

## WEIGHTED GEOMETRIC OBJECT CONSTRAINTS INTEGRATED IN A LINE-PHOTOGRAMMETRIC BUNDLE ADJUSTMENT

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### ABSTRACT

The line-photogrammetric bundle adjustment is a new approach for the 3D reconstruction of polyhedral objects, with several advantages. Observing lines instead of points and integration of additional object information are two of them. Line features of an object are often well visible and better recognizable than corner points. Therefore, image lines belonging to object features are the main type of observations of the line-photogrammetric bundle adjustment. Furthermore, the use of line features makes the reconstruction of occluded object points possible. Due to the coplanarity constraint the mathematical model chosen here results in a valid polyhedral description of the object. Polyhedrality and shape regularity of an object can be expressed as relations between point, line and plane entities, i.e. as geometric object constraints.

In the paper, the approach for integration of weighted constraints into the line-photogrammetric model is presented. Experiments were conducted using the CIPA test data set. The results confirm the need to implement the constraints as weighted pseudo-observations. The paper reports on investigations regarding weighting of constraints. The approach allows reconstruction of a model on the condition that exterior orientation parameters are approximately known. Object constraints are also applied to support initial value computation. The major benefit of the approach is that it allows an accurate object reconstruction using only a few images. In some cases the reconstruction is not possible without the geometric constraints. The least squares adjustment allows a rigorous testing of the weighted constraints. Examples demonstrate the potential of constraint-based processing and the quality improvement of the reconstructed object model.

### 1 INTRODUCTION

Architecture has always been a field where the capabilities of photogrammetric methods are exploited. The line-photogrammetric approach discussed in this paper is a relatively new method still under investigation. The method is used for the reconstruction of objects, which have linear or curved features, or can be generalised as a set of geometric primitives, including boxes, tubes, slabs, etc. Measured lines in images, that are placed to fit to the image features, are the basic type of observation.

Many man-made objects often comply to the assumption above, particularly most architectural objects have been constructed consisting of planar faces and straight edges. We have continued the research published in (van den Heuvel, 1999) where a novel mathematical model designed primarily for the use in architectural applications is proposed. The concept of polyhedral generalisation belongs to the main properties of the model. In the same paper ideas regarding the use of constraints and a detailed list of possible constraints on object features can be found. The mathematical model has been prepared to accommodate constraints in the adjustment via a set of pseudo-observation equations. Considering constraints as observations avoids singularities due to possible linear dependencies, therefore it makes the choice of the constraints less critical. However, the constraints differ from observations in a classical sense. The solution to the differences between diverse types of observations is commonly based on a consistent use of weights. To introduce a rigorous adjustment of image line observations and weighted constraints is the primary goal of our research.

Shape generalisation that is implicitly included in the mathematical model determines the reconstruction method for purposes like architectural visualisation. With respect to this purpose, regularity of the object shape is to be considered as the most important demand. Our effort is targeted on extending the method to be able to provide models with a regular shape. Furthermore, low computational and processing costs are desirable. But a deformation of the object shape appears to be an obvious consequence of processing less images. Geometric object constraints integrated in the line-photogrammetric bundle adjustment are exploited here to support validation of the object shape. In our approach, we aim to obtain a model not as precise as possible, but with a sufficient completeness and regularity. Moreover, with as few images as possible. The approach redefines the meaning of the word "adjustment". A tool for processing observations is then changing to a tool for modelling a regular object supported by image observations, especially if the amount of non-image observations in the form of constraints increases.

A possibility of simple testing of constraints, offered by the adjustment, has been implemented. However, it is difficult to find a realistic precision for each constraint. A possibly weak geometry and the generalisation applied while extracting image lines may affect the truthfulness of constraints more than inaccuracies caused by the construction of the building. In our experiments an “educated guess” for the weights was made with which a good result was obtained in the sense of the goals proclaimed above.

