A HIERARCHICAL REPRESENTATION OF LINE-REGION TOPOLOGICAL RELATIONS

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ABSTRACT:

Topological relations have found applications in many fields such as similarity analysis, spatial query, object matching, inconsistency detection, spatial association rule mining, spatial reasoning, and so on. This paper is concentrated on the hierarchical representation of the topological relations between a line and a region in IR^2. In this study, line-region relations are classified into basic relations and compound relations based on the number of intersections between line and region boundary. The basic relations are further differentiated into 16 types based on the invariants of dimension and local order. These basic relations are then organized into a hierarchy with multiple levels of topological invariants. These invariants are developed from the point of view of set theory, including (a) separation number and dimension at set level, (b) intersection type at element level, and (c) sequence of intersection types at an integrated level. A practical example is provided to illustrate the use of the hierarchical approach presented in this paper.

1. INTRODUCTION

Topological relations have been extensively applied to scene similarity analysis in multi-scale representation (Egenhofer and Clementini, 1994), representation of natural-language relations in spatial query (Clementini *et al*., 1994; Xu, 2007), object matching in multi-sources spatial data integration (Zhang and Meng, 2007), inconsistency detection in map updating (Chen *et al*., 2008), cartographic quality assessment in generalization (Steiniger and Weibel, 2005), spatial association rules mining in spatial data mining and knowledge discovery (Clementini and Di Felice, 2000; Miller and Han, 2001) and spatial reasoning in artificial intelligence (Sharma, 1996).

According to the dimension of spatial objects in a planar space, six types of spatial configurations can be distinguished, including point-point, point-line, point-region, line-line, line-region and region-region. As the configurations involving point objects (i.e. point-point, point-line and point-region) are relatively simple, the corresponding relations are also simple. As a consequence, attention has been paid mainly to the cases involving lines or regions.

A number of general models have been developed to describe the topological relations, the 4-intersection model (4IM) by Egenhofer and Franzosa (1991), the 9-intersection model (9IM) by Egenhofer and Herring (1991), the Voronoi-based 9-intersection model (V9I) by Chen *et al*., 2001), the dimension extended model (DEM) by Clementini *et al*. (1993), the calculus-based model (CBM) by Clementini and Di Felice (1994) and their combinations (e.g. the dimension extended 4-intersection model -- DE-4IM).

	4IM	9IM	V9I	DE-4IM	DE-9IM	CBM	Guo's method	Our method
Numbers of line-region relations	11	19	13	17	31	31	97	16
Are topological components of a line involved?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No

Table 1. Comparison of various description methods

Through the analysis of literature, it can be found that that the numbers of topological relations differentiated by these models can only indicate the capacities of the models but nothing about a complete spectrum of all possible topological relations. For example, there should be infinite number of topological relations for a line and a region. To overcome the deficiency of these general models, efforts have also been made to develop dedicated models to specific types of features, e.g. line-line relations (e.g. Clementini and Di Felice, 1998; Li and Deng, 2006) and region-region relations (e.g. Egenhofer and Franzosa, 1995; Deng *et al*., 2007). However, not much work has been done for line-region relations. Indeed, this paper is devoted to the topological relations between a line and a region in a two-dimensional vector space (IR^2).

The remainder of this paper is structured as follows: Section 2 describes a new strategy (i.e. decomposition and combination) on the hierarchical representation of line-region topological relations. By decomposition, a set of basic relations can be obtained for a complex line-region relation. Section 3 discusses the representation of basic relations in a hierarchy. The combination of basic relations into compound relation and its hierarchical representation are further discussed in Section 4. A practical example is provided in Section 5. Section 6 summarizes the major findings and proposes the future work.

2. A STRATEGY FOR HIERARCHICAL REPRESENTATION OF LINE-REGION TOPOLOGICAL RELATIONS

A line-region topological relation may be very simple (e.g. Figure 1a), also being very complex (e.g. Figure 1b). As a result it is difficult to find a simple model to completely describe all the topological details between a line and a region in IR^2.

