CLOSED-FORM SOLUTION OF SPACE RESECTION USING UNIT QUATERNION

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KEY WORDS: Photgrammetry, Algorithm, Unit Quaternion, Rotation Matrix, Space Resection, Exterior Orientation Parameters (EOPs)

ABSTRACT:

Determining exterior orientation parameters (EOPs) of image is an important task in photogrammetry, and various classical methods are in use. On consideration of some shortcomings in classical methods, a new algorithm based on quaternion is presented. The method uses unit quaternion to represent rotation. Based on the least square principle, the closed-form solution of unit quaternion representing relationship between object space and photo space is solved out first; then the rotation matrix is obtained, and angular elements are derived. Lastly coordinates of exposure station (perspective centre) is computed. The main advantages of the method are that initial values of EOPs need not be provided, and linearization for the collinearity equations is not involved in formula deduction, so iterative computing is avoided. At the same time, the method can directly deal with image with large oblique angle. The numerical experiments are done and results show the correctness and reliability of the method.

1. INTRODUCTION

Single photo space resection is an important task in photogrammetry, and its objective is to determine exterior orientation parameters (EOPs) of an aerial image, including three linear elements, i.e. coordinates of exposure station $\left(X_S, Y_S, Z_S\right)$ and three angular elements, i.e. the orientation omega, phi, kappa of image. Various classical methods are in use, for example, collinearity equation method (ZHU Zhao-guang, SUN Hu etc.2004), direct linear transformation method (ZHANG Zu-xun, ZHANG Jiang-qing, 1997), and method based on pyramid (GUAN Yun-lan, ZHOU Shi-jian etc. 2006). In these methods, some problems exist. Collinearity equation method need to provide initial values of EOPs; pyramid method need to provide linear elements of EOPs, When the initial values are not accurate or can not be obtained at all, results will be wrong or can not be solved. In both methods, collinearity equations need to be linearized, so iterative computation are needed. Using direct linear transformation method, we can not adopt all information of control points because of solving EOPs by six equations selected out of nine.

On consideration of these, a closed-form solution to space resection is presented, one that does not require iteration. The method uses unit quaternion to construct rotation matrix. Based on the least square principle, the closed-form solution of unit quaternion is solved out first. Then the rotation matrix is obtained, and angular elements are derived. Lastly coordinates of exposure station is computed. The main advantage of the closed-form solution is that linearization of collinearity equations is not required, so initial values of EOPs need not be provided and iterative computing is avoided. Thus improves the speed and accuracy of space resection. Another advantage is that using the method, we can directly deal with large inclination angle images which can not be processed in collinearity equation method.

2. OVERVIEW OF QUATERNION

2.1 Basics of quaternion

Quaternion is introduced by Hamilton in 1843. It can be regarded as a column vector containing four elements. A quaternion **q** can be represented as

$$\mathbf{q} = \mathrm{w} + \mathrm{x}\mathbf{i} + \mathrm{y}\mathbf{j} + \mathrm{z}\mathbf{k}$$

where w is a scalar, (x, y, z) is a vector. The relationships among **i, j, k** are

$$\begin{aligned} &\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1 \\ &\mathbf{i} * \mathbf{j} = \mathbf{k} = -\mathbf{j} * \mathbf{i} \\ &\mathbf{j} * \mathbf{k} = \mathbf{i} = -\mathbf{k} * \mathbf{j} \\ &\mathbf{k} * \mathbf{i} = \mathbf{j} = -\mathbf{i} * \mathbf{k} \end{aligned}$$

quaternion **q** can also be represented as （w, **V**）,where w is the scalar, and **V** is a vector consisting of (x, y, z). So using quaternion, a 3D vector **p** can be written as follows

$$\mathbf{P} = 0 + \mathrm{p}_1\mathbf{i} + \mathrm{p}_2\mathbf{j} + \mathrm{p}_3\mathbf{k} = (0, \mathrm{p})$$

a scalar a can be written as

$$\mathbf{a} = \mathrm{a} + 0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k} = (\mathrm{a}, \mathbf{0})$$

