DERIVING SPATIOTEMPORAL RELATIONS FROM SIMPLE DATA STRUCTURE

Ale Raza

ESRI 380 New York Street, Redlands, California 92373-8100, USA
Tel.: +1-909-793-2853 (extension 2009)
Fax: +1-909-307-3067
araza@esri.com

Commission II, WG II/1

KEY WORDS: Data Structure, Operation, Query, Object, Spatial, Temporal

ABSTRACT:

A spatiotemporal data model is incomplete without three components: classes, consistency constraints, and operators. Classes define the structure of the model, constraints enforce consistency in the model, and operators operate on the structure of the model. In the past, many models have been proposed, but most of them discussed the classes. The studies on operators for spatiotemporal data models are not abundant. Operators used to query spatial, temporal, and spatiotemporal relations are the focus of this paper. Relations in the spatiotemporal databases can be categorized into three groups: spatial, temporal, and spatiotemporal relations. Spatial relations that are valid for a certain time period are called spatiotemporal relations. These spatiotemporal relations are based on a cell-tuple-based spatiotemporal data model (CTSTDM). Spatiotemporal relations can be classified into five groups: metric, topological, order, set oriented, and Euclidean. This paper elaborates on the topological relations (spatiotemporal topology) derived from a simple temporal cell-tuple structure. The operator, operand(s), results, and syntax of the spatiotemporal relations are defined. By employing relational algebra, spatiotemporal relations (boundary, contains, overlaps, etc.) can be derived from the cell-tuple-based spatiotemporal data model. In the past, two common approaches have usually been employed to obtain topological relations. The first is called explicit, and the other is implicit. Both approaches have advantages and disadvantages. The cell-tuple-based spatiotemporal data model stores spatiotemporal topology implicitly, which is more appropriate for spatiotemporal and network databases. The paper concludes with limitations of this implicit topology approach and recommendations for future work.

1. INTRODUCTION

A spatiotemporal data model has three components: the classes, consistency constraints, and operators. Classes define the structure of the model, constraints enforce consistency in the model, and operators operate on the structure of the model. These operators can be static or dynamic (Raza, 2004). Dynamic operators change the state of the system, for example, create, kill, delete, or reincarnate operators. Static operators are query operators. Past research on spatiotemporal models (STM) mainly focused on the classes of the model. Research on operators of STM is not abundant. Static operators are the focus of this paper. These operators are utilized to query spatial, temporal, and spatiotemporal relations. Relations in spatiotemporal databases can be categorized into three groups (i.e., spatial, temporal, and spatiotemporal relations). Past studies mainly focused on purely spatial or temporal relations. Spatial relations that are valid for a certain time period are called spatiotemporal relations.

This paper discusses the spatiotemporal relations based on temporal cell-tuple structure of an object-oriented, cell-tuple-based spatiotemporal data model (Raza, 2001; Raza and Kainz, 1999). The object-oriented cell-tuple-based spatiotemporal data model (CTSTDM) consists of three main classes: spatial, attribute, and temporal. A spatiotemporal class is the aggregation of spatial and temporal classes (Figure 1).

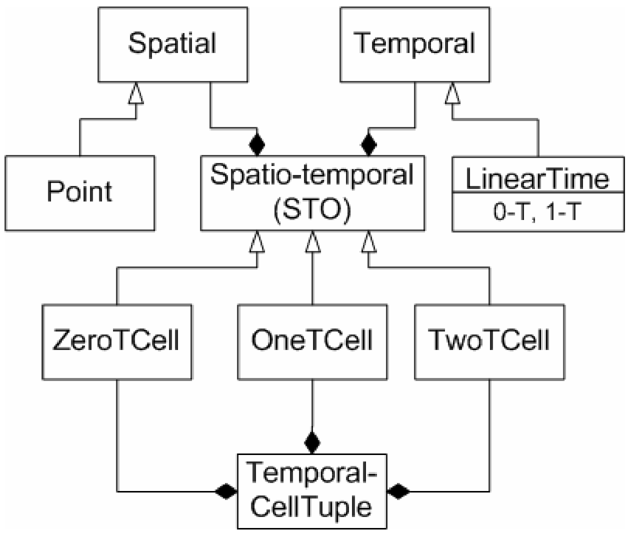

Figure 1. Spatial and temporal class hierarchy for spatiotemporal objects

This spatiotemporal class is also a super class of three classes—ZeroTCellClass (ZeroTCell), OneTCellClass (OneTCell), and TwoTCellClass (TwoTCell). TemporalCellTuple class is the aggregation of three classes: ZeroTCellClass, OneTCellClass, and TwoTCellClass. Operations pertaining to the TemporalCellTuple class are the focus of this paper. This paper elaborates the topological relations (spatiotemporal topology) derived from the simple temporal cell-tuple structure of

TemporalCellTuple class. The details of this structure can be found in Raza and Kainz (1999).