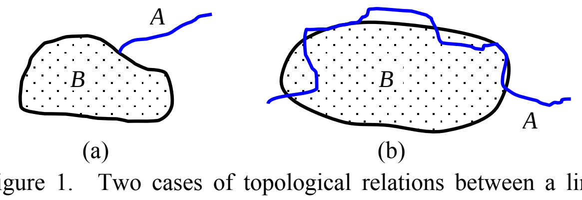


Figure 1. Two cases of topological relations between a line and a region, (a) a simple case with one intersection and (b) a complex case with six intersections

With a hierarchical approach, all the possible topological relations for a line and a region are first of all divided into two levels, i.e. *basic* and *compound* relations. A basic relation is referred as that with the number of intersections being less than or equal to one, i.e. *disjoint* or single-intersection relation. A compound relation is referred as that with the number of intersections being larger than one, also called multi-intersection relation. Therefore, a *compound* topological relation can be decomposed into and described by a combination of a finite number of basic relations. In other words, a *joint* line-region topological relation can be described by an ordered set of finite number of basic relation(s). In Figure 1(b), for instance, the topological relations between *A* and *B* can be decomposed into six basic relations.

These basic relations could then be represented in a hierarchical approach. Such a hierarchical representation will be discussed in next section. By hierarchy, it means that different levels of invariants are employed for the identification of line-region relations with different topological details. The hierarchical representation serves as a base for the representation of compound relations, which will be discussed in Section 4.

3. REPRESENTATION OF BASIC LINE-REGION RELATIONS IN A HIERARCHY

In this section, basic relations between a line and a region are represented at two levels, i.e. coarse level and detailed level, so they are also called coarse and detailed representations.

3.1 A Coarse Representation from the Point of View of Human Spatial Cognition

At the coarse level, basic relations may be roughly classified into six kinds, namely, *disjoint*, *meet*, *cross*, *covered-by*, *contained-by* and *on-boundary*, as shown in Figure 2. These six kinds of relations can be organized into a hierarchy from the point of view of human spatial cognition.

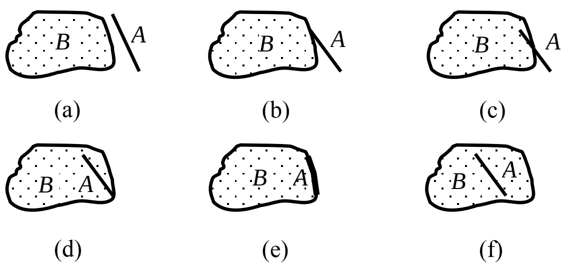


Figure 2. Six kinds of topological relations between a line and a region at the coarse level, (a) *disjoint*, (b) *meet*, (c) *cross*, (d) *covered-by*, (e) *on-boundary*, and (f) *contained-by*

For a line (*A*) and a region (*B*), the first concern for their relations is whether or not there is a connection (or linkage) between them. This can be clarified by the intersection between *A* and *B*'s boundary (i.e. $A \cap \partial B$). If the line *A* is connected with the region *B*, the next concern is whether or not the line belongs to the region. In this case, the difference operation (i.e. $A - B$) can be used to answer such a question. If the line does belong to the region, it can be further differentiated as *on-boundary* and *covered-by* by means of the intersection operation between *A* and the *B*'s interior (i.e. $A \cap B^o$). Otherwise it must be either *meet* or *cross* which can also be differentiated by $A \cap B^o$. A hierarchical decision process is shown in Figure 3.

