FUZZY ANALYTICAL HIERARCHY PROCESS IN GIS APPLICATION

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Commission II, WG II/4

KEY WORDS: Multicriteria Decision Making, Fuzzy Analytical Hierarchy Process, Fuzzy Extent Analysis, α-cut Based Method

ABSTRACT:

Analytical Hierarchy Process (AHP), as a multiple criteria decision making tools especially in the problems with spatial nature or GIS-based. In addition this study treats the steps AHP, its manner to apply and its weaknesses and strengths and ultimately the fuzzy modified Analytical Hierarchy Process (FAHP) which is proposed after that the concepts like of fuzziness, uncertainty and vagueness was broadly posed in expert's decision making. In the last part of this paper the chang's Fuzzy Extent Analysis method and α-cut based method on fuzzy AHP is described to obtain a crisp priority vector from a triangular fuzzy comparison matrix.

1. INTRODUCTION

Some spatial planning or spatial problems like site selection can be considered as a multiple criteria decision making or multiple MCDM problems involve a set of alternatives that are evaluated on the basis of conflicting and incommensurate criteria (Malczewski, 1999). GIS-based multicriteria analysis is used in a wide range of decision and management situations like Environment planning and ecology management, Urban and regional planning, Hydrology and water resources, Forestry, Transportation, Agriculture, Natural hazard management, Health care resource allocation and etc. In GIS technology, usually the alternatives are a collection of point, line and aerial objects, attached to which are criterion values (criterion map). Fig. 1 illustrated a schema of spatial multicriteria decision analysis.

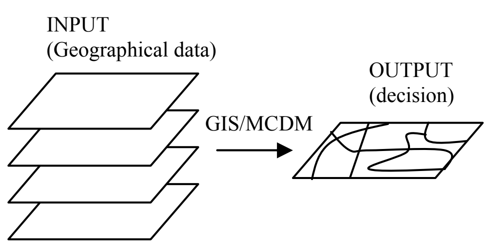

Figure 1. spatial multicriteria decision analysis

MADM as a class of MCDM is the approach dealing with the ranking and selection of one or more sites from the alternatives. Some important characteristics of MADM are having restricted set of alternatives and explicitly defined set of alternatives, requiring a priori information on the decision maker's preferences and being outcome oriented (Chakhar, 2003). In MADM, the aim is to rank a finite number of alternatives with respect to a finite number of attributes. In solving a MADM, one needs to know the importance or weights of the not equally important attributes and also the evaluations of the alternatives with respect to the attributes. There have been different methods on MADM and the most known is Analytical Hierarchy Process (AHP) which especially is based on pairwise comparisons on a ratio scale (Saaty, 1980). According to some AHP limitations the fuzzy modification of AHP (FAHP) was then posed that is the subject of this study. This paper shows implementation of FAHP. The remainder of this paper is structured as follows: Section 2 proposes the definition and application of AHP and the six steps of implementation. Section 3 introduces Fuzzy AHP and its necessity and related concepts like alpha cut. And in section 4 conclusion and future work are discussed.

2. ANALYTICAL HIRARCHY PROCESS (AHP)

AHP is a multi-criteria decision method that uses hierarchical structures to represent a problem and then develop priorities for alternatives based on the judgment of the user (Saaty, 1980). The AHP procedure involves six essential steps (Lee et al., 2008):

1. Define the unstructured problem
2. Developing the AHP hierarchy
3. pairwise comparison
4. Estimate the relative weights
5. Check the consistency
6. Obtain the overall rating

2.1. Define the unstructured problem

In this step the unstructured problem and their characters should be recognized and the objectives and outcomes stated clearly.

2.2. Developing the AHP hierarchy

The first step in the AHP procedure is to decompose the decision problem into a hierarchy that consists of the most important elements of the decision problem (Boroushaki and Malczewski, 2008). In this step the complex problem is decomposed into a hierarchical structure with decision elements

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(objective, attributes i.e. criterion map layer and alternatives). Fig.2 represents this structure.

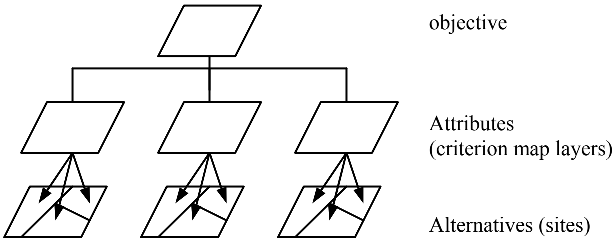

Figure 2.Hierarchical structure of decision problem

2.3. pairwise comparison

for each element of the hierarchy structure all the associated elements in low hierarchy are compared in paiwise comparison matrices as follows:

$$A = \begin{bmatrix} 1 & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & 1 & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & 1 \end{bmatrix} \qquad (1)$$

Where A = comparison pairwise matrix,
w_1 = wheight of element 1,
w_2 = wheight of element 2,
w_n = wheight of element n.

in order to determine the relative preferences for two elements of the hierarchy in matrix A, an underlying semantical scale is employs with values from 1 to 9 to rate (Table 1).

