AN ALGORITHM FOR CENTRELINE EXTRACTION USING NATURAL NEIGHBOUR INTERPOLATION

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ABSTRACT

Data caption and conversion are two of the most costly operations of any GIS, in terms of computer time and manual work needed for spatial data acquisition. They can represent up to 80 percent of the total implementation costs. Manual digitising is a very error prone and costly operation, especially due to the lack of explicit topology in commercial GIS systems. Indeed, each map update might require the batch processing of the whole map. Currently, commercial GIS do not offer completely automatic raster/vector conversion even for simple scanned black and white maps. Various commercial raster/vector conversion products exist for the skeletonisation or thinning of the pixels forming the line, but these approaches have shown difficulties with the extraction of good topology. The spatial feature extraction in raster/vector conversion systems is based on line tracing algorithms. In order to operate they need user defined tolerances settings, what causes difficulties in the extraction of complex spatial features, for example: road junctions, curved or irregular lines and complex intersections of linear features. The approach we use here is based on image processing filtering techniques to extract the basic spatial features from raster data. These spatial features can be used for the reconstruction of the image within the topological data structure - the Voronoi diagram. The novel part of this research is the definition of deterministic topological rules and algorithms for extracting the spatial features from the Voronoi data structure. These spatial features can then be represented in different spatial data structures that can be implemented in a GIS. In this research we use the topological approach to develop new algorithms and data structures for integrated raster/vector models leading to the improvement of data caption and conversion in GIS and to develop a software toolkit for automated raster/vector conversion. The approach is based on computing the skeleton from Voronoi diagrams using natural neighbour interpolation. In this paper we present the algorithm for skeleton extraction from scanned maps. We show that the skeleton extracted from the map features can approximate the centreline of the map object. We apply this algorithm directly on the Voronoi cells, for the extraction of complex spatial features. This research can lead to the improvement of current practices in spatial data acquisition reducing significantly the cost and amount of work needed.

1 INTRODUCTION

In this paper we present an algorithm for raster to vector conversion of scanned maps using skeletonisation from Voronoi diagram. This involves sampling the scanned map irregularly using edge detection algorithms and then applying the natural neighbour interpolation. Since we are considering a scanned map in grayscale, the interpolant is the level of grey.

In spatial interpolation, local techniques have been used in order to get an interpolation continuous at data points, and smooth around data points. In these local techniques, the data points which influence the interpolant are the ones neighbouring the given interpolation point. The interpolation is thus based on the definition of adjacency or of neighbourliness. In 1D, the neighbourliness is given by the natural topology of the real line, induced by its total order. In 2D, there is no such relationship, and the neighbourliness can be defined by some topological structure. Such structures include the Delaunay triangulation, that is the dual of the Voronoi diagram. The Delaunay triangulation has been extensively used in linear interpolation (which corresponds to convoluting with the triangle or Barlett filter (Foley et al., 1996)). Another local technique is the natural neighbour interpolation (Sibson, 1981) based on local coordinates.

These local coordinates were introduced by Sibson (Sibson, 1980).

Local coordinates based on the Voronoi diagram are used in natural neighbour interpolation (also studied in (Gold, 1994) as "stolen area" interpolation), to quantify the "neighbourliness" of data sites. The properties of these local coordinates have been extensively studied by Sibson (Sibson, 1980) and Piper (Piper, 1993), who gave a formula for the gradient of the volume stolen from neighbouring Voronoi regions due to the insertion of a query point, obtained from two directional derivatives. The natural neighbour or stolen area interpolation technique has been extended from ordinary Voronoi diagrams to Voronoi diagrams for sets of points and line segments in (Anton et al., 1998). Anton et al. (Anton et al., 1998) extended the results presented in Gold and Roos (Gold and Roos, 1994), by providing direct vectorial formulas for the first order and second order derivatives for the stolen area. The analysis presented in (Anton et al., 1998) generalises the analysis of Piper (Piper, 1993) based on the formalism of partial derivatives, to the formalism of derivatives of a function on a normed space.

In section 2, we introduce the concept of Voronoi diagrams of a set of points. In section 3 we present the relationship between the skeleton and the Voronoi diagram. In section 4 we present three different techniques to sample the scanned map irregularly for detecting the edges in the map. In section 5, we present the natural neighbour interpolation technique. In section 6, we present

our centreline algorithm that uses the natural neighbour interpolation technique and skeletonisation. In section 7 we present the experimental results. Finally, in section 8 we present discussion.

