

**AN APPROACH TO ESTIMATE OF 2D-3D MOTION OF SURFACES:  
THE FREQUENCY DOMAIN TECHNIQUE**

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**ABSTRACT**

Phase correlation is one of the robustest methods for estimating planar translations: it is a global approach because it operates in the frequency domain using the whole image information and not just a selected subset of the image as the feature-based methods do. The good characteristics of phase correlation for estimating planar translations were found to be a strong motivation for its extension to the estimation of planar rotations. An original algorithm for estimating planar rotations inspired by phase correlation is presented in this work. Practical experimentation on real imagery confirm the expected robustness of the method. Furthermore, a natural extension of the proposed algorithm will be also presented for 3D shapes motion estimation. Free-form 3-D surfaces registration is a fundamental problem in 3-D imaging, typically approached by extensions or variations of the ICP algorithm [2,10]. In this paper an alternative procedure for 3-D motion estimation will be suggested, based on the Fourier transform of the 3-D intensity function, implicitly described by the registered time-sequences of range data. As the proposed method is very suitable for 2D and 3D modeling of cultural heritages, some examples and results of its application to Madonna of Donatello, located in the Basilica of St. Anthony in Padua, will be also presented.

**1. ESTIMATION OF PLANAR ROTATIONS BY MEANS OF PHASE CORRELATION**

Phase correlation has been developed in order to estimate planar translations between images pairs and has been shown to be one of the most robust methods for estimating planar translations [3,5]. This method relies upon the following concept: let  $g_1(\mathbf{x})$ ,  $\mathbf{x} \in R^2$ , be an image and  $g_2(\mathbf{x}) = g_1(\mathbf{x}-\mathbf{t})$ ,  $\mathbf{t} \in R^2$ , be a translated version of  $g_1(\mathbf{x})$ . Denote as  $G_i(\mathbf{k}) = F[g_i(\mathbf{x}) | \mathbf{k}]$ ,  $\mathbf{k} = [k_x, k_y]_r$ , the 2-D cartesian Fourier transform of  $g_i(\mathbf{x})$ ,  $i = 1, 2$ ; then

$$G_2(\mathbf{k}) = G_1 e^{-j2\pi\mathbf{k}^T \mathbf{t}} \quad (1)$$

differs from  $G_1(\mathbf{k})$  only in a phase shift. Therefore from

$$Q(\mathbf{k}) = \frac{G_1^*(\mathbf{k})G_2(\mathbf{k})}{|G_1(\mathbf{k})G_2(\mathbf{k})|} = e^{-j2\pi\mathbf{k}^T \mathbf{t}} \quad (2)$$

and

$$q(\mathbf{x}) = F^{-1}[Q(\mathbf{k}) | \mathbf{x}] = \delta_{R^2}(\mathbf{x} - \mathbf{t}) \quad (3)$$

one can estimate translational vector  $\mathbf{t}$  as the coordinates of the peak of impulsive signal  $q(\mathbf{x})$ . This work extends this idea to the case of planar rotations; in this case let  $f_1(\mathbf{x})$  and  $f_2(\mathbf{x})$ ,  $\mathbf{x} \in R^2$ , respectively denote an image and its version rotated by an angle  $\varphi$ , i.e., in cartesian coordinates

$$f_2(x) = f_1(R(\varphi)^{-1} x) \quad (4)$$

where

$$R(\varphi) = \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix} \in SO(2) \tag{5}$$

and  $SO(2)$  is the group of  $2 \times 2$  special orthogonal matrices. Some examples of images pairs  $f_1(\mathbf{x})$  and  $f_2(\mathbf{x})$  are shown in Fig. 1. (“Madonna con bambino” of Donatello, St Anthony Basilic in Padua).