Preferences expressed in numeric variables	Preferences expressed in linguistic variables
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2,4,6,8	Intermediate values between adjacent scale values

Table 1. scales for pairwise comparison (Saaty, 1980)

2.4. Estimate the relative weights

Some methods like eigenvalue method are used to calculate the relative weights of elements in each pairwise comparison matrix. The relative weights (W) of matrix A is obtained from following equation:

$$(A - \lambda_{\max} I) \times W = 0 \qquad (2)$$

Where λmax = the biggest eigenvalue of matrix A,
I = unit matrix.

2.5. Check the consistency

In this step the consistency property of matrices is checked to ensure that the judgments of decision makers are consistent. For this end some preparameter is needed. Consistency Index (*CI*) is calculated as:

$$CI = \frac{\lambda_{\max} - n}{n - 1} \qquad (3)$$

The consistency index of a randomly generated reciprocal matrix shall be called to the random index (*RI*), with reciprocals forced. An average *RI* for the matrices of order 1–15 was generated by using a sample size of 100 (Nobre et al., 1999). The table of random indexes of the matrices of order 1–15 can be seen in Saaty (1980). The last ratio that has to be calculated is *CR* (Consistency Ratio). Generally, if *CR* is less than 0.1, the judgments are consistent, so the derived weights can be used. The formulation of *CR* is:

$$CR = \frac{CI}{RI} \qquad (4)$$

2.6. Obtain the overall rating

In last step the relative weights of decision elements are aggregated to obtain an overall rating for the alternatives as follows:

$$W_i^s = \sum_{j=1}^{j=m} w_{ij}^s w_j^a \qquad , \mathrm{i} = 1, \ldots, \mathrm{n} \qquad (5)$$

Where W_i^s = total weight of site i,
w_{ij}^s = weight of alternative (site) i associated to attribute (map layer) j,
w_j^a = weight of attribute j,
m = number of attribute,
n= number of site.

3. FUZZY ANALYTICAL HIERARCHY PROCESS (FAHP)

In spite of popularity of AHP, this method is often criticized for its inability to adequately handle the inherent uncertainty and imprecision associated with the mapping of the decision-maker’s perception to exact numbers (Deng, 1999). Since fuzziness and vagueness are common characteristics in many decision-making problems, a fuzzy AHP (FAHP) method should be able to tolerate vagueness or ambiguity (Mikhailov and Tsvetinov, 2004). In other word the conventional AHP approach may not fully reflect a style of human thinking because the decision makers usually feel more confident to give interval judgments rather than expressing their judgments in the form of single numeric values and so FAHP is capable of capturing a human's appraisal of ambiguity when complex multi-attribute decision making problems are considered (Erensal et al., 2006). This ability comes to exist when the crisp judgments transformed into fuzzy judgments.

Number of criteria	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

Table 2. Random inconsistency indices (Saaty, 1980)

Zadeh (1965) published his work Fuzzy Sets, which described the mathematics of fuzzy set theory. This theory, which was a generalization of classic set theory, allowed the membership functions to operate over the range of real numbers [0, 1]. The main characteristic of fuzziness is the grouping of individuals into classes that do not have sharply defined boundaries (Hansen, 2005). The uncertain comparison judgment can be represented by the fuzzy number. A triangular fuzzy number is the special class of fuzzy number whose membership defined by three real numbers, expressed as (l, m, u). The triangular fuzzy numbers is represented as follows (Fig.3):

$$\mu_A(x) = \begin{cases} (x-l)/(m-l), & l \le x \le m \\ (u-x)/(u-m), & m \le x \le u \\ 0 & otherwise \end{cases} \tag{6}$$

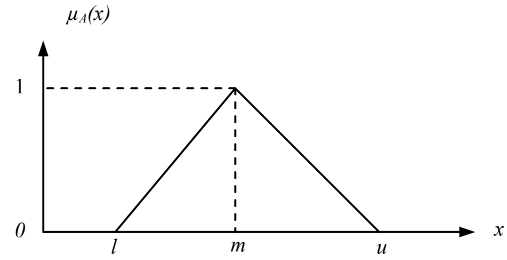

Figure 3. Fuzzy triangular number

In order to construct pairwise comparison of alternatives under each criterion or about criteria, like that was said for traditional AHP, a triangular fuzzy comparison matrix is defined as follows:

$$\widetilde{A} = (\widetilde{a}_{ij})_{n\times n} = \begin{bmatrix} (1,1,1) & (l_{12},m_{12},u_{12}) & \cdots & (l_{1n},m_{1n},u_{1n}) \\ (l_{21},m_{21},u_{21}) & (1,1,1) & \cdots & (l_{2n},m_{2n},u_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (l_{n1},m_{n1},u_{n1}) & (l_{n2},m_{n2},u_{n2}) & \cdots & (1,1,1) \end{bmatrix}$$