2   THE VORONOI DIAGRAM OF A SET OF POINTS

Let us first introduce the definition of the Voronoi diagram for a set of sites (i.e. objects) in the Euclidean plane.

Definition 1. Let $\mathcal{O}$ be a set of sites in the Euclidean plane. For each site o of $\mathcal{O}$, the Voronoi cell $V(o)$ of o is the set of points that are closer to o than to other sites of $\mathcal{O}$. The Voronoi diagram $V(\mathcal{O})$ is the space partition induced by Voronoi cells.

Then let us introduce the definition of the Delaunay triangulation of a set of sites (or objects) in the Euclidean plane.

Definition 2. The Delaunay triangulation of $\mathcal{O}$ is the geometric dual of the Voronoi diagram of $\mathcal{O}$: two sites of $\mathcal{O}$ are linked by an edge in the Delaunay triangulation if and only if their cells are incident in the Voronoi diagram of $\mathcal{O}$.

3   SKELETONS FROM VORONOI DIAGRAMS

A very illustrative definition of the skeleton is given by the prairie-fire analogy: the boundary of an object is set on fire and the skeleton is the loci where the fire fronts meet and quench each other. Imagine a fire along all edges of the polygon, burning inward at a constant speed. The skeleton (Skiena, 1997) is marked by all points where two or more fires meet. This operation is also called the medial-axis transformation and is useful in thinning a polygon, or, as is sometimes called, finding its skeleton (Skiena, 1997). Another approach to compute skeletons is based on the Voronoi diagram. The medial-axis transform of a polygon $\mathcal{P}$ is simply the portion of the line-segment Voronoi diagram that lies within $\mathcal{P}$.

The simplest and most readily implementable thinning algorithm (Skiena, 1997) starts at each vertex of the polygon and grows the skeleton inward with an edge bisecting the angle between the two neighboring edges. Eventually, these two edges meet at an internal vertex, a point equally far from three line segments. One of the three is now enclosed within a cell, while a bisector of the two surviving segments grows out from the internal vertex. This process repeats until all edges terminate in vertices (Skiena, 1997). The Voronoi diagram of a discrete set of points (called generating points) is the partition of the given space into cells so that each cell contains exactly one generating point and it is the locus of all points which are nearer to this generating point than to other generating points.

Each polygon formed by edges of the map has sample points near its boundary. If the density of the boundary points (as generating points) goes to infinity then the boundary of the union of all the Voronoi zones belonging to points of the same polygon converges to that polygon. The boundaries between Voronoi zones belonging to points of different polygons converges towards the skeleton of the objects in the polygon. This property is important in designing the algorithms for sampling the map features on the scanned maps. In the next section we will present the edge detection algorithms that we used for sampling the images.

4   EDGE DETECTION IN SCANNED MAPS

There are a wide variety of edge detection algorithms that exists, but the set of basic tools on which most of the general algorithms

are built are: the derivative in the direction of the gradient, the Laplacian, the directional derivatives and the statistical differencing.

We select the data points where the variation of the intensity is the highest i.e. at the edges. In order to get sample points around the high frequency changes in the image we use two edge detection algorithms based on the Laplacian and one edge detection algorithm based on the derivative in the direction of the gradient.

The gradient is the first order differential of the interpolant at the point. The derivative in the direction of the gradient gives the highest variation of the interpolant (in our case the level of grey) at the point. This derivative in the direction of the gradient equals to the highest magnitude of the derivative, i.e. the square root of the sum of the squares of the derivatives in any pair of orthogonal directions, e.g. $\left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{\frac{1}{2}}$. This is a well behaved function used in image sharpening and edge detection. In the gradient based sampling that we used, we took the square root of the sum of the square of the difference between the row above and the row below the pixel, and the square of the difference between the strip on the left and right of the pixel.

The Laplacian $\left(\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)$ is proportional to the variation of the derivative of the interpolant at the point with respect to an annulus centered at the point. We used two different computations for the Laplacian, the standard Laplacian, and an alternative Laplacian where the "annulus" is composed of eight pixels instead of four. In this alternative Laplacian, a $\frac{1}{\sqrt{2}}$ factor is used to compensate for the wider diagonal pixel separation. These two Laplacian based sampling techniques are the other two sampling techniques we used in our experimentation.
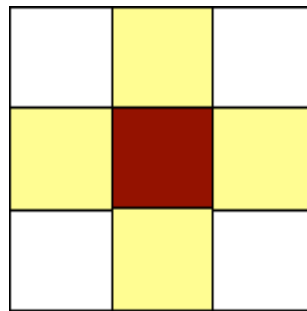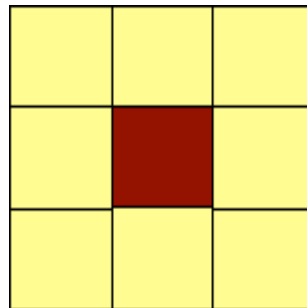