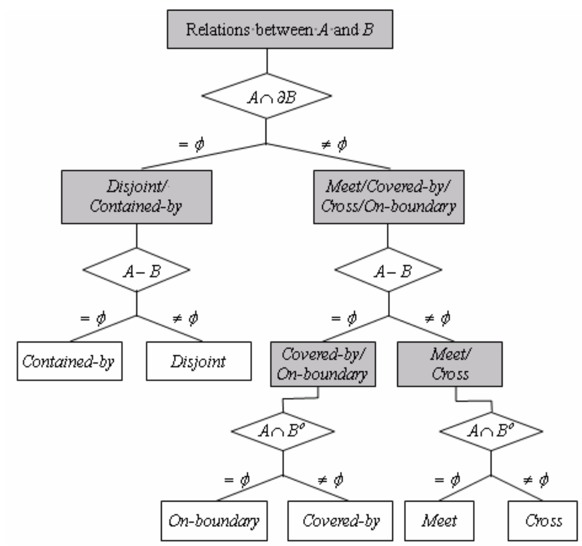


Figure 3. Hierarchical decision tree of topological relations between a line and a region

3.2 A Detailed Representation by Invariants of Dimension and Local Order

Through an analysis, one can find that the three kinds of topological relations (i.e. *meet*, *cross*, and *covered-by*) can be further differentiated by the invariants of dimension and local order. For instance, *meet* relation can be distinguished into 0- and 1-D cases by dimension invariant. Further, 16 basic relations can be identified at the detailed level, as shown in column 2 of Figure 4.

The *meet*, *cross* and *covered-by* have several cases respectively. First, *meet* can be classified into two groups in terms of dimension, namely, (I) 0-D *meet*: *end-meet* and *mid-meet*; (II) 1-D *meet*: *head-meet*, *belly-meet*, and *tail-meet*. And then, the local order of intersection point is used. Specifically speaking, the local order of an intersection point p_i, denoted by $Lo(p_i)$, can be defined to be the order of the intersecting lines within a very small circle centered at p_i. This small circle is equivalent to the concept of neighborhood, and therefore the local order is also an invariant. In this way, the local order of p_b in Figure 4(b) is $Lo(p_b) = <B; A; B>$, and that of p_c in Figure 4(c) can be represented as $Lo(p_c) = <B; A; A; B>$. The three types of 1-D *meet* (i.e. Figures 4(d), (e) and (f)) can be also differentiated by local order. Similarly, *cross* can be subdivided into *point-cross*, *in-cross* and *out-cross*, and *covered-by* into *head-meet-covered-by* (HM-covered-by), *tail-meet-covered-by* (TM-covered-by), *belly-meet-covered-by* (BM-covered-by), *end-meet-covered-by* (EM-covered-by) and *mid-meet-covered-by* (MT-covered-by), as shown in Figure 4.

Further, 13 of these sixteen relations together form a basis for combinational description of a compound relation, with the exception of *disjoint*, *on-boundary* and *contained-by*. The 13 basic relations are thus termed as compoundable relations.

Coarse representation	Detailed representation	Local order
disjoint	(a) disjoint	No
meet	(b) end-meet	$Lo(p_b) = <B; A; B>$
	(c) mid-meet	$Lo(p_c) = <B; A; A; B>$
	(d) head-meet	$Lo(p_{d1}) = <B; A \cap B>$ $Lo(p_{d2}) = <A \cap B; A; B>$
	(e) belly-meet	$Lo(p_{e1}) = <B; A; A \cap B>$ $Lo(p_{e2}) = <A \cap B; A; B>$
	(f) tail-meet	$Lo(p_{f1}) = <B; A; A \cap B>$ $Lo(p_{f2}) = <A \cap B; B>$
on-boundary	(g) on-boundary	$Lo(p_{g1}) = <B; A \cap B>$ $Lo(p_{g2}) = <A \cap B; B>$
cross	(h) point-cross	$Lo(p_h) = <B; A; B; A>$
	(i) in-cross	$Lo(p_{i1}) = <A; B; A \cap B>$ $Lo(p_{i2}) = <A \cap B; A; B>$
	(j) out-cross	$Lo(p_{j1}) = <B; A; A \cap B>$ $Lo(p_{j2}) = <A \cap B; B; A>$
covered-by	(k) HM-covered-by	$Lo(p_{k1}) = <B; A \cap B>$ $Lo(p_{k2}) = <A \cap B; B; A>$
	(l) TM-covered-by	$Lo(p_{l1}) = <A; B; A \cap B>$ $Lo(p_{l2}) = <A \cap B; B>$
	(m) BM-covered-by	$Lo(p_{m1}) = <A; B; A \cap B>$ $Lo(p_{m2}) = <A \cap B; B; A>$
	(n) EM-covered-by	$Lo(p_n) = <A; B; B>$
	(o) MT-covered-by	$Lo(p_o) = <B; B; A; A>$
contained-by	(p) contained-by	No

