

## CONSTRAINED LINE-PHOTOGRAMMETRIC 3D RECONSTRUCTION FROM STEREOPAIRS

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### ABSTRACT

Objects and scenes encountered in close-range photogrammetry, vision metrology and virtual reality applications may often be adequately modeled as specific combinations of straight lines and/or line-defined planes. The descriptive power of lines for 3D object reconstruction and their suitability for automatic extraction justify the strong interest in line-based approaches introducing straight linear features as primitives in fundamental photogrammetric tasks. Notwithstanding the suitability of single image line-based techniques for several calibration/orientation and reconstruction problems, stereopairs provide a higher potential. Here, the question of relative orientation of the stereopair based on straight lines is addressed as a geometrically powerful tool also allowing partial 3D reconstruction. Investigation of the problem showed that solution minimally requires a constraint of four coplanar lines. Besides this "structural" constraint, however, additional constraints are expected to further enhance the ground truth of 3D reconstruction. In this contribution, results from "ordinary" stereopairs are presented with combinations of constraints among planes. It is clearly shown that the overall shape reconstruction is significantly improved, in particular as regards unconstrained relative directions of 3D lines or object symmetries defined by line intersections. Further possible constraints are also discussed. Such approaches allow to fully exploit the potential of the stereopair in line-photogrammetry instead of stereotriplets usually required in this context.

### 1 INTRODUCTION

As a rule, virtual reality and vision metrology applications – but also most close-range photogrammetric tasks – are situated within the man-made environment. Hence, objects and scenes may often be advantageously represented as a specific combination of straight lines and/or planes bounded by lines. The suitability of lines for geometric modeling and CAD-based object reconstruction as well as their usefulness in higher level tasks such as object recognition are often mentioned in this framework (Li and Zhou, 1994; Habib, 1999; van den Heuvel, 1999). For instance, Zhou (1998) points out that space robot vision can benefit from using straight lines rather than points as feature landmarks in calibration/orientation tasks. Besides a "descriptive potential" regarding both geometric and semantic information, advantages of lines include their suitability for automatic extraction and expected higher accuracy compared to isolated point detection. In this view, the course of digital photogrammetry towards automation has aroused a sincere interest in line-based approaches, in which linear features are introduced as the primitives in fundamental photogrammetric tasks. Indeed, algorithms for basic orientation and intersection procedures have been reported and tested, while straight lines have been also used in the context of aerial triangulation (Petsa and Patias, 1995; Barakat et al., 1996; Habib, 1999).

A man-made environment clearly implies that, in general, a priori knowledge about features may be reliably assumed and exploited. Thus, the introduction of geometric constraints among features may play an important role in line photogrammetry. An obvious case is single-image calibration/orientation and reconstruction tasks in which line orthogonality constraints but also constraints among planes are used (Bräuer-Burchardt and Voss, 1999; Karras and Petsa, 1999; Liebowitz et al., 1999). In this case the constraints are of a "structural" nature in the sense that they are indispensable for the solution. Such monocular techniques relying on straight lines can generally meet relatively low accuracy requirements (notably in VR applications); furthermore, reconstruction is here limited to instances where object lines and planes may be conveniently related to each other. In cases more demanding, both in terms of accuracy and object geometry, stereo or multi-image approaches should be adopted.

In this context, control lines (or points) are necessary for full absolute orientation. On the other hand, “partial” absolute information for space lines – known or assumed vertical and horizontal directions – allows “leveling” of the model. Model scaling obviously requires known distances. But for relative orientation, i.e. shape reconstruction, no constraints at all are required if at least *three* images are used. The question of line-based relative orientation of image triplets has been extensively treated and investigated by Petsa and Patias (1994). Indeed, the equivalent of the five (von Gruber) points in a stereopair are six lines in an image triplet allowing the recovery of the eleven parameters of relative orientation with one redundancy. Relative rotations  $R$  and translations  $t$  may be solved for separately: for rotations it is required that homologous projection planes intersect in parallel lines; that lines have a common point constrains the linear parameters. Of course, orientation and line intersection in the model system can well be performed simultaneously; here, object constraints can be imposed on lines and/or planes to enhance reconstruction (van den Heuvel, 1999). Line-based relative orientation had been initially posed by Finsterwalder (1937) and was to re-emerge later in computer vision in the context of the “motion and structure from motion” problem (e.g. Weng et al., 1993).