Figure 1: Annulus of 4 pixels

Figure 2: Annulus of 8 pixels

The sampling consists in selecting all the pixels whose derivative operator value is bigger than some threshold. After the image is irregularly sampled, the Voronoi diagram (and its dual graph: the

Delaunay triangulation) of the set of samples is computed using an incremental algorithm based on the Quad-Edge data structure (see (Guibas and Stolfi, 1985) for an introduction to the Quad-Edge data structure and the algorithms for the construction of the Voronoi diagram based on this data structure).

5   NATURAL NEIGHBOUR INTERPOLATION

In this section we will make a brief introduction to the work for natural neighbour interpolation presented by Gold and Roos (Gold and Roos, 1994). Consider a set of points

$$\mathcal{O} := \{P_1, \ldots, P_n\}.$$

Consider what would be the Voronoi diagram $VD(S \cup \{x\})$ after the insertion of another point $x \in \mathbb{R}^2$. The goal is to compute the areas that x would steal to its neighbours if it was inserted in the Voronoi diagram without actually inserting x in the Voronoi diagram.

The Voronoi diagram of points in the plane forms a network of vertices and edges. The vertices are the points that have at least three nearest neighbours while the edges are the loci of points having at least two closest neighbours.

Let $v_{i,i+1}(x)$ denote the Voronoi vertex whose nearest neighbours are P_i, P_{i+1} and x. Since P_i and P_{i+1} are two nearest neighbours of x, it is clear that $v_{i,i+1}(x)$ lies on the bisector $B_{i,i+1}$ of the points P_i and P_{1+1}:

$$B_{i,i+1(\mu)} := m_{i,i+1} + \mu n_{i,i+1} \text{ with } \mu \in \mathbb{R},$$

$$m_{i,i+1} := \frac{P_i + P_{i+1}}{2} \text{ and}$$

$$n_{i,i+1} := \left(\begin{array}{c} P_{i+1,2} - P_{i,2} \\ P_{i,1} - P_{i+1,1} \end{array} \right) \perp [P_{i+1} - P_i].$$

We can construct the parametric representation of the bisector (Gold and Roos, 1994) and we can compute the position of the Voronoi point $v_{i,i+1}(x)$:

$$v_{i,i+1}(x) := m_{i,i+1} + \frac{[x - P_{i+1}]T[x - P_i]}{2 n_{i,i+1} T[x - P_i]} n_{i,i+1}.$$

Each site P_i has a given height h_i. The height of the inserted point x is determined by the weighted area (Gold and Roos, 1994):

$$h(x) := \sum_{i:v(x) \cap v(P_i) \neq \emptyset} \frac{area[v(x) \cap v(P_i)]}{area[v(x)]} h_i,$$

where $v(\_)$ denotes the Voronoi zone of $\_$. Figures 3, 4 and 5 show the construction of the Voronoi polygons. Both regions $v(x) \cap v(P_i)$ and $v(x)$ are convex and the corners of $v(x)$ are Voronoi points in $VD(S \cup \{x\})$ and the corners of $v(x) \cap v(P_i)$ are Voronoi points in $VD(S)$ or $VD(S \cup \{x\})$.

The areas of the Voronoi zones can be computed as sums of triangles in the following way: let $P_1, \ldots, P_k$ denote the Voronoi neighbours of x in counterclockwise order. The area of $v(x)$ is equal to the sum of the areas of the triangles $\triangle(x, v_{i,i+1}(x), v_{i+1,i+2}(x))$ ie.:

$$area(v(x)) = \sum \frac{1}{2} det[v_{i,i+1}(x) - x, v_{i+1,i+2}(x) - x]$$
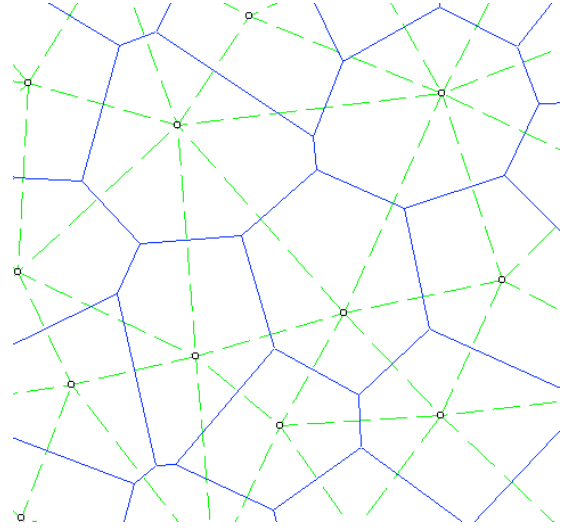