In section 2 of the paper, basic principles of the mathematical model are introduced. Section 3 presents all implemented features regarding the integration of constraints into the model. Reasons for using constraints, and why to use them with weights, are discussed. At the end of the section, the way in which the mathematical model joins the image line observations and the non-measured information, is presented. The example of processing a test data set fills section 4.

## 2 PRINCIPLES OF LINE-PHOTOGRAMMETRY, USED MATHEMATICAL MODEL

### 2.1 Object parametrisation

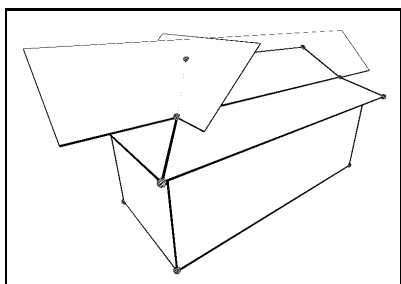


Figure 1: Object parametrisation by planes and points

As the basic type of parameters, 3D coordinates of object points are used to describe the model. The assumption, that the model is polyhedral, is an important part of the concept. It has several consequences. The model must involve a certain level of generalisation. Due to the polyhedrality, each set of points creating a face belongs to a common plane. In our approach the parameters of the planes are used as the second type of parameters. Figure 1 shows possible parameters for a model. In principle, an object could be defined only by planes, whose object points are intersections, and a topology description. For the presented models both point and plane parameters are used. It means a redundancy of parameters and it automatically leads to the need to formulate constraints that force the points to corresponding planes. Although the over-parametrisation solved by constraining could seem unwise, it has advantages. A relationship between an observation and the object point is easy to formulate. But the use of points as the only parameters would cause difficulties in handling plane entities. The use of object plane parameters has the advantage of a simple formulation of relationships and constraints on planes.

A third (optional) group of parameters are the parameters of exterior orientation. Four quaternion elements parametrise a rotation matrix  $\mathbf{R}_i$ , see the equation (2). Replacement of three unknown rotation angles by four quaternions is another example of over-parametrisation. The advantage is the absence of singularities.

### 2.2 Observations

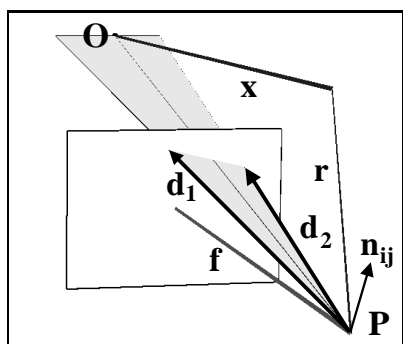


Figure 2: Interpretation plane

The formula

The observations are characterised by image coordinates of beginning and end point of each extracted line. Using the camera system, camera coordinates of such a point are expressed by the spatial direction vector

$$\mathbf{d} = (x, y, -f) \quad , \quad (1)$$

$x, y$  – image coordinates of the image point.

$f$  – focal length.

Deformation of the image and a lens distortion are supposed to be already corrected.

For later formulation of needed relationships, so called interpretation plane is introduced. The plane is generated by the projection center of the camera and by the extracted line. Figure 2 explains the genesis of an interpretation plane.

$$\mathbf{n}_{ij} = \mathbf{R}_i (\mathbf{d}_1 \times \mathbf{d}_2) \quad , \quad (2)$$

$\mathbf{d}_1, \mathbf{d}_2$  – vector (1) of begin and end point of the line.

$\mathbf{R}_i$  – rotation matrix of camera exterior orientation.

$(\times)$  – cross product of vectors.

expresses the interpretation plane, parametrised by its normal vector and rotated to the object system, where  $\mathbf{n}_{ij}$  denotes the normal vector of the interpretation plane, in the object system.