Where $\widetilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij}) = \widetilde{a}_{ij}^{-1} = (1/u_{ji}, 1/m_{ji}, 1/l_{ji})$

$$for \quad i, j, = 1, \ldots, n \quad and \quad i \ne j. \tag{7}$$

Total weighs and preferences of alternatives can be acquired from different method. Two approaches will be posed in resumption.

3.1. Fuzzy Extent Analysis

Different methods have been proposed in the literatures that one of most known of them is Fuzzy Extent Analysis proposed by Chang (1996). The steps of chang's extent analysis can be summarized as follows:

First step: computing the normalized value of row sums (i.e. fuzzy synthetic extent) by fuzzy arithmetic operations:

$$\widetilde{S}_i = \sum_{j=1}^{n} \widetilde{a}_{ij} \otimes \left[\sum_{k=1}^{n} \sum_{j=1}^{n} \widetilde{a}_{kj} \right]^{-1} \tag{8}$$

Where $\otimes$ denotes the extended multiplication of two fuzzy numbers.

Second step: computing the degree of possibility of $\widetilde{S}_i \ge \widetilde{S}_j$ by following equation:

$$V(\widetilde{S}_i \ge \widetilde{S}_j) = \sup_{y \ge x} [\min(\widetilde{S}_j(x), \widetilde{S}_i(y))] \tag{9}$$

which can be equivalently expressed as,

$$V(\widetilde{S}_i \ge \widetilde{S}_j) = \begin{cases} 1 & m_i \ge m_j \\ \dfrac{u_i - l_j}{(u_i - m_i) + (m_j - l_j)} & l_j \le u_i \quad i, j = 1, \ldots, n; j \ne i \\ 0 & otherwise \end{cases}$$

Where $\widetilde{S}_i = (l_i, m_i, u_i)$ and $\widetilde{S}_j = (l_j, m_j, u_j)$ (10)

Fig.4 illustrates this degree of possibility.

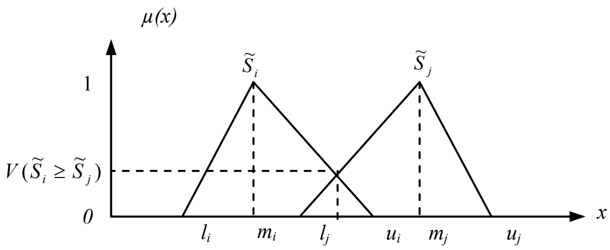

Figure 4.The degree of possibility of $V(\widetilde{S}_i \ge \widetilde{S}_j)$

Third step: calculating the degree of possibility of $\widetilde{S}_i$ to be greater than all the other (n-1) convex fuzzy numbers $\widetilde{S}_j$ by:

$$V(\widetilde{S}_i \ge \widetilde{S}_j \mid j = 1, \ldots, n; j \ne i) = \min_{j \in (1,\ldots,n) j \ne i} V(\widetilde{S}_i \ge \widetilde{S}_j), \quad i = 1, \ldots, n \tag{11}$$

Fourth step: defining the priority vector $W = (w_1, ..., w_n)^T$ of the fuzzy comparison matrix $\widetilde{A}$ as:

$$w_i = \frac{V(\widetilde{S}_i \ge \widetilde{S}_j \mid j = 1, \ldots, n; j \ne i)}{\sum_{k=1}^{n} V(\widetilde{S}_k \ge \widetilde{S}_j \mid j = 1, \ldots, n; j \ne k)}, \quad i = 1, \ldots, n \tag{12}$$

3.2. α-cut-based method

In this method fuzzy extent analysis is applied to get the fuzzy weights or performance matrix for both alternatives under each criteria context and criteria. After that, a fuzzy weighted sum performance matrix (P) for alternatives can thus be obtained by multiplying the fuzzy weight vector related to criteria with the decision matrix for alternatives under each criteria and summing up obtained vectors.

$$\tilde{p} = \begin{pmatrix} (l_1, m_1, u_1) \\ (l_2, m_2, u_2) \\ \vdots \\ (l_n, m_n, u_n) \end{pmatrix} \tag{13}$$

Where n = number of alternative

According to Wang (1997), in order to checking and comparing fuzzy number, α-cut-based method 1 stated that if let A and B be fuzzy numbers with α-cuts, $A_\alpha = [a_\alpha -, a_\alpha +]$ and $B_\alpha = [b_\alpha -, b_\alpha +]$. It says A is smaller than B, denoted by $A \le B$, if $a_\alpha - < b_\alpha -$ and $a_\alpha + < b_\alpha +$ for all α in the range of (0,1].