Given two quaternions $\mathbf{q}_1 = (\mathrm{s}_1, \mathbf{V}_1), \mathbf{q}_2 = (\mathrm{s}_2, \mathbf{V}_2)$, the fundamental rules of algorithm are

(1) addition

$$\mathbf{q}_1+\mathbf{q}_2 = s_1+s_2+\mathbf{V_1}+\mathbf{V_2}$$
$$= s_1+s_2+(x_1+x_2)\mathbf{i}+(y_1+y_2)\mathbf{j}+(z_1+z_2)\mathbf{k}$$

(2) multiplication

$$\mathbf{q}_1\mathbf{q}_2 = (s_1+\mathbf{V_1})(s_2+\mathbf{V_2})$$
$$= s_1s_2-\mathbf{V_1}\cdot\mathbf{V_2}+\mathbf{V_1}\times\mathbf{V_2}+s_1\mathbf{V_2}+s_2\mathbf{V_1}$$

where $\mathbf{V}_1\cdot\mathbf{V_2}$ is dot product, and $\mathbf{V_1}\times\mathbf{V_2}$ is cross product.

(3) norm

$$\mathrm{N}(\mathbf{q}) = \|\mathbf{q}\| = \mathrm{w}^2+\mathrm{x}^2+\mathrm{y}^2+\mathrm{z}^2$$

when N(**q**)=1， i.e. $w^2+x^2+y^2+z^2=1$, it is a unit quaternion.

(4) conjugate quaternions of **q**

$$\mathbf{q}^* = w - x\mathbf{i} - y\mathbf{j} - z\mathbf{k} = [w, -\mathbf{V}]$$

2.2 Using unit quaternion to represent 3D rotation

Given a vector **P**(X, Y, Z) to rotate around a unit quaternion **q**, the rotation can be represented as follows:

$$\mathrm{Rot}(\mathbf{P}) = \mathbf{qPq}^* \qquad (1)$$

in order to compute conveniently, we use matrix to represent multiplication of quaternions.

$$\mathbf{q_1q_2} = \begin{bmatrix} \mathrm{w}_1 & -\mathrm{x}_1 & -\mathrm{y}_1 & -\mathrm{z}_1 \\ \mathrm{x}_1 & \mathrm{w}_1 & -\mathrm{z}_1 & \mathrm{y}_1 \\ \mathrm{y}_1 & \mathrm{z}_1 & \mathrm{w}_1 & -\mathrm{x}_1 \\ \mathrm{z}_1 & -\mathrm{y}_1 & \mathrm{x}_1 & \mathrm{w}_1 \end{bmatrix}\mathbf{q_2} = \mathrm{C}(\mathbf{q_1})\mathbf{q_2} \qquad (2a)$$

or

$$\mathbf{q_2q_1} = \begin{bmatrix} \mathrm{w}_1 & -\mathrm{x}_1 & -\mathrm{y}_1 & -\mathrm{z}_1 \\ \mathrm{x}_1 & \mathrm{w}_1 & \mathrm{z}_1 & -\mathrm{y}_1 \\ \mathrm{y}_1 & -\mathrm{z}_1 & \mathrm{w}_1 & \mathrm{x}_1 \\ \mathrm{z}_1 & \mathrm{y}_1 & -\mathrm{x}_1 & \mathrm{w}_1 \end{bmatrix}\mathbf{q_2} = \overline{\mathrm{C}(\mathbf{q_1})}\mathbf{q_2} \qquad (2b)$$

Obviously, $C(\mathbf{q_1})$, $\overline{C(\mathbf{q}_1)}$ are both antisymmetric matrices. When $\mathbf{q_1}$ is a unit quaternion, we can easily verified that $\mathrm{C}(\mathbf{q}_1)$ and $\overline{\mathrm{C}(\mathbf{q_1})}$ are orthonormal matrices, and $|C(\mathbf{q_1})|=1$, $\left|\overline{C(\mathbf{q_1})}\right|=1$.