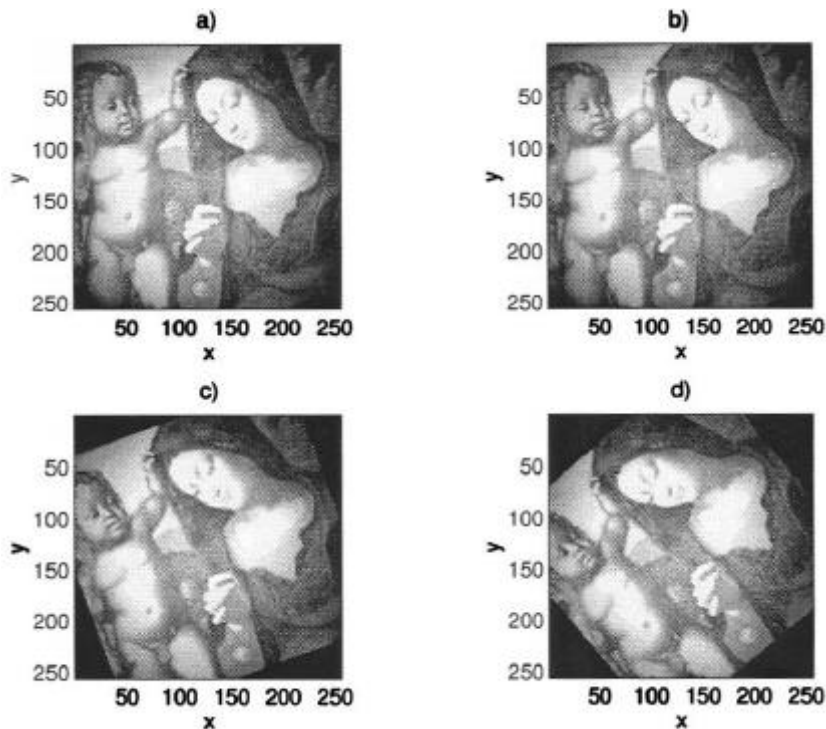


Fig. 1.a) Image  $f_1(x)$ ; b) , c), d) images  $f_2(x)$  , versions of  $f_1(x)$  rotated respectively by  $\varphi=1^\circ$  ,  $\varphi=20^\circ$  ,  $\varphi=40^\circ$ .

Equation (4) can be conveniently rewritten in polar coordinates

$$\begin{cases} x = \rho \cos \vartheta & \rho \geq 0 \\ y = \rho \sin \vartheta & 0 \leq \vartheta \leq 2\pi \end{cases} \tag{6}$$

as

$$\bar{f}_2(\rho, \vartheta) = \bar{f}_1(\rho, \vartheta - \varphi) \tag{7}$$

where

$$\bar{f}_i(\rho, \vartheta) = \bar{f}_i(\rho \cos \vartheta, \rho \sin \vartheta) \quad i=1,2 \tag{8}$$

Let  $F_i(\mathbf{k}) = F[f_i(\mathbf{x}) | \mathbf{k}]$  be the 2-D cartesian Fourier transform of  $f_i(\mathbf{x})$ ,  $i = 1, 2$ ; denote polar coordinates in frequency domain as

$$\begin{cases} k_x = k_\rho \cos k_\vartheta & k_\rho \geq 0 \\ k_y = k_\rho \sin k_\vartheta & 0 \leq k_\vartheta < 2\pi \end{cases} \tag{9}$$

and define the magnitudes of  $F_i(k)$  in this reference system as

$$M_i(k_\rho, k_\vartheta) = |F_i(k_\rho \cos k_\vartheta, k_\rho \sin k_\vartheta)| \tag{10}$$

It can be easily proved [8] that functions  $M_i(k_\rho, k_\theta)$  are related as images  $f_i(\mathbf{x})$ , i.e.,

$$M_2(k_\rho, k_\theta) = M_1(k_\rho, k_\theta - \varphi) \tag{11}$$

However there is a difference between relationships (7) and (11) concerning the periodicity of the functions: functions  $f_i(\rho, \theta)$  have  $2\pi$  as periodicity whereas functions  $M_i(k_\rho, k_\theta)$  have  $\pi$  as periodicity, owing to the hermitian symmetry of the Fourier transform.