First, the spatial and temporal relations are briefly discussed in §2. The temporal cell-tuple class is explained in §3. Section 4 elaborates the spatiotemporal relations. The paper concludes in §5.

2. SPATIAL AND TEMPORAL RELATIONS

Static operators are query operators. These operators are used to query spatial, temporal, and spatiotemporal relations. Relations in spatiotemporal databases can be categorized into three classes:

- Spatial relations
- Temporal relations
- Spatiotemporal relations

These spatial relations are grouped into four classes: set-oriented, metric, topological, and Euclidean relations (Worboys, 1992). Spatial order relations were also introduced (Kainz, 1989). These spatial relations can be grouped into five categories:

- Spatial metric relations
- Spatial topological relations
- Spatial order relations
- Set-oriented spatial relations
- Euclidean spatial relations

	Temporal Relations	**Illustration**
1	Before	
2	After	
3	Equal	
4	Meets	
5	Met	
6	Overlaps	
7	Overlapped	
8	Covers	
9	During	
10	Started	
11	Finishes	
12	Starts	
13	Finished	

Table 1. Adapted temporal relations for bounded interval (Allen, 1984)

Worboys (1992) proposed nine spatial topological relations: interior, closure, boundary, components, extremes, begin, end, inside, and clockwise. These are valid for spatial objects of dimension $0 \leq n \leq 2$. Using the point-set approach, eight topological relations between two spatial objects of dimension 2 and eight topological relations between two spatial objects of dimension 1, respectively, were derived (Egenhofer et al., 1993; Pullar and Egenhofer, 1988). Similarly, 13 temporal topological relations can be realized in a one-dimensional bounded time interval (Allen, 1984). Table 1 shows these relations.

Temporal operations refer to temporal relations. Temporal operations are isomorphic to the spatial relations. These relations could be defined as using metric, topological, and order-theory concepts. For example, "two hours" is a metric relationship, "one hour later" is a topological temporal relationship, and "four weeks in a month" is an order relationship.

Therefore, the temporal relations can be subclassified into five categories:

- Temporal metric relations
- Temporal topological relations
- Temporal order relations
- Set-oriented temporal relations
- Euclidean temporal relations

As mentioned earlier, this paper will focus on temporal topological relations. These relations are associated with TemporalCellTuple class.

3. TEMPORALCELLTUPLE CLASS

The object of a spatiotemporal class is called an *n*-tcell. The boundaries (∂) of an *n*-tcell are its (*n-1*) faces at time t. The coboundary (Φ) of an *n*-tcell produces the (*n+1*) cells incident with *n*-tcell at time t. In the temporal cell complex, *intracell* complex relations (i.e., relations between cells in the cell complex) can be described using boundary and coboundary relations. The boundary and coboundary relations capture two types of topological relationships: adjacency and containment. Relations between spatial objects can be found based on boundary/coboundary relations between cells. The boundary and coboundary relations are encapsulated in a simple temporal cell-tuple structure, which is an extension of the cell-tuple structure of Brisson (1990). A cell-tuple T is an *(n+1)* tuple of cells $\{c_0, c_1, c_2, \ldots, c_n\}$, where any *i*-cell is incident with a *(i+1)*-cell.

TemporalCellTupleClass preserves the temporal cell-tuple structure (Figure 2) and is the aggregation of ZeroTCellClass (ZTC), OneTCellClass (OTC), and TwoTCellClass (TTC) (Figure 1). The object of TemporalCellTupleClass has a unique tuple ID and a unique combination of ZTC, OTC, and TTC. Each tuple must have a ZTC, zero or one OTC, and zero or one TTC. Therefore, a temporal cell-tuple structure encapsulates the spatiotemporal topology of each spatiotemporal object. A temporal cell tuple (TCT) is a set of C and T, as follows:

$$TCT = \{C, T\}$$

where C is a set of cells

$C = \{c_0, c_1, c_2, \ldots c_n \mid c_i \in TCC\}$ and

T is a time interval (1-T)

$T = \{T_{From}, T_{Until} \mid (T_{From} < T_{Until}) \wedge (T_{From}, T_{Until} \in ST)\}$