We also got

$$\mathrm{C}(\mathbf{q_1}^*) = \mathrm{C}(\mathbf{q_1})^{\mathrm{T}} \qquad (3)$$

From equations (2) and (3), we got

$$\mathrm{Rot}(\mathbf{P}) = \mathbf{qPq}^* = \mathrm{C}(\mathbf{q})\mathbf{Pq}^* = \mathrm{C}(\mathbf{q})\overline{\mathrm{C}(\mathbf{q}^*)}\mathbf{P}$$

Therefore, rotation based on unit quaternion **q** is

$$C(\mathbf{q})\overline{C(\mathbf{q}^*)} = \begin{bmatrix} w & -x & -y & -z \\ x & w & -z & y \\ y & z & w & -x \\ z & -y & x & w \end{bmatrix}\begin{bmatrix} w & x & y & z \\ -x & w & -z & y \\ -y & z & w & -x \\ -z & -y & x & w \end{bmatrix}$$
$$= \begin{bmatrix} w^2+x^2+y^2+z^2 & 0 & 0 & 0 \\ 0 & w^2+x^2-y^2-z^2 & 2xy-2wz & 2xz+2wy \\ 0 & 2xy+2wz & w^2-x^2+y^2-z^2 & 2yz-2wx \\ 0 & 2xz-2wy & 2yz+2wx & w^2-x^2-y^2+z^2 \end{bmatrix}$$

where lower right 3*3 matrix is the rotation matrix in 3D space, i.e.

$$\mathbf{R} = \begin{bmatrix} 1-2(\mathrm{y}^2+\mathrm{z}^2) & 2(\mathrm{xy}-\mathrm{wz}) & 2(\mathrm{wy}+\mathrm{xz}) \\ 2(\mathrm{xy}+\mathrm{wz}) & 1-2(\mathrm{x}^2+\mathrm{z}^2) & 2(\mathrm{yz}-\mathrm{wx}) \\ 2(\mathrm{xz}-\mathrm{wy}) & 2(\mathrm{yz}+\mathrm{wx}) & 1-2(\mathrm{x}^2+\mathrm{y}^2) \end{bmatrix} \qquad (4)$$

Because $\mathrm{C}(\mathbf{q})$ and $\overline{\mathrm{C}(\mathbf{q}^*)}$ are both orthonormal matrices, the following formula exists

$$\left(C(\mathbf{q})\overline{C(\mathbf{q}^*)}\right)^T C(\mathbf{q})\overline{C(\mathbf{q}^*)}$$
$$= \overline{C(\mathbf{q}^*)}^T C(\mathbf{q})^T C(\mathbf{q})\overline{C(\mathbf{q}^*)} = \mathbf{I}$$

In other words, $\mathrm{C}(\mathbf{q})\overline{\mathrm{C}(\mathbf{q}^*)}$ is also an orthonormal matrix. According to the arithmetic rules of orthonormal matrix, **R** must be orthonormal, and $|\mathbf{R}|=1$.

3. SPACE RESECTION BASED ON UNIT QUATERNIION

The whole idea of the new solution is: firstly compute distances between exposure station and control points, then using unit quaternion to represent rotation and solve the orientation omega, phi and kappa. Lastly, coordinates of exposure station (X_S, Y_S, Z_S) is computed.

3.1 Computation of distances between exposure station and control points

A, B, C, D are control points whose coordinates are known, a, b, c, d are corresponding image points, S is exposure station, see figure 1.

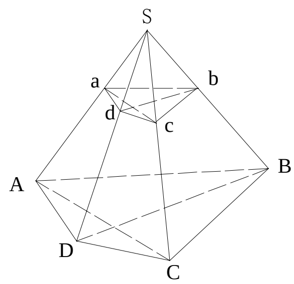

Figure 1 sketch map of space resection

From figure1, we know that any two control points and exposure station form a triangle. Take A, B for example, we got following equation based on law of cosines.

$$AB^2 = SA^2 + SB^2 - 2SA \cdot SB \cdot \cos\angle ASB$$

where

$$\cos\angle ASB = \cos\angle aSb = \frac{Sa^2 + Sb^2 - ab^2}{2Sa \cdot Sb}$$

For n control points, C_n^2 equations like that can be listed. Iterative method or Grafarend method can be used to compute distances (ZHANG Zu-xun, ZHANG Jiang-qing. 1997).