Let  $m_i(k_\rho, \alpha)$ ,  $i = 1, 2$ , be two auxiliary functions, along a generic circumference of radius  $k_\rho$ , defined as

$$m_i(k_\rho, \alpha) = \int_{-\pi}^{\pi} M_i(k_\rho, k_\theta) e^{-j2\pi k_\theta \alpha} dk_\theta \tag{12}$$

Notice that functions  $m_i(k_\rho, \alpha)$  are the partial angular Fourier transforms of  $M_i(k_\rho, k_\theta)$ , i.e.  $m_i(k_\rho, \alpha) = F[M_i(\cdot, k_\theta) | \alpha]$ . From the translational theorem of the Fourier transform we obtain

$$m_2(k_\rho, \alpha) = m_1(k_\rho, \alpha) e^{-j2\pi\alpha\varphi} \tag{13}$$

In order to be independent from the specific circumference, one can build two further auxiliary functions

$$\mu_i(\alpha) = \int_0^R k_\rho m_i(k_\rho, \alpha) dk_\rho \quad i = 1, 2 \tag{14}$$

where  $R$  is a fixed radius.

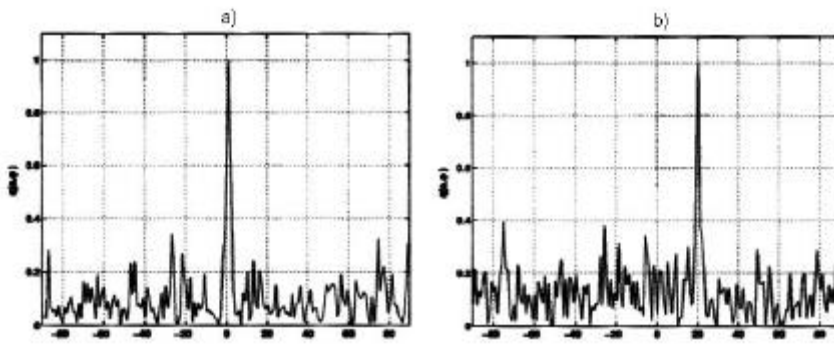


Fig. 2. Angular phase correlation  $q(a, \varphi)$  relative to functions pairs a)-b) and a)-c) of Fig.1. ;the true rotational angles and their estimates are respectively (from left to right):  $\varphi = 1^\circ$  and  $\varphi' = 1^\circ$ ,  $\varphi = 20^\circ$  and  $\varphi' = 20^\circ$ .

Functions  $\mu_i(\alpha)$  are reciprocally related similarly to functions  $m_i(k_\rho, \alpha)$  In fact

$$\mu_2(\alpha) = \int_0^R k_\rho m_2(k_\rho, \alpha) dk_\rho = \int_0^R k_\rho m_1(k_\rho, \alpha) e^{-j2\pi\alpha\varphi} dk_\rho = e^{-j2\pi\alpha\varphi} \int_0^R k_\rho m_1(k_\rho, \alpha) dk_\rho = \mu_1(\alpha) e^{-j2\pi\alpha\varphi} \tag{15}$$

Rotational angle  $\varphi$  (in the exponent of equation (15)) can be computed from normalized product

$$Q(\alpha) = \frac{\mu_1^*(\alpha)\mu_2(\alpha)}{|\mu_1(\alpha)\mu_2(\alpha)|} = e^{-j2\pi\alpha\varphi} \tag{16}$$

Indeed inverse Fourier transform of  $Q(a)$  gives

$$F^{-1}[Q(\alpha) | a] = \delta_r(a - \varphi) \doteq q(a, \varphi) \tag{17}$$

which, for reference convenience, is called *angular phase correlation*. The above mentioned hermitian symmetry of the Fourier transform makes possible estimating angle  $\varphi$  only within  $[-\pi/2, \pi/2]$ . A simple way to extend the estimate beyond this limit is to backrotate the second image by  $\varphi$  and  $\varphi + \pi$  and to subsequently use translational phase correlation in order to disambiguate the correct back rotation [7].