## 2.3 Relations

The possibility to compute coordinates of object points using images with known exterior orientation, is provided by the relationship, which is visualised at the figure 2. An object point (denoted  $O$ ) belongs to an edge, which has a corresponding observation (image line) in the image. Therefore, the object point also belongs to the interpretation plane, that is generated by the image line and the projection center, denoted  $P$ . Let's denote  $\mathbf{r}$  the position vector of the projection center  $P$ ,  $\mathbf{x}$  the position vector of object point  $O$ . Because both the point  $P$  and the point  $O$  belong to the interpretation plane, then the vector connecting  $P$  and  $O$

$$\overrightarrow{PO} = \mathbf{x} - \mathbf{r} \quad , \quad (3)$$

also belongs to the interpretation plane. Perpendicularity of  $\mathbf{n}_{ij}$  to each vector of the interpretation plane gives the inner product

$$\mathbf{n}_{ij} (\mathbf{x} - \mathbf{r}) = 0 \quad , \quad (4)$$

The equation (4) is the basic equation of the model, on which the least squares principle is applied.

## 3 CONSTRAINTS IN THE MODEL

### 3.1 Functionality of constraints

Several groups of constraints can be distinguished in the model. We concentrated on the group of geometrical object constraints, but also other constraints were investigated.

Due to the formulation of the mathematical model, the set of constraints is always non-empty. Each plane leads to a constraint on the length of its normal vector. The exterior orientation parameters cause the presence of a similar constraint on the four quaternion elements. This constraining is a price for the advantages of using redundant parameters.

The control point coordinates are also introduced as weighted constraints. A minimum of seven coordinate constraints is required to define the coordinate system. The co-planarity constraint has a particular significance in the model. It can be classified as the special case of the geometrical object constraint on the distance between a point and a plane. But its main function is to ensure that the faces of the model are (almost) planar. The constraint is set for each point at least once. If the point is in more than one plane a constraint is set for each plane the point belongs to.

All the constraints above are always present in the mathematical model, in contrast to the geometrical object constraints. They are added to the system after manual specification. The main purpose of these constraints can be described as "quality improvement". There are several important factors, which influence the quality of the result. The first factor is the quality of the approximate exterior orientation parameters. Thanks to the integration of geometrical object constraints, convergence of the adjustment is obtained, even for approximate exterior orientations of low quality or bad intersection angles. Quality and quantity of the acquired data is the next factor. The number of images can be limited, due to either restrictions for image acquisition, or due to the need to economise processing. Obstacles and poorly visible features in images also have a quality decreasing effect. All these factors result in a lack of the desired properties of the model, with respect to our proclaimed goal.

We know that architectural objects are constructed with regular, or even perpendicular features, some angles or distances may be known. When the photogrammetric network is too weak to avoid bad intersection angles, the non-measurement information is a welcome new source of information. This information is expressed in the form of geometric object constraints.

### 3.2 Weighting of constraints

The model integrates the image observations, with pseudo-observation equations for the constraints. In section 3.5 it is explained how this integration is done. The importance of a consistent weighting of different types of observations is well known in the adjustment theory.

In principle, the weighting of the image observations is straight forward (part (19) of the model). A propagation derived from (4) is computed for the observations. Then, depending on the stochastic model applied, shorter observed lines are given lower weights, than longer lines. Those weights are absolute in the sense that they are derived from an assumption on the measurement precision in the image.

In order to obtain a well balanced model, we have tried to use realistic standard deviations for computing weights of constraint equations. The constraints should be characterised by standard deviations too. In this application, the constraints cannot be interpreted like geometrical axioms with infinite weights. For an example of the effects of omitting constraints, we point to section 4.1. Experiments showed that hard weighting of constraints can cause the adjustment to

be non-convergent. The loss of convergency is the main proof, that the introduction of weighting for constraints is not only more exact, but sometimes even necessary!

In following sections, standard deviations of the constraints are discussed. In all cases, the constraints have been considered as non-correlated and the formula

$$p_C = 1/m_C^2 \quad , \quad (5)$$

was used for computing weights  $p_C$  from standard deviations  $m_C$  (here of a general constraint) in the adjustment.

