$$A' \cdot \delta_Q + B' \cdot \delta_\psi + C' \cdot \delta_T + l_z = \vartheta_z,$$

(10)

where δ_Q - vector of adjustments to the linear elements of external orientation (spatial co-ordinates of point Q);

δ_ψ - vector of adjustments to the angular elements of external orientation.

δ_T - vector of adjustments to the spatial co-ordinates of object points. A,B,C,A',B',C' – partial derivatives, obtained by differentiation of equations (8).

If co-ordinates of point Q are known precisely then equations (10) are:

$$B \cdot \delta_\psi + C \cdot \delta_T + l_x = \vartheta_x;$$

$$B' \cdot \delta_\psi + C' \cdot \delta_T + l_z = \vartheta_z.$$

(11)

Following constructing of aerial triangulation, as is well known, is implementing with application of least-squares method to the models of type (10) or (11) with arrangement of iterative process. It is classical, good described in the literature method. Second case concerns aerial triangulation when co-ordinates of projection centers are fixing during the flight by GPS. We can consider that co-ordinates of the projection centers are known error-free.

Let formalize the photogrammetrical task by the following way: it is necessary to create the network of block aerial triangulation when linear elements of external orientation X_S, Y_S, Z_S are known for each center of projection. In this case, linearized equations of collinearity is:

$$d_x \delta\alpha + e_x \delta\omega + f_x \delta\kappa + g_x \delta X + h_x \delta Y + k_x \delta Z + l_x = \vartheta_x;$$
$$d_y \delta\alpha + e_y \delta\omega + f_y \delta\kappa + g_y \delta X + h_y \delta Y + k_y \delta Z + l_y = \vartheta_y,$$

(12)

where $\delta\alpha, \delta\omega, \delta\kappa$ - adjustments to sought values of angular elements of external orientation,

$\delta X, \delta Y, \delta Z$ - adjustments to sought values of co-ordinates of the aerial triangulation network points;

l_x, l_y - absolute terms of equations of adjustments;

$d_x, \ldots, k_y$ - partial derivatives.

Classical approach to creation of block aerial triangulation demands availability of control points located on the perimeter of block. Then for control points in equations type (12) $\delta X = \delta Y = \delta Z = 0$, in other words adjustments to the co-ordinates of control points are not determining and with application of least-squares method the task has solution. Our approach consists in the following: if co-ordinates of the projection centers are known then block aerial triangulation network can be created without control points at all. Prima facie this is paradoxical conclusion as for solution of the direct photogrammetrical intersection (fundamental photogrammetrical task) we have to know both linear and angular elements of external orientation of images. When linear elements (spatial co-ordinates of projection centers) are known angles of images tilt can be arbitrary. For completed task solution it is proposed to use additional conditions as photogrammetrical mutual connections between images belonged to one flight strip (in the first priority conditions for connecting points) and between images of adjacent flight strips (for points that lie into interstrip overlap) .
Geometrical essence of the task solution is shown on fig. 4.

Tilts of images with centers S_1 and S_2 result to changes of the vector R_1 and and R_2.. Let impose the condition, that vectors R_1 and R_2 are crossing when lengths of these vectors are fixed. It does not guarantee the success as then point A_1 will circumscribe trajectory what reminds the trajectory of rocking-chair in direction which is approximately perpendicular to the plane XZ. Angles ω for images are not being fixed (are not being determined). But there are other vectors belonged to points located in interstrip overlap, for exaample, R_{100}, and R_{101}.. And here we have full analogy