Therefore,

$TCT = \{c_0, c_1, c_2, \ldots c_n, T_{From}, T_{Until}\}$

Any *i*-tcell (c_i) is incident with an *(i+1)*-tcell (c_{i+1}). Every c_i cell is a boundary of a c_{i+1} cell, where $0 \le i \le n$ and n+1 is the maximum number of cells in each tuple. For n = m, the first cell c_0 is a ZTC, the second cell c_1 is an OTC, the third cell c_2 is a TTC, and *m*-cell c_m is an *m*TC. In $\{c_0, c_1, c_2, \ldots c_n, T_{From}, T_{Until}\}$ at time t, any *i*-tcell (c_i) is either a boundary of an *(i+1)*-tcell (c_{i+1}) or coboundary of an *(i-1)*-tcell (c_{i-1}). The advantage of TCT is that it stores topology implicitly. It is dimension-independent, that is, it can accommodate objects of dimension *k* ($k \ge 1$), and it encapsulates boundary and coboundary and order relations over time. We can formulize the spatiotemporal relations (Φ and ∂) history of *k*-tcell at time T_i as

$$\Phi(k\text{-tcell})_{Ti} = \{\forall(k+1)\text{tcell} \mid T_{From} \le T_i\}$$
$$\partial(k\text{-tcell})_{Ti} = \{\forall(k-1)\text{tcell} \mid T_{From} \le T_i\}$$

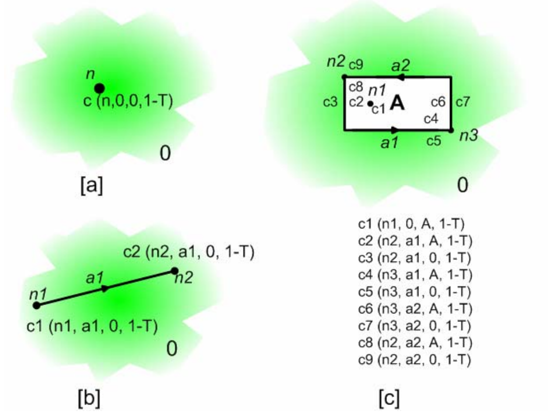

Figure 2. Process of assigning temporal cell tuples to spatiotemporal cells of dimensions ($0 \le n \le 2$)

The process of assigning cell tuples to a ZTC is illustrated in Figure 2. A temporal cell tuple is a unique combination of ZTC, OTC, and TTC. The world TTC W = {0} is defined NULL. In principle, every ZTC gets a TCT. If this OTC is a member of W, then there is only one TCT; if it belongs to OTC or TTC, the TCTs are assigned accordingly. Every OTC has two TCTs if it is not a boundary of TTC (except W), and it gets four TCTs if it is a boundary of TTC (except W). The number of TCTs for TTC depends on the number of TTC boundaries.

If the ZTC ∈ {W} ∧ ∉ {OTC, TTC}, where ZTC, OTC, and TTC ⊂ {W}, then Figure 2[a] shows this configuration for ZTC (n), that is, c (n, 0, 0, 1-T). The duration or lifetime of this relation is indicated by time interval 1-T. Figure 2[b] shows the configuration when the ZTC n1 and n2 are the boundary of OTC (a1). In other words, {n1, n2} ∈ {a1} and {a1} ∈ {W}. The tuple c1 (n1, a1, 0, 1-T) shows that this tuple belongs to ZTC (n1), OTC (a1), and TTC (0). The c2 shows that it belongs to ZTC (n2), OTC (a1), and TTC (0).

4. SPATIOTEMPORAL RELATIONS

Spatial relations that are valid for a certain time period are called spatiotemporal relations. Topological relations are considered for further discussion. Most of these relations (spatiotemporal topology) can be derived from the TCT structure. These spatiotemporal relations are preserved in the TCT structure. Spatiotemporal relations derived from the TCT structure are based on two primary relations: boundary and coboundary. In the following subsections, each operation, operands, results, and syntax in unified modeling language (UML) is presented.

4.1 Boundary (∂) and Coboundary (Φ)

The boundary (∂) of an *n*-tcell is its (*n-1*) faces at time t. The coboundary (Φ) of an *n*-tcell produces the (*n+1*) cells incident with *n*-tcell at time t.