## 2 AN ANGULAR PHASE CORRELATION ALGORITHM

An algorithm for estimating rotational displacement  $\varphi$  can be structured as follows:

- i) compute magnitudes  $M_i(k_x, k_y)$ ,  $i = 1, 2$ , of the Fourier transforms in cartesian coordinates of the two images  $f_i(\mathbf{x})$ ;
- ii) interpolate magnitudes  $M_i(k_x, k_y)$  on a polar grid in order to obtain  $M_i(k_\rho, k_\theta)$ ;
- iii) compute  $m_i(k_\rho, \alpha)$  by means of equation (12);
- iv) evaluate functions  $\mu_i(\alpha)$  and normalized product  $Q(\alpha)$ ;
- v) compute *angular phase correlation*  $q(\alpha, \varphi)$ ;
- vi) estimate rotational angle  $\varphi$  as the location of the highest peak in  $q(\alpha, \varphi)$ .

In practice image noise and border effects will not lead to a perfectly impulsive function  $q(\alpha, \varphi)$ . However extensive experimentation has shown that *angular phase correlation* functions  $q(\alpha, \varphi)$  exhibit a distinctive peak in correspondence of the true rotational angle  $\varphi$ . Fig.2 shows functions  $q(\alpha, \varphi)$  relative to the images of Fig.1: the estimates are very accurate. This kind of performance is typical of the proposed algorithm.

## 3 THE 3D CASE

As for the estimate of planar rotations, the angular phase correlation technique can be profitably used also for the estimate of the 3D motion of free-form surfaces. This procedure is based on the Fourier transform of the 3-D intensity function, implicitly described by the registered time-sequences of range data. With respect to memory occupancy, the use of a time-sequence of a 3-D intensity function represents a considerable data reduction with respect to a pair of time-sequences of 2-D functions.

Let  $l_1(\mathbf{x})$ ,  $\mathbf{x} \in \mathbb{R}^3$ , be a 3-D object data and let  $l_2(\mathbf{x})$  be a rigidly rotated version of  $l_1(\mathbf{x})$  (these data can be obtained from registered range and intensity data captured at different times). An example of  $l_1(\mathbf{x})$  and  $l_2(\mathbf{x})$  is depicted in Fig. 3. It can be shown that  $l_1(\mathbf{x})$  and  $l_2(\mathbf{x})$  relate as:

$$l_2(\mathbf{x}) = l_1(R^{-1}\mathbf{x} - \mathbf{t}) \tag{18}$$

where  $R \in \text{SO}(3)$ ,  $\mathbf{t} \in \mathbb{R}^3$ , and  $\text{SO}(3)$  is the group of the 3x3 special orthogonal matrices. According to (18)  $l_2(\mathbf{x})$  is first translated by the vector  $\mathbf{t}$  and then rotated by the matrix  $R$ . Denote as

$$L_i(\mathbf{k}) = F[l_i(\mathbf{x}) | \mathbf{k}] = \iiint_{-\infty}^{+\infty} l_i(\mathbf{x}) e^{-j2\pi\mathbf{k}^T \mathbf{x}} d\mathbf{x} \tag{19}$$

with  $\mathbf{k} = [k_x, k_y, k_z]^T$  the 3-D Fourier transform of  $l_i(\mathbf{x})$ ,  $i = 1, 2$ ; it is straightforward to prove that, similarly to 2D case, the two transforms are related as

$$L_2(\mathbf{k}) = L_1(R^{-1}\mathbf{k}) e^{-j2\pi\mathbf{k}^T R\mathbf{t}} \tag{20}$$

From (20) one sees that the translation  $\mathbf{t}$  affects only phases and not magnitudes. Magnitudes are related as

$$|L_2(\mathbf{k})| = |L_1(R^{-1}\mathbf{k})| \tag{21}$$

and (21) can be used in order to determine  $R$ . Therefore in the frequency domain the estimation of  $R$  and  $\mathbf{t}$  can be decoupled and one can estimate first  $R$  from (21) and  $\mathbf{t}$  from (20), according to the following two passes procedure.

Fig. 3 shows an example of functions  $l_1(x)$  and  $l_2(x)$ , related to Apollo's head, located at Padua University Collection.