### 3.3 Formulation of implemented constraints and their standard deviations

**3.3.1 Constraints weighted as hard constraints** Two constraints creating this group have very simple weighting due to their character. Namely the quaternion constraint on quaternions  $q_1 - q_4$ , defining a rotation matrix

$$\sqrt{q_1^2 + q_2^2 + q_3^2 + q_4^2} - 1 = 0 \quad , \quad (6)$$

and a similar constraint on the length of the normal vector  $\mathbf{n} = (n_x, n_y, n_z)$  of an object plane

$$\sqrt{n_x^2 + n_y^2 + n_z^2} - 1 = 0 \quad , \quad (7)$$

are the exceptions, because only these two constraints are defined as axioms. These constraints are given very small, but non-zero standard deviations. These constraints do not affect the geometry of the model at all.

**3.3.2 Co-planarity constraints** The formulation of co-planarity of a point using parameters allowed by the model is

$$(\mathbf{x}_i, \mathbf{n}_j) - l_{n_j} = 0 \quad , \quad (8)$$

$l_{n_j}$  – distance between the plane and the origin of a chosen coordinate system.

$(, )$  – notation for the inner (dot) product.

For its weighting,  $m_{COP}^2$  is used, entered as a system parameter. It expresses two effects. One of them is the position uncertainty of the point with respect to the plane due to a face that is not perfectly planar. An error caused by the generalisation is the other effect, and probably a more significant one.

Therefore, the weight of this constraint expresses and prevents an influence of a possible irregularity, which is either difficult to describe or even to estimate. If we want to use a polyhedral generalisation, then decreasing the weight of the co-planarity equations results in a violation against this principle. Therefore, weighting of this constraint may remain a subject of further investigation.

**3.3.3 Constraints for control points** In contrast with the previous constraint, the precision of control points has a clear meaning. Generally, a triple of constrained point coordinates has a (3x3) full co-variance matrix. The constraint on one particular coordinate (here for a coordinate  $x$ ) is realised as an equation of the type

$$x_i = const \quad . \quad (9)$$

A combination of at least seven control coordinates is required to avoid the singularity of the system (22). Theoretically, for a full co-variance matrix of the coordinates one obtains the full weight matrix, which is its inverse. We have implemented a common standard deviation  $m_{CP}$  characterizing a complete file of control points. In practice, co-variance information about control points is hardly ever available.

**3.3.4 Parallelogram constraint** This special type of constraint was discussed in (van den Heuvel, 1997), where it was applied in image space. Here, it is applied to the co-ordinates of the corner points of a parallelogram in object space. The assumption results to a set of three equations

$$\begin{aligned} x: & -x_1 + x_2 + x_3 - x_4 = 0 \\ y,z: & \text{similarly} \end{aligned} \quad . \quad (10)$$

The set has the advantage of linearity and can also be useful for a simple computation of approximate values of coordinates. Assuming non-correlated  $x_1 - x_4$ , for simplicity with the same variance  $m_x^2$  for each coordinate, then computing the propagation gives the variance of the constraint

$$m_{PLL}^2 = 4 m_x^2 \quad . \quad (11)$$

In practice, it is a difficult task to find a good estimation for  $m_x$ . We use the value of standard deviation of the co-planarity constraint  $m_{COP}$  for  $m_x$ .

**3.3.5 Constraint on the angle between two planes** inserts an assumption on the angle between two planes into the system (20). It benefits from the parametrisation by planes. Thanks to the unit length of the normal vectors  $\mathbf{n}_i, \mathbf{n}_j$  of the planes, the constraint assuming  $\alpha$  as the angle between them, has the linear form

$$(\mathbf{n}_i, \mathbf{n}_j) - \cos \alpha = 0 \quad , \quad (12)$$

For a given standard deviation  $m_\alpha$  of the assumed angle, it is necessary to compute a propagation on the variance of  $\cos$  of the angle  $\alpha$

$$m_{\cos \alpha}^2 = \sin^2 \alpha m_\alpha^2 \quad , \quad (13)$$

**3.3.6 Constraint on the distance between two parallel planes** is restricted in its application on such planes, on which the plane-plane parallelism constraint has been applied simultaneously. Again we benefit from over-parametrisation; the constraint has the simple form

$$|l_{\mathbf{n}_i} - l_{\mathbf{n}_j}| - d_{ij} = 0 \quad , \quad (14)$$

$d_{ij}$  – assumed distance between the planes.