Figure 4. Sixteen basic line-region topological relations and their differentiations

3.3 A Conceptual Neighborhood Graph of Basic Relations at Two Levels

From Figure 4 it can be noted that some basic relations are more closely related than others. For example, *disjoint* relation will possibly first be transformed into *meet* or *on-boundary* relations by moving the line or both line and region. Then other basic relations like *cross* and *covered-by* may be obtained by further moving the line or both line and region. The closeness of these basic relations can be arranged into a graph, called conceptual neighborhood graph here, as shown in Figure 5. In Figure 5(a), the neighborhood graph of six kinds of basic relations at the coarse level is represented, and a comprehensive neighborhood graph of all 16 basic relations at the detailed level can be obtained by combining Figure 5(a) with Figures 5(b), (c) and (d). From this neighborhood graph, one can see clearly a transformation order among these basic relations, which is very useful to predict the most likely relations at the next moment in spatio-temporal reasoning (Egenhofer and Al-Taha, 1992) and similarity assessment in multiple representations (Egenhofer and Clementini, 1994).

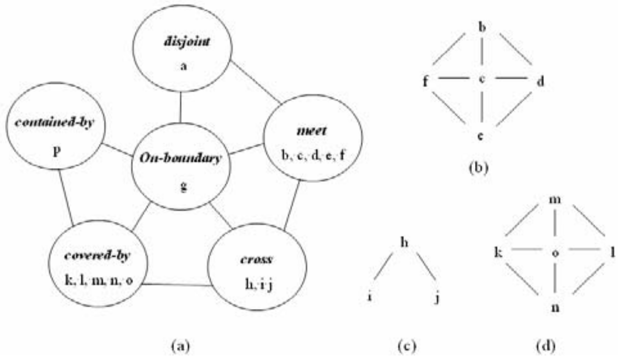


Figure 5. A comprehensive conceptual neighborhood graph of basic topological relations, where (a) the neighborhood graph of six kinds of relations at the coarse level, (b), (c) and (d) corresponding to the neighborhood graphs of *meet*, *cross* and *covered-by* relations, respectively.

4. REPRESENTATION OF COMPOUND LINE-REGION RELATIONS IN A HIERARCHY

4.1 A Categorization of Compound Relations

As mentioned in Section 2, a compound line-region relation is such a relation that the number of intersections between the line and the region boundary is larger than one. A compound relation can be formed with a combination of two or more basic relations, more precisely compoundable basic relations. According to its compositions, three kinds of compound line-region relations are differentiated as follows:

- Compound *meet* relation: composed of *meet* relations only.
- Compound *covered-by* relation: composed of *covered-by* relations only.
- Compound *cross* relation: composed of *cross* and possible compoundable relations.

From above categorization, four algebraic operations among these three kinds of compound relations can be defined as follows:

(i) *meet* + *meet* = *meet*
(ii) *covered-by* + *covered-by* = *covered-by*
(iii) *cross* + *cross* = *cross*
(iv) *cross* + *other* = *cross*

Therefore, these three kinds of compound relations can be regarded as the extensions of corresponding basic relations. Combined with the categorization of basic relations at the coarse level, all the line-region topological relations in IR^2 may be uniformly classified into six kinds, including *disjoint*, *meet*, *on-boundary*, *cross*, *covered-by* and *contained-by*.