3.2 Calculation of exterior orientation parameters (EOPs)

There are six EOPs for a single image, including three linear elements and three angle elements. As we know, collinearity equation can be expressed as

$$\lambda_i \mathbf{R} \begin{bmatrix} x_i \\ y_i \\ -f \end{bmatrix} = \begin{bmatrix} X_i - X_S \\ Y_i - Y_S \\ Z_i - Z_S \end{bmatrix} \tag{5}$$

where **R=** rotation matrix, often expressed as

$$\mathbf{R} = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$$

$a_i, b_i, c_i\,(i = 1,2,3)$ =direction cosine
(X_i, Y_i, Z_i) =coordinates of control points
(x_i, y_i) = corresponding image coordinates
f = focal length
(X_S, Y_S, Z_S) = coordinates of exposure station
λ_i =ratio of distance of exposure station to control point and to its photo point, i.e.

$$\lambda_i = \frac{SI}{Si} = \frac{SI}{\sqrt{x_i^2 + y_i^2 + f^2}} \tag{6}$$

when the distance between exposure station to control point is computed, λ_i for each point is also known.
Equation (5) can be written as

$$\begin{bmatrix} \lambda_i x_i \\ \lambda_i y_i \\ -\lambda_i f \end{bmatrix} = \mathbf{R}^T \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} - \mathbf{R}^T \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix}$$

Let

$$\mathbf{s_i} = \begin{bmatrix} \lambda_i x_i \\ \lambda_i y_i \\ -\lambda_i f \end{bmatrix} \qquad \mathbf{p_i} = \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} \qquad \mathbf{T} = \mathbf{R}^T \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix} \tag{7}$$

Then

$$\mathbf{s_i} = \mathbf{R}^T \mathbf{p_i} - \mathbf{T}$$

On consideration of the error, we can get error equation

$$\mathbf{V_i} = \mathbf{R}^T \mathbf{p_i} - \mathbf{s_i} - \mathbf{T} \tag{8}$$

Using least-squares principle

$$\min \sum_i \mathbf{V_i}^T \mathbf{V_i} \tag{9}$$

Combination of Eq. (8) and (9), we got

$$\begin{aligned} \sum_i \mathbf{V_i}^T \mathbf{V_i} &= \sum_i \left(\mathbf{R}^T\mathbf{p_i} - \mathbf{s_i} - \mathbf{T}\right)^T \left(\mathbf{R}^T\mathbf{p_i} - \mathbf{s_i} - \mathbf{T}\right) \\ &= \sum_i \left(\mathbf{p_i}^T\mathbf{R} - \mathbf{s_i}^T - \mathbf{T}^T\right)\left(\mathbf{R}^T\mathbf{p_i} - \mathbf{s_i} - \mathbf{T}\right) \\ &= \sum_i \left(\mathbf{p_i}^T\mathbf{p_i} + \mathbf{s_i}^T\mathbf{s_i} - 2\mathbf{p_i}^T\mathbf{R}\mathbf{s_i} + 2\mathbf{s_i}^T\mathbf{T} - 2\mathbf{p_i}^T\mathbf{R}\mathbf{T} + \mathbf{T}^T\mathbf{T}\right) \end{aligned} \tag{10}$$

Put partial derivatives of (10) to **T** and let derivative be zero

$$\mathbf{T} = \frac{1}{n}\mathbf{R}^T \sum_i \mathbf{p_i} - \frac{1}{n}\sum_i \mathbf{s_i} \tag{11}$$

Where n is the number of control points.
Combination of Eq. (11) and (8)

$$\begin{aligned} \mathbf{V_i} &= \mathbf{R}^T\mathbf{p_i} - \mathbf{s_i} - \mathbf{T} = \mathbf{R}^T\mathbf{p_i} - \mathbf{s_i} - \frac{1}{n}\mathbf{R}^T\sum_i \mathbf{p_i} + \frac{1}{n}\sum_i \mathbf{s_i} \\ &= \mathbf{R}^T\left(\mathbf{p_i} - \frac{1}{n}\sum_i \mathbf{p_i}\right) - \left(\mathbf{s_i} - \frac{1}{n}\sum_i \mathbf{s_i}\right) \end{aligned}$$