$l_{\mathbf{n}_i}, l_{\mathbf{n}_j}$  – distances between the planes and the origin of chosen coordinate system.

The signs of  $l_{\mathbf{n}_i}, l_{\mathbf{n}_j}$  have to respect corresponding or the opposite direction of the vectors. The constraint allows the use of a direct measurement of the object size. Standard deviation  $m_d$  of  $d_{ij}$  reflects the precision of the distance, and thus it is to be assessed by the user.

**3.4 Constraint on parallelism of two planes**

The equation (12) cannot be used for a value  $\alpha \rightarrow 0$ , when the planes turn to be parallel, because the adjustment will not converge. Despite the problem, the parallelism constraint is very useful for validating a regular object shape, as experiments show. Therefore, we have developed a new original solution to the plane-plane parallelism constraint, using a singularity free formulation. The angle constraint fails due to the need to fix two degrees of freedom in this case. We formulate two constraints using two vectors derived from  $\mathbf{n}_i$ , and here denoted  $\mathbf{n}_{i_1}^\perp, \mathbf{n}_{i_2}^\perp$ . For  $\mathbf{n}_i$  and the new pair of vectors following relationship is valid:

$$\mathbf{n}_i = \frac{\mathbf{n}_{i_1}^\perp \times \mathbf{n}_{i_2}^\perp}{\|\mathbf{n}_{i_1}^\perp \times \mathbf{n}_{i_2}^\perp\|} \quad , \quad (15)$$

The pair of constraints for plane parallelism is then

$$\begin{aligned} (\mathbf{n}_{i_1}^\perp, \mathbf{n}_j) &= 0 \\ (\mathbf{n}_{i_2}^\perp, \mathbf{n}_j) &= 0 \end{aligned} \quad . \quad (16)$$

For  $\mathbf{n}_{ij} = (n_x, n_y, n_z)$ , a proper pair is chosen from the triplet

$$(n_z, 0, -n_x) \quad , \quad (0, n_z, -n_y) \quad , \quad (n_y, -n_x, 0) \quad , \quad (17)$$

with the aim to have the angle  $\varphi$  between the two vectors as closer as possible to  $90^\circ$ .

How to set weights of these newly introduced constraints? For a given standard deviation of  $0^\circ$  angle between  $\mathbf{n}_i, \mathbf{n}_j$ , we have to compute standard deviations for  $90^\circ$  angles between decomposed vectors  $\mathbf{n}_{i_1}^\perp, \mathbf{n}_{i_2}^\perp$  and  $\mathbf{n}_j$ . The situation is shown in figure 3, the parameters are explained bellow. Applying rules of sphere trigonometry, we obtained an exact formula for the relation between  $\varepsilon, \varepsilon_1, \varepsilon_2, \varphi$ . We have made the assumption  $m_{\varepsilon_1} = m_{\varepsilon_2} = m_{\varepsilon^\perp}$  and their correlation as the most unfavourable. Then we estimate the variances  $m_{\varepsilon_1}^2, m_{\varepsilon_2}^2$  of the  $90^\circ$  angles with

$$m_{\varepsilon^\perp}^2 = \frac{\sin^2 \varphi}{2(1 - \cos \varphi)} m_\varepsilon^2 \quad . \quad (18)$$

$\varphi$  – angle between  $\mathbf{n}_{i_1}^\perp$  and  $\mathbf{n}_{i_2}^\perp$ .

$\varepsilon$  – real error in the  $0^\circ$  angle between  $\mathbf{n}_i, \mathbf{n}_j$ .

$\varepsilon_1, \varepsilon_2$  – real errors of  $90^\circ$  angles between  $\mathbf{n}_{i_1}^\perp, \mathbf{n}_j$  and  $\mathbf{n}_{i_2}^\perp, \mathbf{n}_j$  respectively.