4.2 Hierarchy of Topological Invariants Based upon Set Theory

It can be noted here that a compound relation is determined by the number and type of intersections between the line and the region boundary. A compound relation can be considered as an ordered set with its elements being basic relations.

For a set, three levels of topological information can be defined, namely, set level, element level, and an integrated level (with consideration of the information at both the set level and the element level).

- At the set level, the dimension and separation number of the intersections between the line and the region boundary (abbreviated as *LRBIS*) are utilized;
- At the element level, type of intersection in the *LRBIS* is considered, and
- At the integrated level, a sequence of intersection types is defined.

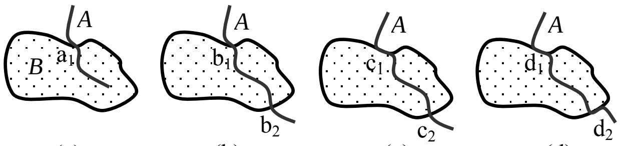
Indeed, the measures at these three levels are topological invariants. In the following, these three invariants are respectively utilized to differentiate line-region topological relations with different levels of details.

4.2.1 Separation Number and Dimension of the *LRBIS* at the Set Level: Separation number can be measured by the number of parts of a line separated by the intersections (with region boundary). For example, the separation number of the *LRBIS* in Figure 6 is equal to 1 in (a), 2 in (b), (c) and (d), respectively. In other words, the topological relation between *A* and *B* in Figure 6(a) is different from those in Figures 6(b), (c) and (d). In practice, the separation number invariant can be used to answer such queries as "how many times does a line (e.g. a river) pass through a region (e.g. a city)".

The dimension of a set *S*, denoted by dim(*S*), may be defined as the maxima of the dimensions of its elements (Clementini and Di Felice, 1998), i.e.

$$\dim(S) = \max\{\dim(s_1), \dim(s_2), \ldots, \dim(s_n)\} \quad (1)$$

By using Equation 1, the dimension of the *LBRIS* is 0 in Figures 7(a) and (b), 1 in (c) and (d), respectively. In practice, the dimension invariant may be used to answer such queries as "whether a line (e.g. a river) acts as the boundary of a region (e.g. a county)".



(a) (b) (c) (d)

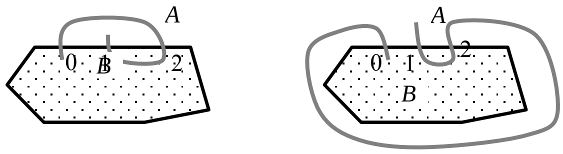
Figure 6. Separation number and dimension of the intersection set between a line (*A*) and a region (*B*), where (a) $A \cap \partial B = \{a_1\}$, (b) $A \cap \partial B = \{b_1, b_2\}$, (c) $A \cap \partial B = \{c_1, c_2\}$, and (d) $A \cap \partial B = \{d_1, d_2\}$.

4.2.2 Intersection Type in the *LRBIS* at the Element Level: Apparently, if one knows all types of intersections, one can derive the separation number and dimension. The former is the number of all the intersections, and the latter can be obtained by Equation 1. In Figure 6(c), for instance, two intersections (i.e. c_1 and c_2) belong to types (h) and (j) according to Figure 4, respectively. Therefore, it is not difficult to compute and obtain the separation number and dimension as 2 and 1. However, the reverse is not applicable. Figures 6(c) and (d) are such examples. The relations between *A* and *B* in Figures 6(c) and (d) can be differentiated by the intersection type invariant. As a result, intersection type at the element level can provide more topological details than the separation number and dimension at the set level. It means that the description at the element level is deeper than that at the set level. In practice, one can determine that a river passes through a city or that a river passes by the city according to the type of intersections between them.