Let

$$\mathbf{s_g} = \frac{1}{n}\sum_i \mathbf{s_i} \qquad \mathbf{p_g} = \frac{1}{n}\sum_i \mathbf{p_i}$$
$$\overline{\mathbf{p}}_\mathbf{i} = \mathbf{p_i} - \mathbf{p_g} \qquad \overline{\mathbf{s}}_\mathbf{i} = \mathbf{s_i} - \mathbf{s_g} \tag{12}$$

then

$$\mathbf{V_i} = \mathbf{R}^T\overline{\mathbf{p}}_\mathbf{i} - \overline{\mathbf{s}}_\mathbf{i} \tag{13}$$

We can see from Eq. (13) that using centralized coordinates, translation parameters **T** are removed and **R** can be computed individually. Then using equation (11) to computer **T** and Eq. (7) can be used to solve coordinates of exposure station.

3.3 Calculation rotation matrix based on unit quaternion

There are many kinds of methods to represent rotation matrix, including Euler angle method, Cayley-Klein parameters, rotation axis and rotation angle and unit quaternion etc.(Ramesh Jain, Rangachar Kasturi, etc. 2003). Among these, Euler angle is most used in remote sensing and Photogrammetry, computer vision. But unit quaternion method has a simple concept and is easy to get closed form solution, and we use it in the paper.

Substitute equation (13) into (9), and rearrange, we got

$$\sum_i \mathbf{V_i}^T\mathbf{V}_i = \sum_i \left(\overline{\mathbf{p}}_\mathbf{i}^T\overline{\mathbf{p}}_\mathbf{i} + \overline{\mathbf{s}}_\mathbf{i}^T\overline{\mathbf{s}}_\mathbf{i} - 2\overline{\mathbf{p}}_\mathbf{i}^T\mathbf{R}\overline{\mathbf{s}}_\mathbf{i}\right)$$

In order to make above formula minimum, it must meet the following formula

$$\max_R \sum_i \left(\overline{\mathbf{p}}_\mathbf{i}^T\mathbf{R}\overline{\mathbf{s}}_\mathbf{i}\right) \tag{14}$$

Using quaternion to represent $\overline{\mathbf{p}}_\mathbf{i}$ and $\overline{\mathbf{s}}_\mathbf{i}$, that is,

$\mathbf{P_i} = (0, \overline{\mathbf{p}}_\mathbf{i})$ and $\mathbf{S_i} = (0, \overline{\mathbf{s}}_\mathbf{i})$.
Given **Q** is a unit quaternion, represented as

$$\mathbf{Q} = w + \mathbf{i}q_1 + \mathbf{j}q_2 + \mathbf{k}q_3$$

then Eq. (14) is rewritten as

$$\begin{aligned}&\sum_i \mathbf{P_i}^T \mathbf{Q}\mathbf{S_i}\mathbf{Q}^* = \sum_i \mathbf{P_i} \cdot (\mathbf{Q}\mathbf{S_i}\mathbf{Q}^*) = \sum_i (\mathbf{P_i}\mathbf{Q}) \cdot (\mathbf{Q}\mathbf{S_i}) \\ &= \sum_i [C(\mathbf{P_i})\mathbf{Q}]^T [\overline{C(\mathbf{S_i})}\mathbf{Q}] = \mathbf{Q}^T \left(\sum_i C(\mathbf{P_i})^T \overline{C(\mathbf{S_i})} \right) \mathbf{Q} \\ &= \mathbf{Q}^T \mathbf{N} \mathbf{Q}\end{aligned} \tag{15}$$