It is known that simply homologue lines do not allow relative orientation of the stereopair since – unlike model points constrained as intersections of homologue rays – corresponding projection planes always do intersect. This requirement for three overlapping images for shape reconstruction represents a drawback of line photogrammetry as photogrammetric practice is still expected to make extensive use of the basic stereopair configuration. Petsa and Patias (1995) have outlined how relative orientation of the stereopair is indeed possible if a minimum of constraints are forced upon lines; to these, further constraints may be added to improve object shape recovery by “tying” model features together according to a priori information. Main aspects of constrained line-photogrammetric 3D reconstruction from stereopairs are treated in the next sections.

## 2 MATHEMATICAL MODEL

### 2.1 Minimal constraints

As mentioned above, relative orientation of stereopairs is made feasible via simple a priori knowledge constraining the involved object lines. Inclusion of unconstrained lines in the adjustment is trivial, since they add nothing to the solution. If required, however, such lines could also be included to be reconstructed in a one-step process. Here, only constrained lines have been used in the solutions.

Regarding the type of constraints, Barakat et al. (1996) categorize information on lines in “relative” (parallel, perpendicular, coplanar) and “partial absolute” (vertical, horizontal). For relative orientation to retain its independence of external control, mere “relative” constraints come in question. However, in the present case one needs to make a further distinction within the former category between “relative directional” (parallel, perpendicular) and “relative positional” (mainly coplanar). The reason is that known angular relations among space lines are inadequate for solution since directional constraints cannot fix the translational degrees of freedom, i.e. allow recovery of the base components (Petsa and Patias, 1995). Indeed, a shift of the cameras does not affect the directions of intersected model lines. Among “relative positional” constraints of lines, their coplanarity (in one or several planes) is most important, a condition commonly met in urban scenes with objects presenting a polyhedral structure.

### 2.2 Basic formulation

Given a stereopair, let  $R$  denote the rotation matrix and  $t$  the translation vector (in an arbitrary model scale) of relative orientation with respect to the first image. In this system, model line  $L$  is described here through a reference point  $X_L$  and a direction vector  $d = [L \ M \ N]^t$  upon which two conditions – one on  $X_L$  and one on  $d$  – have to be imposed to account for the four independent parameters of a 3D line. Line  $L$  is formed as the intersection of the projection planes  $e_{1,2}$  of two homologue image lines  $l_{1,2}$  as in Fig. 1. Furthermore, model line  $L$  also needs to be constrained on a model plane  $s$  expressed as  $AX + BY + CZ + D = 0$  in the system of the first image. Let  $n_s = [A \ B \ C]^t$  be the normal vector of  $s$  and  $n_{1,2}$  denote, in the respective image systems, the normal vectors of  $e_{1,2}$  (expressed through the interior orientation and the image lines parameters). Thus,

$$n_s^t d = n_s^t (n_1 \times R^t n_2) = 0 \tag{1}$$

In the current implementation, reference point  $X_L$  is automatically chosen among the intersections of line  $L$  with the three planes  $XY$ ,  $YZ$ ,  $ZX$  of the coordinate system, according to the magnitude of the angles of intersection.

If  $n_o$  denotes the orthonormal vector of the model coordinate system plane on which  $X_L$  is defined:

$$n_o^t X_L = 0 \tag{2}$$

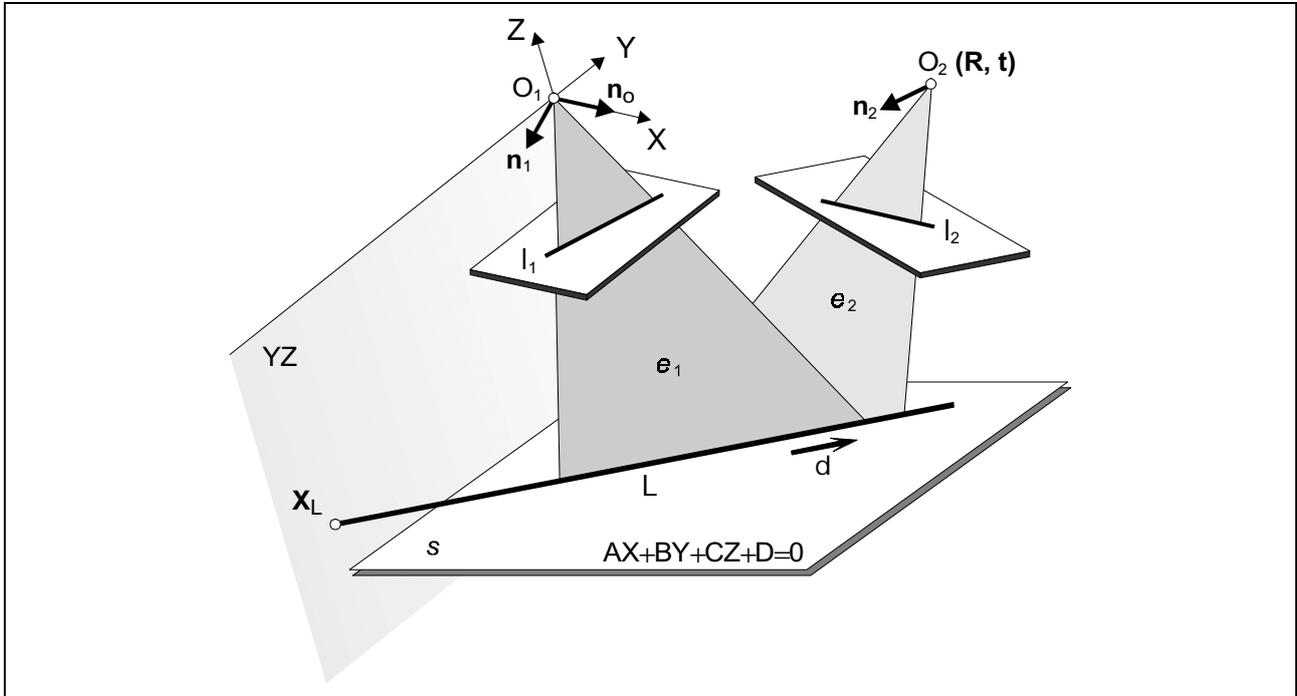


Figure 1. Basic geometry of constrained relative orientation of the stereopair.

Since  $X_L$  is a common point of planes  $e_1, e_2$  satisfying Eq. (2) but also lies on model plane  $s$ , then:

$$X_L = \frac{n_2^t R t (n_o \times n_1)}{n_o^t (n_1 \times R^t n_2)} \quad \text{and} \quad n_s^t X_L + D = 0 \tag{3}$$

from which the following equation is obtained:

$$n_2^t R t n_o^t (n_1 \times n_s) - D n_o^t (n_1 \times R^t n_2) = 0 \tag{4}$$

Obviously, one of the model plane parameters has to be arbitrarily fixed. Indeed, parameter  $D$  can be conveniently fixed in model space as  $D \neq 0$  since no perspective centre belongs to an imaged plane. Thus, two equations are available for each line – Eqs. (1) and (4) – allowing to solve for the five relative orientation elements of the stereopair. In this adjustment the 2D line parameters are treated as weighted observations. The question regarding initial parameter values presents a certain difficulty. Of course, good values for the rotational parameters could be extracted from the vanishing points of lines in two perpendicular directions (Karras and Petsa, 1999); for the base components, however, a different approach has to be worked out. Given initial values for orientation, approximate values for the plane coefficients are computed in the process.

A crucial advantageous aspect of the approach is that model planes are simultaneously estimated in the process and, beyond being auxiliary entities simply constraining orientation, in fact represent basic elements of 3D object reconstruction. The model line parameters are also estimated by intersection within the process.

### 2.3 Investigation

If  $N$  observed lines are constrained on  $M$  planes, solution is possible when  $2N \geq (5 + 3M)$ . For  $M = 1$  (all lines

coplanar),  $N \geq 4$  lines are required. For  $M = 2$ ,  $N \geq 6$  lines are necessary (one redundancy). Investigations on critical geometries have indicated that in the minimal case  $N = 4$ ,  $M = 1$  solution is not possible when: a) one of the lines is coplanar with the base of the stereopair; b) three lines have a common point of intersection, including intersection at infinity (i.e. when three lines are parallel). Generally, for  $M = 1$  at most  $N-2$  lines may be parallel or have a common point (it is irrelevant whether the remaining two lines are parallel or not). Besides, lines running nearly-parallel to the stereopair base are generally to be avoided.