The Φ and ∂ history of *k*-tcell at time T_i can be formalized as

$$\partial(k\text{-tcell})_{Ti} = \{\forall(k-1)\text{tcell} \mid T_{From} \le T_i\}$$
$$\Phi(k\text{-tcell})_{Ti} = \{\forall(k+1)\text{tcell} \mid T_{From} \le T_i\}$$

Whereas, the boundary of *k*-tcell at time T_i is

$$\partial(k\text{-tcell})_{Ti} = \{\forall(k-1)\text{tcell} \mid T_{From} = T_i \wedge k\text{-tcell}_1 \ne k\text{-tcell}_2\}$$

Where

$$\Phi(k\text{-}1)\text{-tcell} = \{k\text{-tcell}_1, k\text{-tcell}_2\}$$

The coboundary *k*-tcell at time T_i can be defined as

$$\Phi(k\text{-tcell})_{Ti} = \{\forall(k+1)\text{tcell} \mid T_{From} = T_i\}$$

In Figure 3, the boundary of A1 at time T2 can be calculated as

$$\partial(A1)_{T2} = \{a1, a2, a3\}$$
$$\Phi(a1) = \{\varnothing, A1\}$$
$$\Phi(a2) = \{\varnothing, A1\}$$
$$\Phi(a3) = \{A1, A1\}$$

where symbol ∅ represents null.

Therefore, *a3* is excluded from the boundary of A1 because the coboundary of *a3* is the same.

$$\partial(A1)_{T2} = \{a1, a2\}$$

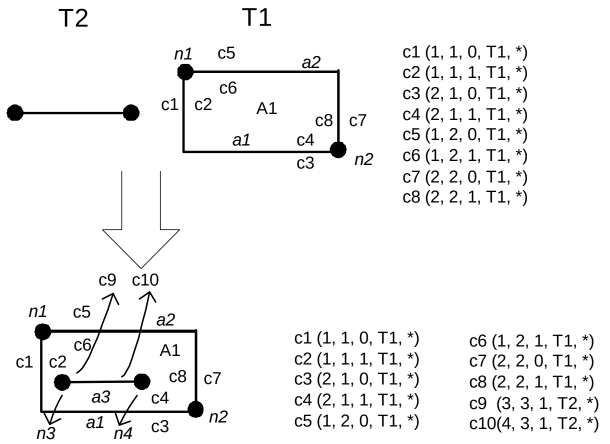

Figure 3. OTC intersects with TTC.

In Figure 4[b], the coboundary of ZTC (*n1*) at time T1 and T2 is

$\Phi(n1)_{T1} = \{\forall \text{ OTC} \mid T_{From} = T_1\} = \{a1\}$
$\Phi(n1)_{T2} = \{\forall \text{ OTC} \mid T_{From} = T_2\} = \{a2, a3, a4\}$

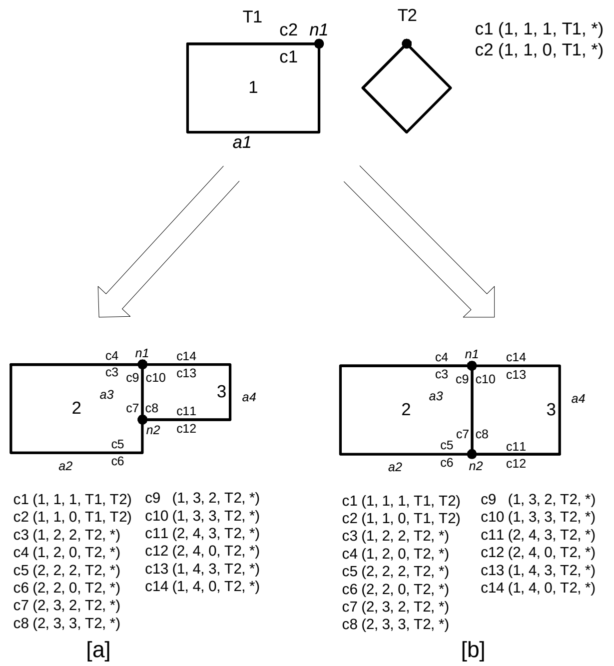

Figure 4. TTC intersects with TTC.