where

$$\mathbf{N} = \sum_i C(\mathbf{P_i})^T \overline{C(\mathbf{S_i})} = \sum_i \begin{bmatrix} 0 & -x_{pi} & -y_{pi} & -z_{pi} \\ x_{pi} & 0 & -z_{pi} & y_{pi} \\ y_{pi} & z_{pi} & 0 & -x_{pi} \\ z_{pi} & -y_{pi} & x_{pi} & 0 \end{bmatrix}^T \begin{bmatrix} 0 & -x_{si} & -y_{si} & -z_{si} \\ x_{si} & 0 & z_{si} & -y_{si} \\ y_{si} & -z_{si} & 0 & x_{si} \\ z_{si} & y_{si} & -x_{si} & 0 \end{bmatrix}$$

$$= \sum_i \begin{bmatrix} x_{si}x_{pi} + y_{si}y_{pi} + z_{si}z_{pi} & y_{si}z_{pi} - z_{si}y_{pi} & z_{si}x_{pi} - x_{si}z_{pi} & x_{si}y_{pi} - y_{si}x_{pi} \\ y_{si}z_{pi} - z_{si}y_{pi} & x_{si}x_{pi} - y_{si}y_{pi} - z_{si}z_{pi} & x_{si}y_{pi} + y_{si}x_{pi} & x_{si}z_{pi} + z_{si}x_{pi} \\ z_{si}x_{pi} - x_{si}z_{pi} & x_{si}y_{pi} + y_{si}x_{pi} & -x_{si}x_{pi} + y_{si}y_{pi} - z_{si}z_{pi} & z_{si}y_{pi} + y_{si}z_{pi} \\ x_{si}y_{pi} - y_{si}x_{pi} & x_{si}z_{pi} + z_{si}x_{pi} & z_{si}y_{pi} + y_{si}z_{pi} & -x_{si}x_{pi} - y_{si}y_{pi} + z_{si}z_{pi} \end{bmatrix} \tag{16}$$

N is a real symmetry matrix, and its eigenvector corresponding to the maximum eigenvalue can make equation maximum.Unit quaternion **Q** is the eigenvector of maximum eigenvalue of **N** (Berthold K.P.Horn. 1987). According to formula (4), we can get rotation **R**.

Using the relationships of direction cosine and angular elements (ZHU Zhao-guang, SUN Hu,2004), we get

$$\tan\varphi = -\frac{a_3}{c_3} \qquad \sin\omega = -b_3 \qquad \tan k = \frac{b_1}{b_2} \tag{17}$$

Procedure of calculation is as follows:

1) compute distances between exposure station and each control points; calculate λ_i using equation (6);
2) calculate $\mathbf{s}_i$ using equation (7);
3) computer centralized coordinates using equation (12);
4) according to equation (16) to obtain matrix **N**;
5) computer eigenvalue of **N**;
6) select maximum eigenvalue of **N** and compute its eigenvector; this is the unit quaternion wanted;
7) use the unit quaternion to construct rotation matrix (equation 4), and derive angle elements of EOPs (equation 17);
8) computer translation matrix **T** (equation 11);
9) calculate linear elements of EOPs (equation 7).

Now, we got all six EOPs.

4. NUMERIC EXPERIMENTS

In order to testify the correctness of the method, we use data in reference [6] to computer. The initial data are listed in table 2.

No.	Image coordinates		object coordinates in ground coordinate system		
	x/mm	y/mm	X/m	Y/m	Z/m
1	-86.15	-68.99	36589.41	25273.32	2195.17
2	-53.40	82.21	37631.08	31324.51	728.69
3	10.46	64.43	40426.54	30319.81	757.31
4	-14.78	-76.63	39100.97	24934.98	2386.50

Table 2. Initial data

We use matlab (PU Jun, JI Jia-feng, YI Liang-zhong. 2002) to calculate results. First we obtained distance between perspective center and control points, see table 3. Results of EOPS are in table 4.