$m_{\varepsilon_1}^2, m_{\varepsilon_2}^2$  – variances – mean value  $E(\varepsilon_1 \varepsilon_1), E(\varepsilon_2 \varepsilon_2)$  respectively.

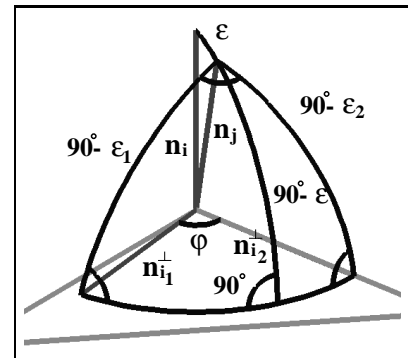


Figure 3: Errors in the relation of parallel planes

It has to be noted, that for weighting of equations (16),  $m_{\epsilon_{\perp}}^2$  has to be multiplied by the length of  $\mathbf{n}_{i_1}^{\perp}$ ,  $\mathbf{n}_{i_2}^{\perp}$  respectively. This is caused by the non-unit length of decomposition vectors (17).

### 3.5 Integration of constraints into the model

The mathematical model integrates constraints with the observation equations. The basic equation for observations is (4). With (2) we see that it contains both parameters and observations. In more general point of view, (4) can be rewritten into a formal equation, here in a form with adjusted values

$$\varphi_1(\bar{\mathbf{I}}, \bar{\mathbf{x}}) = 0 \quad , \quad (19)$$

where  $\varphi_1(, )$  denotes a set of functions – conditions. Arguments of the functions are the vector of adjusted parameters  $\bar{\mathbf{x}}$  and the vector of adjusted observations  $\bar{\mathbf{I}}$ . So-called least squares *adjustment of condition equations with observations and parameters* is the result of applying the least squares principle on the equation (19). Similarly, the formal equation for the set of constraints is expressed by

$$\varphi_2(\bar{\mathbf{x}}) = 0 \quad , \quad (20)$$

with the same notation as for (19). This set can be interpreted as pseudo-observation equations. With this formulation constraints can be given weights by means of a submatrix of the weight matrix of the observations. The final step of the task is to find a least squares solution respecting (19), (20). Linearisation of the equations (19), (20) gives

$$\begin{aligned} \mathbf{B} \cdot \mathbf{v} + \mathbf{A} \cdot d\mathbf{x} + \mathbf{I}'_A &= 0 \\ \mathbf{C}^T \cdot d\mathbf{x} + \mathbf{I}'_C &= 0 \end{aligned} \quad , \quad (21)$$

$\mathbf{v}$  – vector of corrections to observations.

$d\mathbf{x}$  – vector of corrections to parameters.

$\mathbf{A}, \mathbf{B}$  – matrices of partial derivatives of (19).

$\mathbf{C}^T$  – matrix of partial derivatives of (20).

$\mathbf{I}'_A, \mathbf{I}'_C$  – vectors of misclosures.

The system (21) is solved in two steps. In the first step, the vector  $d\mathbf{x}$  is computed from the system

$$\mathbf{z} = \begin{bmatrix} \mathbf{z}_A \\ \mathbf{z}_C \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{C}^T \end{bmatrix} \cdot d\mathbf{x} + \begin{bmatrix} \mathbf{I}'_A \\ \mathbf{I}'_C \end{bmatrix} \quad , \quad (22)$$

on condition  $\mathbf{z}^T \mathbf{Q}_z^{-1} \mathbf{z} = \min$ , where  $\mathbf{Q}_z$  is the co-factor matrix of the system. Then the left part

$$\mathbf{B} \cdot \mathbf{v} = \mathbf{z}_A \quad , \quad (23)$$

is solved with  $\mathbf{v}$  as unknown, with a similar condition  $\mathbf{v}^T \mathbf{Q}_l^{-1} \mathbf{v} = \min$  for the co-factor matrix  $\mathbf{Q}_l$  of observations. The computation has to be done iteratively due to non-linearities of the model. For more details, see (van den Heuvel, 1999) or (Mikhail, 1976).