Similarly, in Figure 5, the coboundary of OTC (a1) at time T1 and T2 is

$\Phi(a1)_{T1} = \{\forall \text{ TTC} \mid T_{From} = T_1\} = \{1, 0\}$
$\Phi(a1)_{T2} = \{\forall \text{ TTC} \mid T_{From} = T_2\} = \{1,2\}$

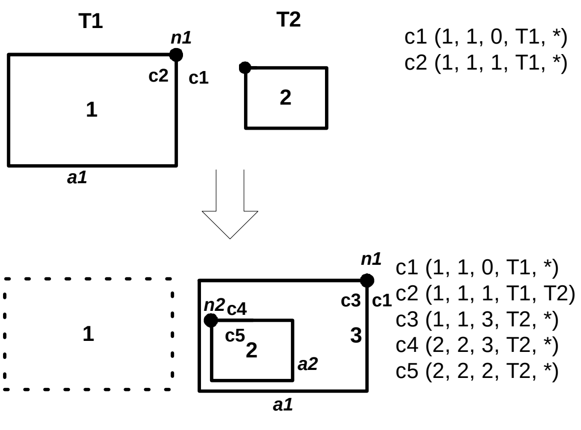

Figure 5. Interior of TTC intersects with boundary-interior of TTC'.

4.2 Disjoint (Ω)

The two *n*-tcells (n = 1,2) are disjoint if the intersection of their faces is empty. Disjoint relations of point and ZTC are straightforward. The Ω relations of OTC and TTC can be expressed as

Disjoint(P:TTC$_{T1}$, P:TTC$_{T2}$): Boolean
$\{\text{2-tcell}_{T1}\ \Omega\ \text{2-tcell}_{T2} = \text{true} \mid \partial(\text{2-tcell}_{T1}) \cap \partial(\text{2-tcell}_{T2}) = \varnothing\}$

Disjoint(P:OTC$_{T1}$, P:OTC$_{T2}$): Boolean
$\{\text{1-tcell}_{T1}\ \Omega\ \text{1-tcell}_{T2} = \text{true} \mid \partial(\text{1-tcell}_{T1}) \cap \partial(\text{1-tcell}_{T2}) = \varnothing\}$
For example, consider Figure 4[a],
$\Omega(\text{TTC2}_{T2}, \text{TTC3}_{T2})$ = FASLE because
$\Omega(\text{TTC2}_{T2}, \text{TTC3}_{T2}) = \partial(\text{TTC2})_{T1} \cap \partial(\text{TTC3})_{T2}$
$= \{(a2, a3) \cap (a3, a4)\}$
$= \{a3\}$

4.3 Contains (α)

The containment relations can be between spatiotemporal objects of the same spatial dimension or different spatial dimensions. For example, a TTC can contain a TTC, an OTC, or a ZTC; these relations are depicted in Figure 5, Figure 3, and Figure 6, respectively.

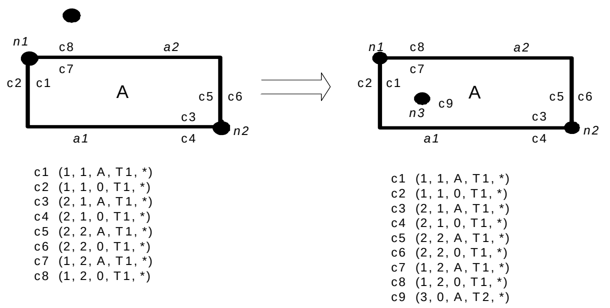

Figure 6. ZTC interior of TTC

At time T_i, 2-tcell$_j$ contains 2-tcell$_k$;
Contains(P:TTC, P:TTC): Boolean

$\{\text{2-tcell}_j\ \alpha\ \text{2-tcell}_k = \text{true} \mid T_{From} = T_i \wedge \partial(\text{2-tcell}_j) \cap \partial(\text{2-tcell}_k) = \partial(\text{2-tcell}_k)\}$

At time T_i, 2-tcell contains 1-tcell;
Contains(P:TTC, P:OTC): Boolean

$\{\text{2-tcell}\ \alpha\ \text{1-tcell} = \text{true} \mid T_{From} = T_i \wedge \partial(\partial(\text{2-tcell})) \cap \partial(\text{1-tcell}) = \partial(\text{1-tcell})$

At time T_i, 2-tcell contains 0-tcell;
Contains(P:TTC, P:ZTC): Boolean

$\{\text{2-tcell}\ \alpha\ \text{0-tcell} = \text{true} \mid T_{From} = T_i \wedge \partial(\partial(\text{2-tcell})) \cap \text{0-tcell} = \text{0-tcell}\}$

$\{\text{2-tcell}\ \alpha\ \text{0-tcell} = \text{true} \mid T_{From} = T_i \wedge \partial(\partial(\text{2-tcell})) \cap \text{0-tcell} = \text{0-tcell}\}$

For example, to check whether TTC contains a TTC or not, consider Figure 5, where at time T2, TTC(3) α TTC (2).