## 2.4 Additional object constraints

Under the above conditions, a minimum of four image line homologues defining one model plane allows to recover relative orientation of the pair with no redundancy. Redundant configurations (more lines on more planes) strengthen the overall performance of the adjustment. However, this algorithm converges to a solution basically (but not exclusively) determined by the precision of image lines. If shape reconstruction is indeed considered as important in this context, the next step is to incorporate further constraints derived from the polyhedral form of the objects. These – provided they conform to object truth – allow more “coherent” reconstruction which reproduces basic, a priori available, information on shape. Constraints may include assumed relative information connecting planes, lines and planes with lines (in this context of relative orientation no “absolute” information is considered). In a next step, wire-frame representations may be materialised through the intersections of reconstructed features.

At the current stage of implementation, only the angular conditions of perpendicularity and parallelity (treated as “strict” constraints) have been imposed on model planes. Their effect on the relative positions of the unconstrained lines and planes was then practically evaluated.

## 3 PRACTICAL EXAMPLES

### 3.1 Input data

It was chosen to evaluate the approach with “ordinary” stereopairs, i.e. images not taken for this specific purpose. Of the three image pairs  $P_1$ ,  $P_2$ ,  $P_3$  used the first two showed indoor scenes. Their enlarged prints from a 35 mm amateur camera ( $f = 35$  mm) were scanned at resolution equivalent to  $10 \mu\text{m}$  at negative scale. The third pair had been acquired at the highest resolution of an Agfa ePhoto 1680 digital camera with  $f = 38$  mm (35 mm equivalent). As no precise inner orientation data were available, “nominal” values ( $c = f$ ,  $x_0 = y_0 = 0$ ) were used. Although most image lines seemed to have been acceptably extracted automatically through the Sobel operator, the lines actually employed had been fitted to points carefully digitised manually to minimize input noise. The image lines used are seen in Fig. 2; the stereopairs are presented in Fig. 3.

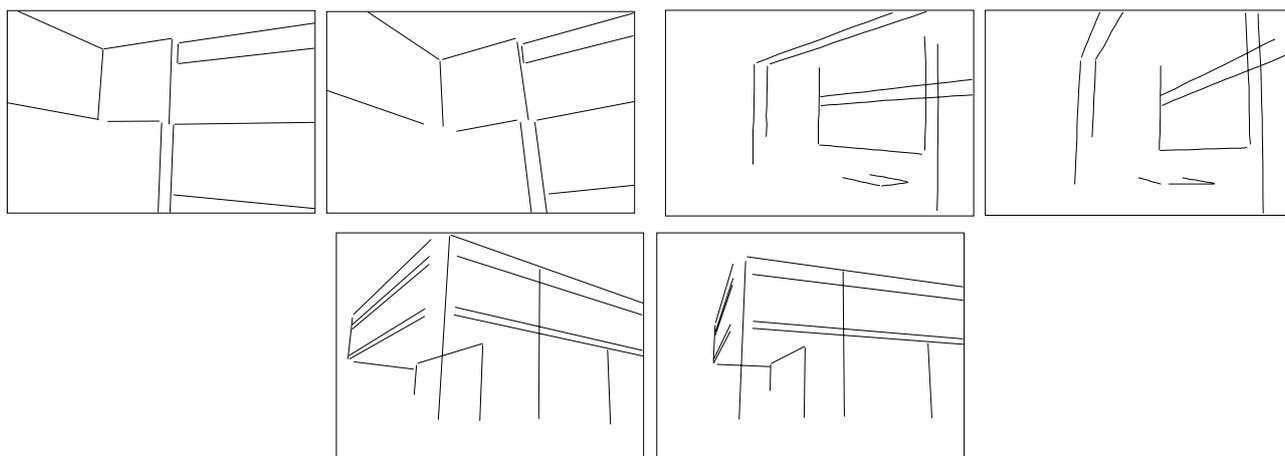


Figure 2. The measured image lines of the three stereopairs  $P_1$  (top left),  $P_2$  (top right) and  $P_3$  (bottom).