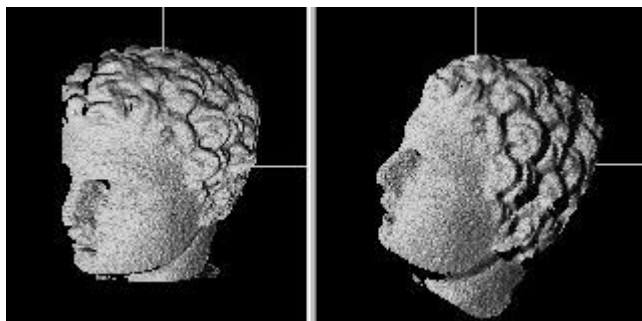


Fig. 3: Example of image data  $l_1(x)$  and its corresponding rotated version  $l_2(x)$  in 3D space.

#### 4 ESTIMATION OF THE 3-D ROTATIONAL MATRIX $R$

Write the 3-D rotational matrix  $R$  as follow [6]

$$R = R(\hat{\omega}, \theta) = e^{\hat{\omega}\theta} \tag{22}$$

where

$$\hat{\omega} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \in \text{so}(3) \tag{23}$$

$\omega = [\omega_x, \omega_y, \omega_z]^T \in \mathbb{R}^3$ , is a unit vector determining the rotational axis,  $\hat{\omega}$  is a skew-symmetric matrix obtained from the vector  $\omega$  and  $\theta \in \mathbb{R}$  is the rotational angle in radians. Define the difference function  $\Delta(\mathbf{k})$  between transforms as

$$\Delta(\mathbf{k}) = \Delta(\mathbf{k}_x, \mathbf{k}_y, \mathbf{k}_z) = \left| \frac{|L_1(\mathbf{k})|}{L_1(\mathbf{0})} - \frac{|L_2(\mathbf{k})|}{L_2(\mathbf{0})} \right| = \left| \frac{|L_1(\mathbf{k})|}{L_1(\mathbf{0})} - \frac{|L_1(R^{-1}\mathbf{k})|}{L_1(\mathbf{0})} \right| \tag{24}$$

It can be proved [6] that  $R$ , as rotational matrix, has eigenvalues  $\lambda_1 = 1$ ,  $\lambda_2 = e^{j\theta}$  and  $\lambda_3 = e^{-j\theta}$ . Call  $\omega$  the eigenvector corresponding to  $\lambda_1 = 1$ . The vector  $\omega$  is a solution of the equation  $\Delta(\mathbf{k}) = 0$ , indeed  $\Delta(\mathbf{k}) = 0$  if  $R^{-1}\mathbf{k} = \mathbf{k}$  or equivalent  $R\mathbf{k} = \mathbf{k}$ . In other words the locus  $\Delta(\mathbf{k}) = 0$  includes a line through  $\omega$ . For objects without special symmetries (as natural objects typically are) this property of the function  $\Delta(\mathbf{k})$  can be exploited in order to determine the vector  $\omega$  and then the angle  $\theta$  by means of the following procedure:

- 1) express  $\Delta(k_x, k_y, k_z)$  in spherical coordinates

$$\begin{cases} k_\rho = \sqrt{k_x^2 + k_y^2 + k_z^2} & k_\rho \geq 0 \\ k_\varphi = \arctan \frac{k_y}{k_x} & 0 \leq k_\varphi \leq 2\pi \\ k_\vartheta = \arccos \frac{k_z}{\sqrt{k_x^2 + k_y^2 + k_z^2}} & 0 \leq k_\vartheta \leq \pi \end{cases} \tag{25}$$

as  $\bar{\Delta}(k_\rho, k_\varphi, k_\vartheta)$ ; notice that this function can be represented only in a hemisphere because of the hermitian symmetry of the Fourier transform;

2) compute the radial projection of  $\bar{\Delta}(k_\rho, k_\varphi, k_\theta)$  as

$$P(K_\varphi, K_\theta) = \int_0^\infty \bar{\Delta}(K_\rho, K_\varphi, K_\theta) dK_\rho \tag{26}$$

3) the angular coordinates of  $\omega$  in spherical representation (see Fig. 4) can be found as

$$(\varphi, \theta) = \arg \min_{K_\varphi, K_\theta} P(K_\varphi, K_\theta) \tag{27}$$

since, from the inclusion of  $\omega$  within the locus  $\Delta(\mathbf{k}) = 0$ ,  $P(k_\varphi, k_\theta) \geq 0$  with  $P(\varphi, \theta) = 0$ ;

4) define  $\omega$  the versor of the direction  $(\varphi, \theta)$ .