$\{\partial(3) \cap \partial(2)\} = \{\partial(2)\}$
$\{(a1, a2) \cap (a2)\} = \{(a2)\}$
$\{a2\} = \{a2\}$

4.4 Inside (χ)

At time T_i, a ZTC, OTC, or TTC can be inside a TTC. The same logic is employed to discern the χ relations between two n-tcells. For example:

At time T_i, 2-tcell$_j$ is inside 2-tcell$_k$;

Inside(P:TTC, P:TTC): Boolean

$\{$2-tcell$_j$ χ 2-tcell$_k$ = true $|$ $T_{From} = T_i \wedge \partial$(2-tcell$_j$) $\cap$ ∂(2-tcell$_k$) = ∂(2-tcell$_j$) $\}$
At time T_i, 1-tcell is inside 2-tcell;

Inside(P:OTC, P:TTC): Boolean

$\{$ 1-tcell χ 2-tcell = true $|$ $T_{From} = T_i \wedge \partial$ (∂(2-tcell)) $\cap$ ∂(1-tcell) = ∂(1-tcell)

At time T_i, 0-tcell is inside 2-tcell;
Inside(P:ZTC, P:TTC): Boolean

$\{$ 0-tcell χ 2-tcell = true $|$ $T_{From} = T_i \wedge \partial$ (∂(2-tcell)) $\cap$ 0-tcell = 0-tcell $\}$

4.5 Equal (=)

Checking Equal relations between two points or ZTCs is straightforward. TTC at time T1 is in equal relation to TTC at time T2 if the boundaries of both are the same.

$\{$2-tcell$_{T1}$ = 2-tcell$_{T2}$ $|$ ∂(2-tcell)$_{T1}$ = ∂(2-tcell)$_{T2}$ $\}$

Although it is a topological relation, the Equal relation between two OTCs may not be checked correctly in the TCT structure (based on boundary/coboundary relations) because these OTCs can be defined by different intermediate points, regardless of the same boundary. A geometric calculation is needed to check this relation.

4.6 Meet (δ)

A TTC at time T1 can meet with TTC, OTC, or ZTC at time T2. Similarly, an OTC at time T1 can meet with OTC or ZTC at time T2.

Meet(P:TTC, P:TTC): Boolean
$\{$2-tcell$_{T1}$ δ 2-tcell$_{T2}$ $|$ (∂(∂(2-tcell)$_{T1}$) $\cap$ ∂ (∂(2-tcell)$_{T2}$) $\neq \emptyset$ $\}$

Meet(P:TTC, P:OTC): Boolean
$\{$2-tcell$_{T1}$ δ 1-tcell$_{T2}$ $|$ (∂(∂(2-tcell)$_{T1}$) $\cap$ ∂(1-tcell)$_{T2}$ $\neq \emptyset$ $\}$

Meet(P:TTC, P:ZTC): Boolean
$\{$2-tcell$_{T1}$ δ 0-tcell$_{T2}$ $|$ ∂(∂(2-tcell)$_{T1}$) $\cap$ (0-tcell)$_{T2}$ $\neq \emptyset$ $\}$

Meet(P:OTC, P:OTC): Boolean
$\{$1-tcell$_{T1}$ δ 1-tcell$_{T2}$ $|$ ∂(1-tcell)$_{T1}$ $\cap$ ∂(1-tcell)$_{T2}$ $\neq \emptyset$ $\}$

Meet(P:OTC, P:ZTC): Boolean
$\{$1-tcell$_{T1}$ δ 0-tcell$_{T2}$ $|$ ∂(1-tcell)$_{T1}$ $\cap$ (0-tcell)$_{T2}$ $\neq \emptyset$ $\}$

For example, consider Figure 4[b]. At time T2, TTC (2) and TTC (3) have Meet relations.

$\{\partial(\partial(2)_{T2}) \cap \partial(\partial(3)_{T2})\} \neq \emptyset$
$\{\partial(a2, a3) \cap \partial(a3, a4)\} \neq \emptyset$
$\{(n1, n2) \cap (n2, n1)\} \neq \emptyset$
$\{(n2, n1)\} \neq \emptyset$

Similarly, consider Figure 7. At time T2, TTC (A1) and OTC (*a3*) have Meet relations.