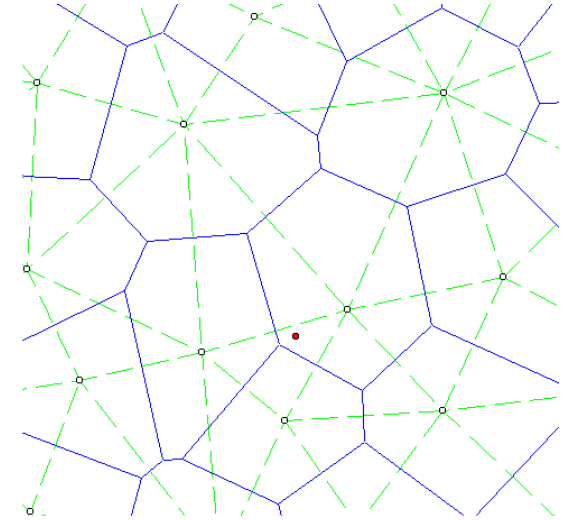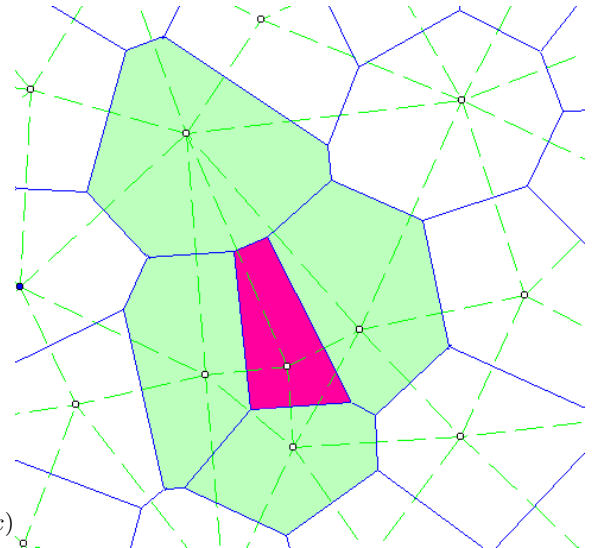Figure 3: Voronoi diagrams of data points

Figure 4: Interpolation point

Figure 5: Area stolen by the new point

In fact, assuming that $v(x)$ is bounded, i.e., x lies within the interior of the convex hull S, we can obtain in the same way the area of the region $v(x) \cap v(P_i)$ (Gold and Roos, 1994).

Thus this allows us to finally compute the variable height $h(x)$. The interpolated height is in fact the weighted average of the levels of grey of the neighbouring sample points with the weights being the areas that would be stolen to the neighbouring sampled points.

6 THE ALGORITHM

Now we will present the algorithm we developed for the skeletonisation of sampled map features. Even though there is a similar algorithm for skeleton extraction from scanned maps proposed by Gold (Gold, 1999) this algorithm is treating only black and white images. Our algorithm is different in this sense because we are interpolating the level of grey. We call "edges" of the picture, the boundaries of the zones where the level of grey changes continuously. The edges of the picture are first detected as a subset of the Voronoi diagram and of the Delaunay triangulation of the set of sampled points. The edges are detected through several criterions on the edges of the Voronoi diagram of the sampled pixels. The first criterion is that the difference of the levels of grey of the extremities of the Voronoi edge should be smaller than the difference of the levels of grey of the extremities of the dual Delaunay edge. The second criterion is that the dual Delaunay edge and the Voronoi edge (considered as line segments) intersect. The third criterion is that the levels of grey of the extremities of the Voronoi edge and of the dual Delaunay edge are not all the same.

The resulting subset of the Voronoi diagram is the set of all the edges of the picture, which we call the border set. We flag the Voronoi edges adjacent to a Voronoi edge of the border set that do not belong to the border set.