The use of radial projection (26) simplifies the 3-D search for a line of the locus  $\Delta(\mathbf{k}) = 0$  into the minimization of a 2-D function which can be solved by standard numerical methods.

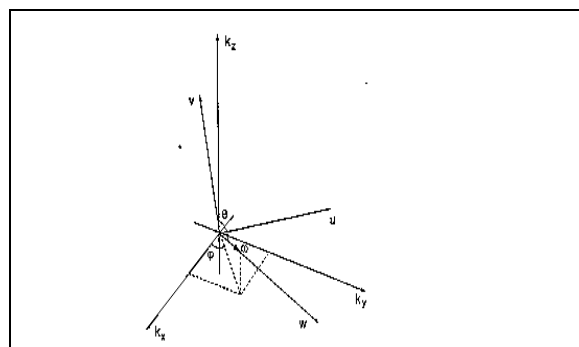


Fig. 4: Change of coordinate reference systems

#### 4 ESTIMATION OF THE ROTATIONAL ANGLE $\psi$

Once the rotational axis has been determined, the estimate of the rotational angle  $\psi$  can be conveniently approached in a cartesian coordinate system with an axis along  $\omega$ , as shown in Fig. 4. The determination of this coordinate system and the representation in this system of relationship (21) between magnitudes can be obtained as follows. Define the cartesian coordinate system

$$\mathbf{u} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = C^T \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} \tag{28}$$

with

$$C = \begin{bmatrix} -\sin\varphi & -\cos\theta \cos\varphi & \sin\theta \cos\varphi \\ \cos\varphi & -\cos\theta \sin\varphi & \sin\theta \sin\varphi \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \tag{29}$$

In the new cartesian reference system defined through (28) and (29), equation (21) becomes

$$|L_2(C\mathbf{u})| = |L_1(R^{-1}C\mathbf{u})| = |L_1(CC^{-1}R^{-1}C\mathbf{u})| \tag{30}$$

Define for convenience  $|\bar{L}_i(\mathbf{u})| = |L_i(C\mathbf{u})|$ ,  $i = 1, 2$ , from which (30) can be written as

$$|\bar{L}_2(\mathbf{u})| = |L_1(C^{-1}R^{-1}C\mathbf{u})| = |\bar{L}_1(R\omega^{-1}(\psi)\mathbf{u})| \tag{31}$$

where

$$R_\omega(\psi) = C^{-1}RC = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} r(\psi) & 0 \\ & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{32}$$

Matrix  $R_\omega(\psi)$  clearly shows the structure of a rotation by  $\psi$  around the  $\omega$  axis as

$$p_i(u, v) = \int_{-\infty}^{+\infty} |\bar{L}_i(u, v, w)| dw \quad i=1,2 \tag{33}$$

given the structure of  $R_\omega(\psi)$ , it is easy to prove that the axial projections defined in (33) relate as

$$p_2\left(\begin{bmatrix} u \\ v \end{bmatrix}\right) = p_1\left(r^{-1}(\psi)\begin{bmatrix} u \\ v \end{bmatrix}\right) \tag{34}$$

from which  $\psi$  can be obtained resolving a 2-D rotation estimate problem. From  $\omega$  and  $\theta$  the rotational matrix  $R(\omega, \theta)$  can be reconstructed by means of Rodrigues' formula as

$$R(\hat{\omega}, \psi) = e^{\hat{\omega}\psi} = I + \hat{\omega} \sin\psi + \hat{\omega}^2 (1 - \cos\psi) \tag{35}$$