4.2.3 Sequence of Intersection Types in the *LRBIS* at the Integrated Level: At the integrated level, individual intersection types and the orders which are defined by all intersection types in the *LRBIS* are considered together. Here, the combination of them is called the sequence of intersection types. Since the boundary of a region is a closed line, the sequence of intersection types in the *LRBIS* can be defined similar to those for line-line relations in the literature (Li and Deng, 2006). It includes the orders of intersection types, of loop types, and/or linkage relations. Figures 7 to 9 illustrate the needs of these individual orders. Compared with the sequence of intersection types for line-line relations, the order of characteristic points is not included. This is mainly because the interior and boundary of a region are differentiated in the representation of line-region relations.

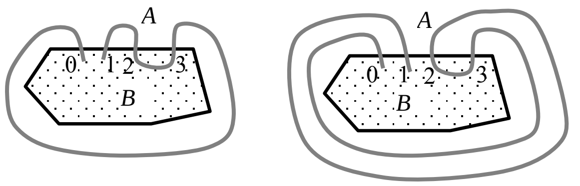
(a) (b)

Figure 7. Need of the order of intersection types in the *LRBIS* to differentiate the compound line-region relations, where (a) *Oet*(*A*) = < 0(h), 1(j), 2(h) >, and (b) *Oet*(*A*) = < 0(j), 1(h), 2(h)



(a) (b)

Figure 8. Need of the order of loop types for the compound line-region topological relations, where (a) *Oet*(*A*) = < 0(h), 2(h), 1(h) >, *Olt*(*A*) = < *ml*, *pl* >, and (b) *Oet*(*A*) = < 0(h), 2(h), 1(h) >, *Olt*(*A*) = < *pl*, *pl* >



(a) (b)

Figure 9. Need of the order of linkage relations between consecutive loops for topological relations of spatial configurations, where (a) *Oet*(*A*) = < 0(h), 3(h), 2(h), 1(h) >, *Olt*(*A*) = < *pl*, *pl*, *pl* >, *Olr*(*A*) = < *m*, *m* >, and (b) *Oet*(*A*) = < 0(h), 3(h), 2(h), 1(h) >, *Olt*(*A*) = < *pl*, *pl*, *pl* >, *Olr*(*A*) = < *m*, *c*>

5. AN EXAMPLE FOR HIERARCHICAL ANALYSIS OF LINE-REGION TOPOLOGICAL RELATIONS

As mentioned in the introduction, topological relations are very useful for spatial analysis, e.g. change detection, assessment of scene similarity, automatic detection of spatial inconsistency. Indeed, topological change indicates geometric modification of the involved spatial objects, but it is not true for vice versa. This section is devoted to illustrate the practical usefulness of the hierarchical approach for the description and determination of the topological relations between a line and a region by a case study of change detection.

In this case study, two data layers (i.e. river and an administrative boundary layer) are selected. Figure 10 (a) and (b) represent the local map of a river (as line *A*) and a county (as region *B*) for a same area in 1998 and in 2003, respectively. One can find the changes in geometry of the river and the county, which may be caused by data uncertainty due to measurement error, acquisition means and/or cartographic scale, or caused by a real change of the location and shape of the river in reality with time. Such a geometric change leads to changes in topological relations, which are marked by broken circles in Figure 10(b).

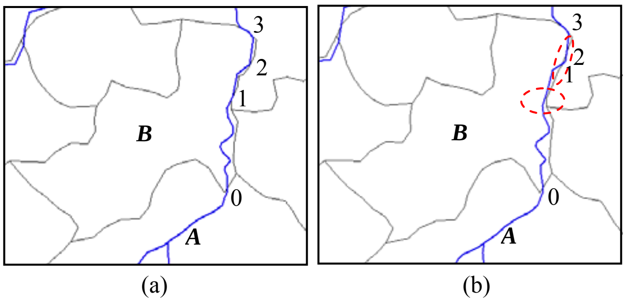


Figure 10. A practical example for hierarchical analysis of line-region topological relations in support of change detection

At first, the topological relations between the river and the county can be determined by a hierarchical decision tree (in Figure 3) as compound *cross* in both (a) and (b), where line algorithm (e.g. intersection of lines) and polygon algorithm (e.g. point-in-polygon analysis) are involved in the intersection and difference operations. However, such descriptions only indicates the river passing through the county but are not capable of telling the topological difference between (a) and (b), needless to say the topological inconsistency.