## 4 EXAMPLE

Two reconstructions have been selected from a number of experiments. The results of other experiments are briefly discussed in the conclusions. Presented examples illustrate the contrast between the result obtained almost without constraints and the constraint-based reconstruction of the same model. Both examples use a stochastic model, which gives a shorter line relatively lower weight. The precision is derived from the standard deviation of the coordinates of the end points of the lines, estimated as the size of one pixel. For the used images, this size is approximately  $0.0065 \times 0.0065$  mm. The stochastic model is described in (van den Heuvel, 1998). The CIPA test data set of Zürich City Hall was used, particularly four images taken with an OLYMPUS C1400L camera (8.2 millimeters focal length) were selected. Figure 4 shows the used images and gives an impression of the data. For more details regarding the data set, consult CIPA web site (CIPA, 1999).

|          | # of image lines/params. | # of parallelogram const./others | st. dev. of result coords. mean/max. | st. dev. of cons. angle+parall. | common settings    |
|----------|--------------------------|----------------------------------|--------------------------------------|---------------------------------|--------------------|
| Model #1 | 112/154                  | 0/0                              | 0.17 m/0.47 m                        | –                               | $m_{COP} = 0.05$ m |
| Model #2 | 393/426                  | 99(33)/41(27)                    | 0.14 m/0.25 m                        | 25'/50'/80'/150'                | $m_{CP} = 0.10$ m  |

Table 1: Statistics of the models

In table 1 the basic statistics of the models can be found. The summary contains the real number of constraints. In brackets there is the number of constraints specified by user, the difference is caused by the fact, that some constraints, however considered as one entity, are rewritten as a set of several constraints (3 for the parallelograms, 2 for the plane parallelism). Standard deviations used for weighting of the angle and the parallelism constraints are given in minutes (denoted  $'$ ), where  $60' = 1^\circ$ .



Figure 4: Zürich data set – used images

#### 4.1 Models

##### Model #1

Figure 5 presents the result of the adjustment of the geometrically weak data set, resulting in an irregular model. There were no constraints employed, apart from coplanarity constraints that force the points in their planes. The result obviously lacks regularity in the shape. Besides other facts presented in table 1, let us emphasise that *only 4 images* with significant occlusions of upper parts of the building were used. The dormer windows, that distinguish this model from the second one, could not be reconstructed at all. The occlusions cause a lack of observations resulting in singularity of the system of normal equations.

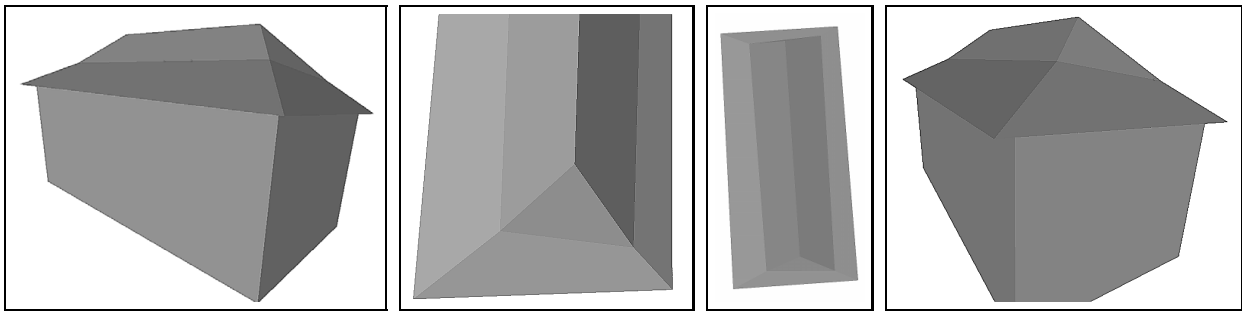


Figure 5: Model #1 – no constraints

##### Model #2

Figure 6 shows the result of computation with the same data set as used for the previous model, but with a strong support of geometrical object constraints. A summary of them is included in table 1. The mean standardised residual of the constraints is equal .27, convergency was reached within 11 iterations. A regularly shaped object model was obtained. The constraint-based approach enables to obtain a complete model, despite occlusions and despite image measurements with poor redundancy.