## 5 ESTIMATION OF THE 3-D TRANSLATIONAL VECTOR $\mathbf{t}$

The translational vector  $\mathbf{t}$  can be estimated as follows:

a) De-rotate the image  $l_2(\mathbf{x})$  as

$$d_2(\mathbf{x}) = l_2(R\mathbf{k}) = l_1(\mathbf{x} - \mathbf{t}) \tag{36}$$

b) apply a 3-D cartesian phase correlation algorithm [9] on  $L_1(\mathbf{k})$  and  $D_2(\mathbf{k}) = F[d_2(\mathbf{x}) | \mathbf{k}] = L_1(\mathbf{k})e^{-j2\pi\mathbf{k}^T\mathbf{t}}$ , i.e. compute the normalized product between transform

$$Q(\mathbf{k}) = \frac{L_1^*(\mathbf{k})D_2(\mathbf{k})}{|L_1(\mathbf{k})D_2(\mathbf{k})|} = e^{-j2\pi\mathbf{k}^T\mathbf{t}} \tag{37}$$

c) evaluate its inverse Fourier transform

$$q(\mathbf{x}) = F^{-1}[Q(\mathbf{k}) | \mathbf{x}] = \delta(\mathbf{x} - \mathbf{t}) \tag{38}$$

The translational vector  $\mathbf{t}$  can be estimated from the peak of impulsive function  $\gamma(\mathbf{x})$ . The method can be efficiently implemented by using MD FFT algorithms [9]. The registration of 3-D views is a natural application of methods for estimating 3-D rigid rotations and translations.

## CONCLUSIONS

This work presents an extension to the rotational case of the well known translational phase correlation. Robustness and accuracy of the proposed algorithm have been tested and experimental evidence of these characteristics is reported. The proposed algorithm retains the good properties typical of the translational phase correlation. The only drawback of the 2D method is that it requires cartesian to polar coordinates conversion which is a numerically delicate and computationally intensive operation. This drawback can be bypassed by resorting to an alternative algorithm for estimating

rotations proposed in [8]. As regards the application of proposed algorithms to the estimate of 3D shapes motion, the available results show that the presented method is suitable to give unsupervised estimates within  $1^0$  degree precision of the angular parameters. It can be applied to 3-D views registration in tasks where this precision is adequate or it can be used in order to obtain effective starting points for standard feature-based methods, which as well-known, can give accurate solutions once they are properly initiated. The novelty of presented procedure relies upon the fact that it is a frequency domain approach; therefore it uses the global information of the data and not sets of features. This is probably one of the causes of its robustness.

## REFERENCES

- [1] Alliney, S., 1993. Digital Analysis of Rotated Images. IEEE Trans. Pattern. Anal. Machine Intell., Vol. 15, No 5, pp. 499-504.
- [2] Besl, P.J. and McKay, N. D. , 1992. A Method for Registration of 3-D Shapes. IEEE Trans. on PAMI, Vol. 14, No.2, pp. 239- 259,
- [3] De Castro, E., Morandi, C., 1987. Registration of Translated and Rotated Images Using Finite Fourier Transforms. IEEE Trans. on Pattern. Anal. Machine Intell.,Vol. PAMI-9, pp. 700-703.
- [4] Jain, Anil K., 1989. Fundamental of Digital Image Processing. Prentice Hall Information and System Sciences Series, Thomas Kailath, Series Editor,.
- [5] Kuglin, C.D. and Hines, D.C., 1975. The phase correlation image alignment method. Proc. IEEE Int. Conf. Qybern. Soc., , pp. 163-165.
- [6] Lucchese, L., Cortelazzo, G.M., Vettore, A., 1997. Estimating 3-D Rototranslations from Range Data by a Frequency Domain Technique. Optical 3-D Measurement Techniques IV, A. Gruen / H. Khamen, Ed. Wichmann, Zuerich, pp. 444-453 .
- [7] Lucchese, L., Cortelazzo, G. M., Monti, C., 1996. Estimation of Affine Transformations between Images via Fourier Transform. Proc. of ICIP '96, Lausanne, Switzerland, Vol. III, pp. 715-718.
- [8] Lucchese, L., Cortelazzo, G.M., Monti, C., 1996. A Frequency Domain Technique for Estimating Rigid Planar Rotations. Proc. of ISCAS '96, Atlanta, Georgia, Vol. 2, pp. 774-777.
- [9] Murray, R., Li, Z. and Sastry, S., 1994. Mathematical Introduction to Robotic Manipulation, CRC Press,
- [10] Weik, S., 1997. Registration of 3-D Partial Surface Models Using Luminance also Depth Information. Proc. of International Conference on Recent Advances in 3-D Digital Imaging and Modeling, Ottawa, Canada, pp. 93-100.