with rocking-chair: images with centers S_1 and S_2,, S_{10} and S_{11} (and all others) are tilted on angles α, ω, κ while vectors R_1, and R_2,, R_{100}, and R_{101} (and all other pairs) will cross in correspondent points A_1, A_{100} and all others. So interstrip connections allow to eliminate vagueness with angles ω and to solve factually the task of determination of angular elements of external orientation.
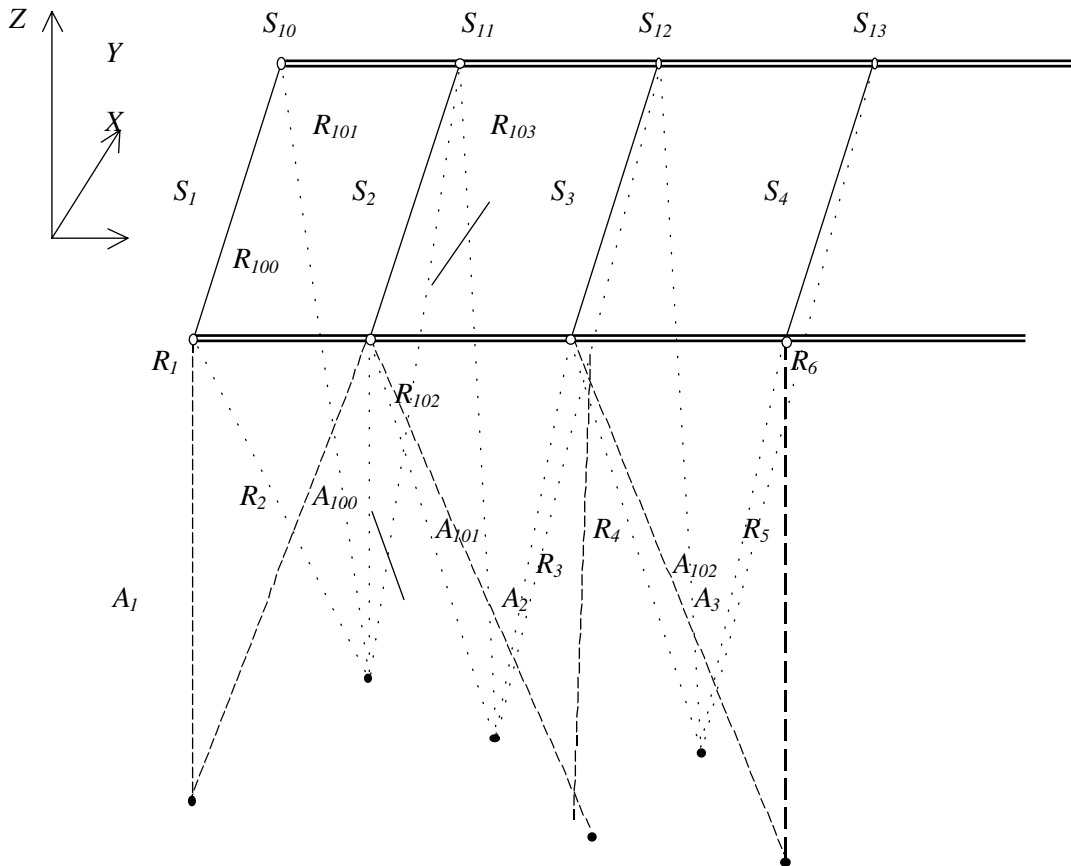

Figure 4. Scheme of photogrammetrical intersections in the aerial surveying overlapping strips.
$S_1, S_2, S_3, \ldots$ – projection centers of i-th flight strip,
$S_{10}, S_{11}, S_{12}, \ldots$ – projection centers of (i+1)-th flight strip,
$A_1, A_2, \ldots$ – points represented on the images of i-th flight strip,
$A_{100}, A_{101}, \ldots$ – points represented on the images of (i+1)-th flight strip.