Iterative number	S1	S2	S3	S4
1st	-742.477	977.305	822.485	-760.923
2nd	-229.483	-177.614	-137.745	-324.246
3rd	-13.67	1.356	3.245	-37.865
4th	-0.436	0.143	0.211	-1.3455
5th	$9.64*10^{-4}$	$-3.99*10^{-4}$	$-5.76*10^{-4}$	$-2.99*10^{-5}$
distance	6636.994	8144.297	7411.651	5817.274

Table 3. Computed distances between projective center S to each control point（unit: m）

From table 4, we notice that there exists maximum difference about 0.37 meter in linear element and 13 second in angular element. There are probably caused by following factors. Firstly, collinearity equation method need to provide initial values of unknowns. When different initial values are given, the results may be slightly different (on condition of converge). At the same time, linearization process may introduce model error, thus have effect on results. Secondly, distances computation of exposure station to control points is essential in the paper; it will affect the accuracy of results. Thirdly, errors exist in calculation can also make results different.

In order to verify the correctness and reliability of the method, simulation data are used. First, give six exterior orientation elements and other parameters of image, and coordinates of four control points, see table 5 and table 6. Using collinearity equation and these data to computer corresponding image coordinates. Then we use coordinates of control points and image points to computer exterior orientation parameters. Obviously, if the method is correct, we can get result similar to table 5.

Based on simulated coordinate of control points and image points, and interior elements, we got results of exterior orientation parameters, see table 7.From table 7, we know that the results are more accurate than those of collinearity equation method.

Further, we simulate data of large inclination angle image, and using the method proposed to process it. Simulation data and computation results are listed in table 8. However, we can not processes this set of data by collinearity equation method.

5. CONCLUSIONS

On consideration of the problems exist in traditional space resection method, we proposed a new method based on unit quaternion, and a closed form solution is derived. The method need not to linearize the collinearity equation, and need not to provide initial values of EOPs. The computation is simple and we verify the correctness in numerical examples. In the method, we don not limit the angle, so it can also deal with large oblique image.

The accuracy of the method depends on such factors as measurement accuracy of control points and image points, distance computation of control points to exposure station, and computation accuracy of eigenvalues and eigenvectors. Because methods for eigenvalues and eigenvectors computation are mature in mathematics, so in order to improve the accuracy of the algorithm, we must improve measurement accuracy of control points and image points, distance computation of control points to exposure station.

Although the method proposed here have some advantages in comparison with other methods, there still have shortcomings, for examples, we can deal with the gross errors, theoretical accuracy etc.. These problems still need to study further.

ACKNOWLEDGEMENTS

The research has been funded by National Natural Science Foundation of China (40574008), and science and technology project of Jiangxi Province ([2007]237). It is also funded by science and technology key project of Jiangxi Province.

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methods	coordinates of perspective center/ m			orientation / rad		
	X_S	Y_S	Z_S	φ	ω	κ
Results using proposed method	39795.08	27476.75	7572.81	-0.003926	0.002075	-0.067556
Results in ref.[6]	39795.45	27476.46	7572.69	-0.003990	0.002110	-0.067581

Table 4. Computing result of exterior orientation parameters

parameters	linear elements/ m			angular elements/ rad			interior elements / mm		
	X_S	Y_S	Z_S	φ	ω	κ	x_0	y_0	f
given value	39795	27477	7573	0.002778	0	0	0	0	153.24

Table 5. Simulation data for photo parameters

No.	coordinates of image points		coordinates of control points		
	x/mm	y/mm	X/m	Y/m	Z/m
1	22.1893	-34.2927	40589	26273	2195
2	-27.4380	-26.9674	38589	26273	728
3	-27.5530	17.9049	38589	28273	757
4	23.0217	23.5064	40589	28273	2386

Table 6. Simulated coordinates of control point and image point

parameters	linear elements / m			angular elements / rad		
	X_S	Y_S	Z_S	φ	ω	κ
results using the method proposed	39795.009	27477.007	7572.997	0.002777	0	0
results using collinearity equation	39795.028	27476.105	7573.084	0.002772	0.000150	0.000064

Table 7. Results for simulation data

parameters	linear elements / m			angular elements / rad		
	X_S	Y_S	Z_S	φ	ω	κ
given value	39795	27477	7573	0. 069813	0	0. 174533
computed value	39795. 005	27477. 009	7572. 998	0. 069812	−0. 000001	0. 174533

Table 8 result for simulated oblique image