At the set level, numbers of the intersections between the river and the county in Figures 10(a) and (b) are both four, labeled by numbers such as 0, 1, 2 and 3, respectively. According to Equation (1), the dimension of *LRBIS* in (a) and (b) is both equal to one, indicating that part of the river acts as the boundary of the county. In this case, there is still unable to detect topological change between (a) and (b) by a comparison of both separation number and dimension of the *LRBIS*.

Further, at the element level, the four intersections (labeled by 0, 1, 2 and 3) in the *LRBIS* correspond to types (h), (m), (o) and (j) in Figure 10(a), and types (h), (h), (h) and (j) in Figure 10(b). From the description results, the end user(s) can see that topological changes happen between the river and the county in 2003 compared with their relation in 1998. According to the types of the intersections in the *LRBIS*, the end user(s) also can know the times of the river passing through the county, as indicated by the numbers of basic *cross* relations (e.g. types (h), (j) in this example). In Figure 10, for instance, the river passes through the county 2 times in (a), while four times in (b).

The last is to utilize the sequence of intersection types to describe the topological relations between the river and the county, which are respectively represented as

(a) $Oet(A, B) = [\ 0(h), 1(m), 2(o), 3(j)\ ]$, and
(b) $Oet(A, B) = [\ 0(h), 1(h), 2(h), 3(j)\ ]$.

This information is very important to further deal with topological inconsistency. On the one hand, the inconsistency can be detected by using the set of ordered intersection types. For instance, it can be determined that a part of the river is not in the county within some section according to the description result of (b), as may have contradiction with the reality. Here the inconsistency is caused due to positional uncertainty of different sources of spatial data. On the other hand, it can be used to resolve the inconsistency. Taking displacement as example, the set of ordered intersection types can be used to determine the direction of displacement. Indeed, one can displace parts of vertices of the river in between intersections 1 and 2 into the county to remove these two intersections.

6. CONCLUSIONS

In this paper, a hierarchical approach is proposed for the topological relations between a line and a region in IR^2. The topological relations for a line and a region are classified into basic relations and compound relations. Basic relations are further classified at two levels: coarse level and detailed level. Compound relations are classified at three levels: set level, element level and integrated level. At each level, topological invariants are developed based on the intersections between line and region boundary. With these invariants developed, compound line-region topological relations are differentiated to meet the needs of topological information at various levels. A practical example is given for the illustration of the approach presented.

This paper has the contributions in the following aspects:

- Topological relations between a line and a region in IR^2 may be classified into six kinds at a coarse level, i.e. *disjoint*, *meet*, *on-boundary*, *covered-by*, *contained-by* and *cross*. It is further classified into a total of sixteen basic relations at a detailed level, and thirteen of them form the basis of compound line-region relations.
- Complexity of line-region topological relations is dependent upon the identification of the intersection set between a line and the boundary of a region.
- Hierarchical topological invariants are developed based upon set theory, which are utilized to differentiate compound line-region relation in a hierarchy with different levels of topological details. By hierarchy, only some of the invariants need to be computed to describe the topological relations which are required in practical applications.
- The approach used in this study is directly based on the line objects themselves, instead of its topological components (a detailed comparison is listed in Table 1). Therefore, there is no such problem as the inadequacy of adopting the boundary definition in IR^1 into IR^2, as argued by Li et al. (2000).

It has been mentioned in the example that spatial inconsistencies occur with a joint analysis of multi-sources and/or multi-temporal spatial data. Such inconsistency is often embodied as the contradiction of multiple types of spatial relations, e.g. topological, directional, and distance. Therefore, the next step of work is to develop an integrated approach to represent various spatial relations between spatial objects for an automated detection and resolution of spatial inconsistency between them.

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