Then from this border set, we draw the skeleton using a traversal of the Voronoi zones belonging to the interior of the border set. For each Voronoi edge e of the border set, we traverse the Voronoi edges of the Voronoi zone that belongs to the interior of the border set starting from the Voronoi edge following e.

We mark each one of the traversed Voronoi edges as visited. For each one of those Voronoi edges f that belongs to the border set and is not adjacent to e, we draw the bisector of e and f in the skeleton.

For each one of those Voronoi edges f that are not in the border set and such that one of its neighbours is flagged, then the Voronoi edge is drawn in the skeleton.

7 EXPERIMENTAL RESULTS

In this section we present experimental results of our algorithm that are applied to processing of scanned maps. The original scanned map is shown in Figure

The sample points are shown in Figure

The Delaunay triangulation of the sample points is shown in Figure

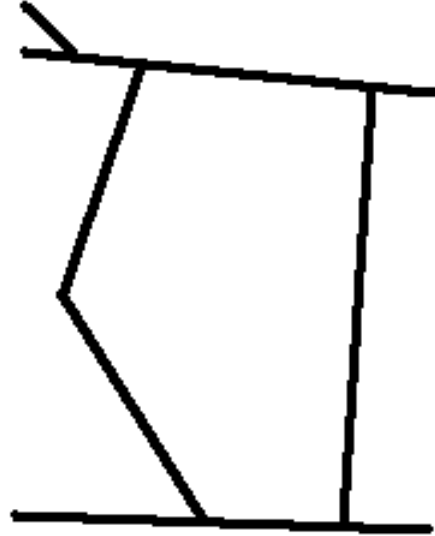The border set and the skeleton are shown in Figures
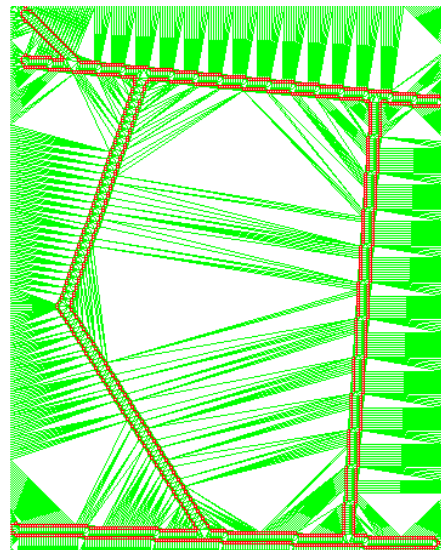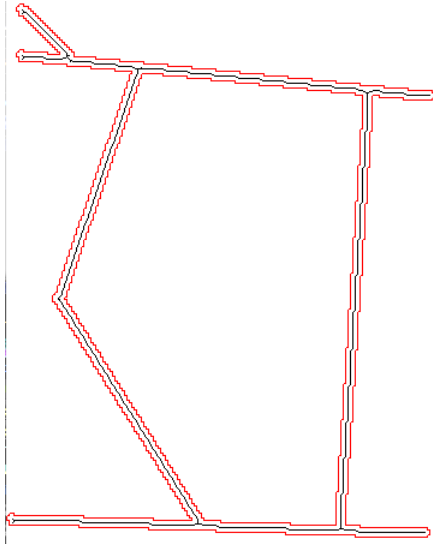

Figure 6: Original picture

Figure 7: Delaunay triangulation

Figure 8: Border set and skeleton

Figure 9: Skeleton

8 DISCUSSION

We have shown in this paper an application of the natural neighbour interpolation for skeletonisation of scanned maps. We have presented an example of use of this interpolation technique for the centreline extraction. Our future work will try to prove the use of this interpolation technique for automated conversion of scanned maps.

9 ACKNOWLEDGMENTS

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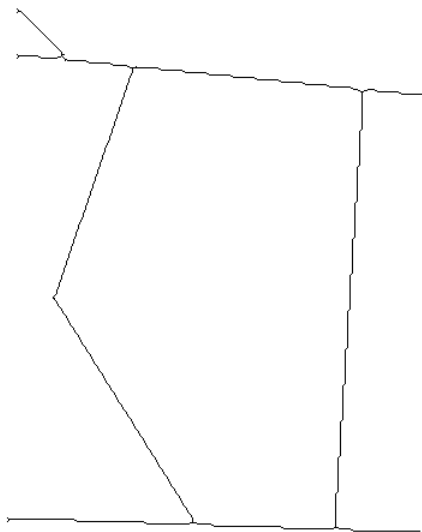Skiena, S., 1997. The Algorithm Design Manual. Springer-Verlag, New York, NY.