$\{\partial(\partial(A1)_{T2}) \cap \partial(a3)_{T2}\} \neq \emptyset$
$\{\partial(a4, a5) \cap (n2, n3)\} \neq \emptyset$
$\{(n3, n4) \cap (n2, n3)\} \neq \emptyset$
$\{(n3)\} \neq \emptyset$

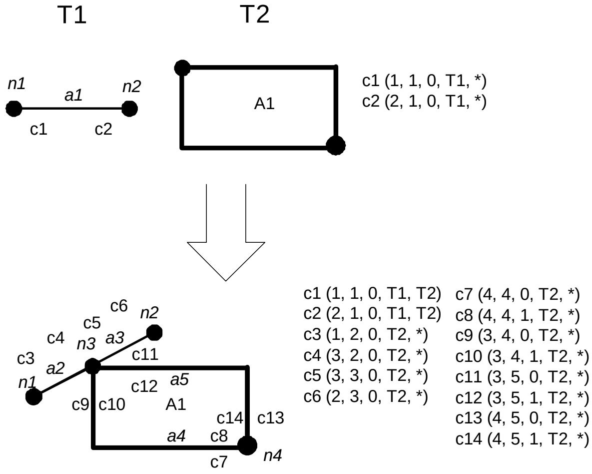

Figure 7. Boundary of TTC intersects with interior of OTC.

4.7 Covers (γ)

A TTC at time T2 can cover a TTC or OTC at time T2. Similarly, an OTC at time T1 can cover OTC at time T2. However, this relation could not be captured in the TCT structure because it does not maintain the interior of OTC.

Covers(P:TTC, P:TTC): Boolean
$\{$2-tcell$_{T1}$ γ 2-tcell$_{T2}$ $|$ (∂(∂(2-tcell)$_{T1}$) $\cap$ ∂(∂(2-tcell)$_{T2}$) $\neq \emptyset$) $\wedge$ (Φ(∂(2-tcell)$_{T1}$) $\cap$ (2-tcell)$_{T2}$ $\neq \emptyset$) $\}$

Covers(P:TTC, P:OTC): Boolean
$\{$2-tcell$_{T1}$ γ 1-tcell$_{T2}$ $|$ (∂(∂(2-tcell)$_{T1}$) $\cap$ ∂(1-tcell)$_{T2}$ $\neq \emptyset$) $\wedge$ (∂(2-tcell)$_{T1}$ $\cap$ (1-tcell)$_{T2}$ $\neq \emptyset$)$\}$

Consider Figure 8. At time T2, TTC (2) covers TTC (3).

$\{ (\partial(\partial(2)_{T1}) \cap \partial(\partial(3)_{T2}) \neq \emptyset) \wedge (\Phi(\partial(2)_{T1}) \cap (3) \neq \emptyset) \}$
$\{ (\partial(a4, a5) \cap \partial(a2, a3, a4) \neq \emptyset) \wedge (\Phi(a4, a5) \cap (3) \neq \emptyset) \}$
$\{ ((n2, n3) \cap (n1, n2, n3) \neq \emptyset) \wedge ((2,3) \cap (3) \neq \emptyset) \}$
$\{ ((n2, n3) \neq \emptyset) \wedge ((3) \neq \emptyset) \}$

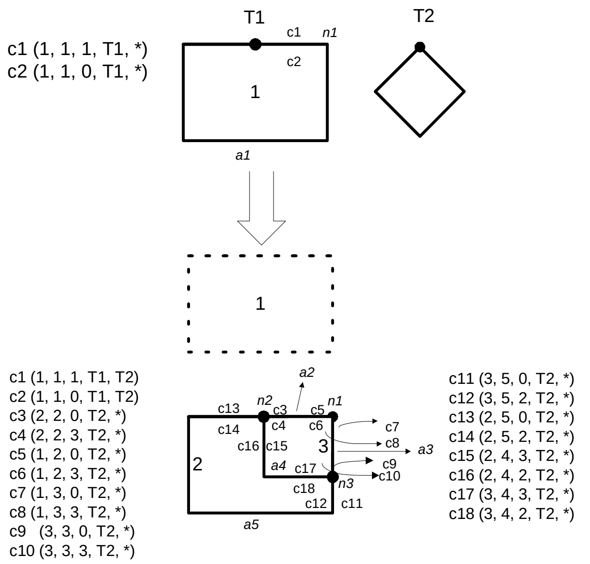

Figure 8. Interior of TTC intersects with boundary-interior of TTC', and boundary of TTC intersects with boundary of TTC'.

4.8 CoveredBy (η)

An OTC or TTC at time T1 can be covered by a TTC at time T2. Similarly, an OTC at time T1 can be covered by OTC at time T2. However, this relation is not captured in the TCT structure. The η relation is similar to γ relations and is not discussed further.