### 3.2 Results

Adjustments were performed both without and with different constraints on model planes employing the lines seen above. Following quantities, subsequently computed for all cases, are concentrated in the Table 4:

- The angles formed between all available pairs of reconstructed model lines. These yielded a mean  $m_L$  of their deviations from the assumed angles ( $0^\circ$  or  $90^\circ$ ).
- The angles formed by all unconstrained reconstructed model planes with all other planes excepting those "represented" by other mutually constrained planes. These provided the mean  $m_P$  of their deviations from the assumed angles ( $0^\circ$  or  $90^\circ$ ).
- Symmetric distances in model space (two for  $P_1$ ,  $P_2$  and three for  $P_3$ ) expected to be equal, namely defined by mutually perpendicular pairs of parallel lines. In this first implementation, lines had only been constrained as regards coplanarity to provide best-fitting planes; as a rule, therefore, reconstructed lines ran skew to each other. Hence, the end-points of line segments were materialised as "interpolated" line intersections. Here, the proportional differences  $d_S$  of distances were used.
- Finally, 8–10 well distributed points were selected on all pairs to check independently the quality of relative orientation. In each case the determined orientation parameters were used to intersect the corresponding projection rays. This resulted in mean precisions  $\sigma_P$  of point intersections.



Figure 3. Stereopairs  $P_1$  (top),  $P_2$  (middle) and  $P_3$  (bottom) used in the examples.

Table 4. Results of constrained shape reconstruction

Pair	Lines	Planes	Constraints on planes	Deviations of lines $m_L$ (°)	Deviations of planes $m_P$ (°)	Deviations of distances $d_S$ (%)	Point precision $\sigma_P$ (pixel)
P <sub>1</sub>	17	5	⊥	0.8	1.1	0.3 0.8	1.0
			⊥ ⊥	0.6	1.1	0.2 0.6	1.6
			⊥ ⊥ ⊥	0.6	0.9	0.1 0.8	1.2
				0.6	1.1	0.1 0.7	1.3
P <sub>2</sub>	17	6	//	3.3	3.5	10.5 0.1	1.9
			// //	1.4	1.0	0.1 0.4	6.7
			// // ⊥	1.2	0.9	0.0 0.1	5.7
			// // ⊥ ⊥	0.9	0.7	0.1 0.1	4.8
			0.9	0.4	0.2 0.5	4.6	
P <sub>3</sub>	22	6	⊥	1.7	1.9	2.6 1.0 0.4	0.9
			⊥ ⊥	0.6	0.6	0.2 0.1 0.7	1.3
			⊥ ⊥ ⊥	0.7	0.6	0.6 0.2 0.8	1.3
			⊥ ⊥ ⊥ //	0.7	0.6	0.6 0.1 0.8	1.3
			0.3	0.6	0.1 0.4 1.2	1.5	

The results for image pair P<sub>1</sub> show that the precision of reconstruction was relatively high from the start and constraints had nothing much to offer. In the other two pairs very significant improvements are apparent. Deviations in relative directions of both lines and planes drop drastically by a factor of 3–5. For symmetric line segments, too, there emerges a clear overall improvement (the extreme case is the first P<sub>2</sub> segment. Precision drops for the last P<sub>2</sub> and P<sub>3</sub> segments; this apparently is the “cost” paid here to constrain overall shape). It may also be pointed that, in general, just one or two constrains among model planes already seem to almost exhaust the limits of improvement. In fact, it appears that addition of more constraints – particularly in case they are contradictory – might well lead into the opposite direction.

On the other hand, as the adjustment is forced to partly reproduce a given shape thus increasing “morphological” accuracy, statistical precision is expected to drop. This was clear in the residuals of the adjustment but can also be seen in the last column above: orientation parameters estimated in constrained situations result in poorer intersections of projective rays. This is most clear in pair P<sub>2</sub> where the initial “free” adjustment produced far better intresections but a considerably distorted shape; respect of shape led to poorer orientation. This contradiction (also witnessed in the examples of van den Heuvel, 1999) is inherent to such approaches. Obviously, the phenomenon is aggravated in the presence of noisy data or of invalid constraints.

#### 4 CONCLUSIONS

The line-photogrammetric approach formulated, presented and tested here allows to exploit the simple stereopair for relative orientation and partial 3D reconstruction. If further constraints are added to the “structural” ones indispensable for solution, overall shape recovery comes considerably nearer to ground truth. At this stage only angular constraints among planes have been used. Next steps will include directional and positional constraints on lines and planes. More generally, the approach should be further elaborated to exploit a given scene topology. Criteria have to be developed for automatic selection/rejection of possible constraints to produce optimal combinations of statistical precision and morphological accuracy. It is believed that research in this path has much to offer to line photogrammetry, CAD-based modeling and vision metrology.

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