As it is known, condition of intersection of rays in limits of images pair is the condition of complanarity. So to the equations (12) the linearised equations of complanarity is added. It increases the geometrical quality of photogrammetrical constructions.

$$d_q \delta\alpha_i + e_q \delta\omega_i + f_q \delta\kappa_i + d_q' \delta\alpha_{i+1} + e_q' \delta\omega_{i+1} + f_q' \delta\kappa_{i+1} + l_q = \vartheta_q \tag{13}$$

here I, $i+1$ – numbers of images forming the stereopair,

    $d_q, \ldots f_q'$ - partial derivatives obtained by differentiation of complanarity equation,

    l_q - absolute term of equation of adjustments;

Realisation of the task is implementing in usual iterative regime. Accuracy estimation of obtained magnitudes according to diagonal elements of inverse matrix of normal equations and mean quadratic errors of unit of weight is the concluding stage.

4 EXPERIMENTAL TESTING OF AERIAL TRIANGULATION

Proposed solution of block aerial triangulation task demands detailed testing. It was done with application of methodology of block aerial triangulation networks modelling on PC by engineer Kolb I. under supervisions of author.

There were 100 various versions of networks used in experimental investigation. In this paper we will present only some of results of modelling of three types of photogrammetrical blocks (3x5 images):
- surveying scale 1:10 000;
- elements of internal orientation: f=100.0 mm, x_0 =0.0 mm, y_0 =0.0 mm;
- forward and side overlaps: Px= Py= 60% ;
- accuracy of determination of linear elements of external orientation is 0.1m, accuracy of photogrammetrical measurement is 0.01 mm;
- each stereopair is provided by 6 points located in standard way.

Variant A. Aerial triangulation network consists of 25 points. Control network is not used. Co-ordinates of the projection centers are known.

Variant B. Aerial triangulation network consists of 24 points. Set of data was added by one control point located in the centre of block. Considerable sidelap of images allows to form stereopair of adjacent flight strips images. Such side images is possible to include into photogrammetric block.

Variant C. Aerial triangulation network consists of 21 points. Network was built on classical method and linear elements of external orientation are unknown. Block is provided by 4 control points located in the corners of the block.

Results of modelling is shown in tables 1 and 2. Mean quadratic errors of determination of angular elements of external orientation is presented in minutes, of plane co-ordinates of network points - in millimetres of image scale. For variants A and B in table 1 there are standards of errors which were put in the linear elements of external orientation.

Variants of networks	Accuracy of linear elements (m.)			Accuracy of angular elements (minutes)		
	m_{Xs}	m_{Ys}	m_{Zs}	m_α	m_ω	m_γ
Variant A	0.10	0.10	0.10	0.16±0.05	0.32±0.10	0.35.011
Variant B	0.10	0.10	0.10	0.18±0.06	0.19±0.06	0.22±0.07
Variant C	0.14±0.004	0.45±0.12	0.19±0.05	0.36±0.15	0.45±0.13	0.40±0.10

Table 1. Accuracy of elements of external orientation determination

Variants of networks	Accuracy of spatial co-ordinates (mm.)		
	m_X	m_Y	m_Z /H
Variant A	0.010v0.002	0.012±0.002	1/9700
Variant B	0.009±0.002	0.011±0.002	1/10400
Variant C	0.015±0.022	0.024±0.002	1/7100

Table 2. Accuracy of aerial triangulation on control points.

Negligible increasing of initial data (variant B) leads to some improving of determination of angular elements of external orientation; with it accuracy of determination of spatial co-ordinates of aerial triangulation points is not improving as it is in the limits of modelled errors of photogrammetrical measurements (0.01 mm. in image scale). Renunciation of known co-ordinates of projection centers (variant B) leads to reduction of accuracy of photogrammetric control in 1.5-2 times with comparison of variant A. Implemented experiments let to make conclusions that developed method of aerial triangulation when linear elements of external orientation of images and created programming complex of block aerial triangulation provides high accuracy of photogrammetric constructions.

5 CONCLUSIONS

GPS-data in considerable degree changes the technology of aerial survey and photogrammetrical works. Now there are conditions for review of existed technological schemes of aerial survey, aerial triangulation and mapping of objects and terrain.

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