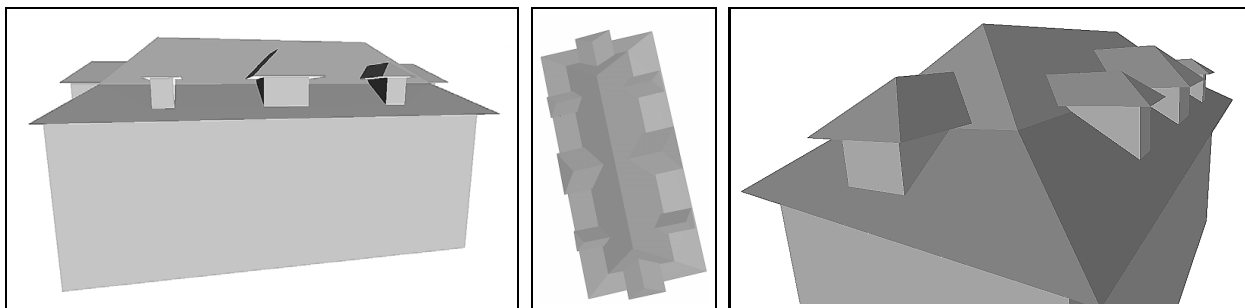


Figure 6: Model #2 – constraint support

#### 4.2 Advantages of the use of weighted constraints

**4.2.1 Enhanced solution to occlusions** The possibility to reconstruct occluded parts of an object using only line measurements has been described in (van den Heuvel, 1999). It presents also an analysis of the minimum image line

configuration for the determination of an object point. Now, with sufficient constraints and well organised computing of approximate coordinates, we can theoretically involve a point, which has no image line measurements related. However, the extreme case when a point is determined by constraints only, we have not encountered yet. A more typical case occurred for *Model #2*, where only vertical lines were visible for lower features of the dorms. Approximate coordinates were computed with the support of linear parallelogram constraints. A valid solution was reached by forcing the points to the planes of the roof by means of the co-planarity constraints.

**4.2.2 Enhanced computation of approximate values of coordinates** Regression model (21) needs initial values for all parameters in the model. Our approach uses exterior orientation parameters approximately known, determined in a separate step. Then the object point coordinates and the object plane parameters remain to be determined. For the points related to a sufficient number of diversely oriented line observations, approximate values of object points are computed by the intersection of interpretation planes. In following steps of the iterative process, points are intersected also using preliminary determined object planes.

Besides the announced possibility of modelling with the use of a priori geometric information, this information can also be used for computation of initial values. In the presented models, the initial values of exterior orientation were separately computed outside the adjustment software. Computation of approximate coordinates of weak points was supported with the use of parallelogram constraints, where this constraint was relevant. Although not yet implemented, other constraints can also support the computation of initial values of object points for which less than three interpretation planes are available.

## 5 CONCLUSION

The integration of weighted constraints in a line-photogrammetric bundle adjustment is presented. A new solution to the weighted parallelism constraint on planes has been developed. Based on the presented approach, we developed a tool integrating adjustment and constraint-based modelling. With this approach we are able to reconstruct models using a geometrically weak set of image line observations, with exterior orientation parameters only approximately known. Despite the weakness and occlusions, a regular shape of the model can be obtained. Furthermore, the number of images needed is minimised.

Regarding our experiments, the used weights provide the convergence and the desired regularity. However, low values of residuals of constraint observations (so-called pseudo-observations) indicate that an optimal balance of weights of image observations and the pseudo-observations is still unclear and needs further investigation. Automation of some steps of processing is one of the possible future improvements. Semi-automatic creation of topology, based on automatically extracted image lines is an example of such an improvement. An efficient tool was developed for the reconstruction of valid, regularly shaped models suitable for an architectural visualisation. An integration of all steps of processing into a common environment would make the tool even more efficient.

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