In next step the alpha cut analysis is applied to transform the total weighted performance matrices into interval performance matrices which is showed with αLeft and αRight for each alternative as follows:

$$\tilde{p}_\alpha = \begin{pmatrix} (\alpha\mathrm{left}_1, \alpha\mathrm{Right}_1) \\ (\alpha\mathrm{left}_2, \alpha\mathrm{Right}_2) \\ \vdots \\ (\alpha\mathrm{left}_n, \alpha\mathrm{Right}_n) \end{pmatrix} \tag{14}$$

$\alpha Left = [\alpha * (m - l)] + l$,
$\alpha Right = u - [\alpha * (u - m)]$

The alpha cut is to account for the uncertainty in the fuzzy range chosen. In this case, the decision maker expressed personal confidence about this range. The confidence value ranges between 0 and 1, from the least confidence to the most confidence.

Last step is devoted to convert interval matrices into crisp values. It is done by applying the Lambda function which represents the attitude of the decision maker that is maybe optimistic, moderate or pessimistic. Decision maker with optimistic attitude will take the maximum lambda; the moderate person will take the medium lambda and the pessimistic person will take the minimum lambda in the range of [0,1] as follows:

$$C_\lambda = \begin{pmatrix} C_{\lambda 1} \\ C_{\lambda 2} \\ \vdots \\ C_{\lambda n} \end{pmatrix}, \tag{15}$$

$C_\lambda = \lambda * \alpha Right + [(1 - \lambda) * \alpha Left]$,
Where C_λ = crisp value

These values should be normalized because of different scales.

4. CONCLUSION

In this paper the application of multicriteria decision making in spatial problems and GIS application was discussed and in resumption AHP as a most applicable tool in this context was introduced. Due to the disregarding of uncertainty in traditional AHP, the fuzzy form of AHP (i.e. FAHP) and two known approaches of FAHP means Fuzzy Extent Analysis and α-cut-based method were treated. According to somewhat which is said in this paper, the advantage of α-cut-based method is that the conclusion is less controversial and also the uncertainty and the different attitude of decision maker can be took into account in this method but the fuzzy extent analysis is more easy in computation.

REFERENCES:

Boroushaki, S., Malczewski, J., 2008. Implementing an extension of the analytical hierarchy process using ordered weighted averaging operators with fuzzy quantifiers in ArcGIS. Computers & Geosciences 34, pp. 399–410.

Chakhar, S., 2003. Enhancing Geographical Information Systems Capabilities with Multi-Criteria Evaluation Functions. Journal of Geographic Information and Decision Analysis 7(2), pp. 47 – 71.

Chang, D.Y., 1996. Applications of the extent analysis method on fuzzy AHP. European Journal of Operational Research 95 pp. 649–655.

Deng, H., 1999. Multicriteria analysis with fuzzy pairwise comparisons, Int. J. Approximate Reasoning 21, pp. 215–231.

Erensal, Y.C., Oncan, T., Demircan, M.L., 2006. Determining key capabilities in technology management using fuzzy analytic hierarchy process: A case study of Turkey. Information Sciences 176, pp. 2755–2770.

Hansen, H.S., 2005. GIS-based Multi-Criteria Analysis of Wind Farm Development. In: Hauska, H. & Tveite, H. (Eds.): ScanGIS'2005. Proceedings of the 10th Scandinavian Research Conference on Geographical Information Science, Stockholm, Sweden, Royal Institute of Technology, pp. 75-87.

Lee, A.H.I., Chen, W.C., Chang, C.J., 2008. A fuzzy AHP and BSC approach for evaluating performance of IT department in the manufacturing industry in Taiwan. Expert Systems with Applications 34, pp. 96–107.

Malczewski, J., 1999. GIS and Multicriteria Decision Analysis. Wiley & Sons INC, 395 pp.

Mikhailov, L., Tsvetinov, P., 2004, Evaluation of services using a fuzzy analytic hierarchy process. Applied Soft Computing 5, pp. 23–33.

Nobre, F.F., Trotta, L.T.F., Gomes, L.F.A.M., 1999. Multi-criteria decision making: An approach to setting priorities in health care. Symposium on statistical bases for public health decision making: from exploration to modeling, 18(23), pp. 3345 – 3354.

Saaty, T.L., 1980. The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation. McGraw-Hill, New York, NY, 437 pp.

Wang, L.X., 1997. A Course in Fuzzy Systems and Control. United States of America: Prentice-Hall, 448 PP.

Zadeh, L., 1965. Fuzzy sets. Information and Control, 8(3), pp. 338-353.