4.9 Overlap (κ)

A TCC is a partition of spaces; therefore, TTC or OTC cannot overlap each other. However, a two spatiotemporal object (TSTO) and one spatiotemporal object (OSTO) at time T1 can overlap with TSTO and OSTO at time T2, respectively, if their intersection with TTC or OTC is nonempty ($\neg\varnothing$).

Consider Figure 9. Let $TSTO_1 = \{2, 3\}$ and $TSTO_2 = \{3, 4\}$ at time T2. These two TSTOs overlap because

$\{(2,3) \cap (3,4) \neq \varnothing\}$
$\{(3) \neq \varnothing\}$

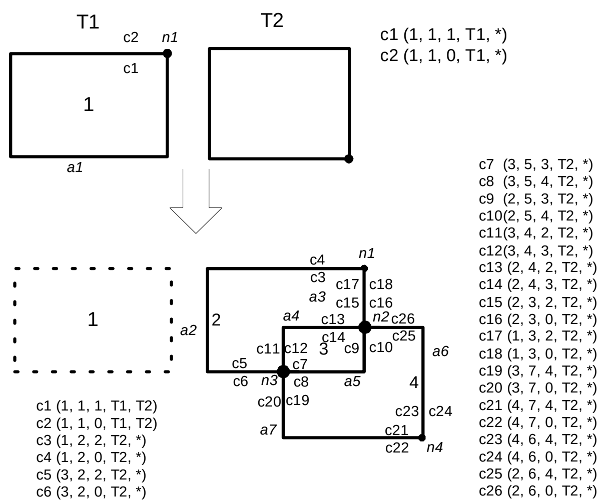

Figure 9. Boundary of TTC intersects with boundary-interior of TTC', and interior of TTC intersects with boundary of TTC'.

5. CONCLUSION

In this paper, operators pertaining to a simple temporal cell-tuple structure are presented. These operators are formulized by employing relational algebra. Examples are provided to derive these relations (spatiotemporal topology) from temporal cell-tuple structure. It has been proved that almost all spatiotemporal relations between OTC-OTC and TTC-TTC in the spatial domain and some other relations can be derived from temporal cell-tuple structure, which is based on the boundary and coboundary of cells. However, depending on time, some relations cannot be derived because of the inherent nature of temporal cell-tuple structure. For example, Overlap and CoveredBy relations for the same time cannot be derived because temporal cell complex is a partition of spaces. Similarly, for the Equal relation, geometric calculation is still needed. More research is needed to evaluate the performance of operators derived from the TCT structure. The composite B-tree index on the elements of this structure may perform better.

REFERENCES

Allen, J., 1984. Towards a general theory of action and time. *Artificial Intelligence,* Vol. 23, pp. 123–154.

Brisson, E., 1990. Representation of d-dimensional geometric object. Ph.D. thesis, University of Washington, WA, USA.

Egenhofer, M. J., J. Sharma, and D. M. Mark, 1993. A critical comparison of the 4-intersection and 9-intersection models for spatial relations: Formal analysis. In: *Proceedings of Auto-Carto 11,* Minneapolis, MN, USA, pp. 1–11.

Kainz, W., 1989. Order, topology and metric in GIS. *ASPRS/ACMS Annual Convention 1989, Technical Papers,* Vol. 4, pp. 154–160.

Pullar, D., and M. J. Egenhofer, 1988. Towards formal definition of topological relations among spatial objects. In: *Proceedings of the International Symposium on Spatial Data Handling (SDH),* Sydney, Australia, pp. 225–239.

Raza, A., 2001. Object-oriented temporal GIS for urban applications. Ph.D. thesis, ITC and University of Twente, The Netherlands.

Raza, A., 2004. Operators for cell tuple-based spatiotemporal data model. In: *XXth Congress of the International Society for Photogrammetry and Remote Sensing (ISPRS),* Istanbul, Turkey, 12–23 July 2004.

Raza, A., and W. Kainz, 1999. Cell tuple based spatio-temporal data model: An object oriented approach. In: *The Proceedings of the Eighth ACM Conference on Information and Knowledge Management (CIKM'99) and Symposium on Geographic Information Systems (GIS'99),* Kansas City, MO, USA, pp. 20–25.

Worboys, M., 1992. Object-oriented models of spatio-temporal information. In: *The Proceedings of the GIS/LIS,* San Jose, CA, USA, pp. 825–834.