Article

An Update on the Use of Rational Functions for Photogrammetric Restitution

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During the past four or five years the photogrammetric community at large has become aware of the use of rational functions for photogrammetric restitution. This has been largely due to the need to use these for setting up Ikonos data, as Space Imaging does not provide a physical sensor model with the image data. Rational functions have been used for some time for military use and their widespread acceptance in that area has led to the proposal to the OGC for a standard for image restitution based on rational functions on the basis of its universality: it can be used with any sensor.

A number of papers have been published, reporting on tests of rational function models which indicate that high accuracy can be achieved. However controversy over the use of rational functions is by no means over. In February and March 2001 an extended email correspondence took place between a number of people who have worked with rational functions. This correspondence revealed first a lack of full understanding about the terminology and methods used with rational functions, and second, fundamental disagreement on the application of rational functions. This topic is still one of hot debate.

This paper will briefly review the background, up to 2000. Dowman and Dolloff (2000) gave a full review at the Amsterdam Congress. This was the result of an initiative by ISPRS Council to track the development and use of rational functions and which summarises the method and tests carried out. The paper then reviews developments since 2000, including investigations into the accuracy and use of rational functions and methods of circumventing their use, and in particular their application to Ikonos data.

Terminology and Background

As with all things new, the terminology used varies according to who is discussing the topic. Dowman and Dolloff (2000) used the term 'replacement sensor model' as the generic term for methods which use ratios of polynomial functions fitted to a spatial grid to define the transformation from object to image space. They give four generic techniques:
- Polynomial or warping approach
- Grid interpolation or anchor point method
- Rational functions
- Universal Sensor Model (USM), an extension of the basic rational function model, developed by the Open GIS Consortium (OGC) Abstract specification (OGC, 1999)

Space Imaging (Grodecki and Dial, 2001) refer to the camera model as the 'Rational Polynomial Camera (RPC) model'; they also refer to the RPC model coefficients. RPC can also stand for rational polynomial coefficients, a terminology which is also used by Fraser et al (2002a). In the OGC document (OGC, 1999), the polynomial coefficients are also called Rapid Positioning Capability (PRC) data. Tao in all his publications refers to the rational function model (RFM). In this paper the term rational function model will be used and also the term rational polynomial coefficients (rpcs). Rational function model will be used as a generic term, to include the Universal Sensor Model.

The formulae for rational function give an image co-ordinate as a ratio of two polynomial functions in the ground co-ordinate:

$$r_n = \frac{p_1(X_n, Y_n, Z_n)}{q_1(X_n, Y_n, Z_n)} \qquad [1]$$

$$c_n = \frac{p_2(X_n, Y_n, Z_n)}{q_2(X_n, Y_n, Z_n)} \qquad [2]$$

$where:$

$r_n = normalised\ row\ index\ of\ pixel\ in\ image$

$c_n = normalised\ column\ index\ of\ pixel\ in\ imag$

$X_n, Y_n, Z_n = normalised\ ground\ coordinate\ values$

The rpcs are often determined from a 3D object grid constructed from the rigorous sensor model. It is at least the initial application of rpcs. The rpcs can also be determined using ground control points (GCPs). This solution is largely dependent on the terrain relief and the number and distribution of the GCPs. Tao and Hu (2001a and 2001b) use the term, terrain-dependent and terrain-independent solutions, to differentiate these two computation scenarios. It would cause some confusion if the computation solution to rpcs, namely terrain dependent and independent, is not clearly defined.

An important use of RFM during the past 2 years has been to restitute Ikonos data. Ikonos images can be set up using these rpc's in digital photogrammetric systems such as SOCET Set and Z/I Imaging and this use will be reported

on below. Dial, Gibson and Poulsen (2001), from Space Imaging, state that: "The ….. equations require five offsets, five scale factors, forty coefficients for line, and forty more coefficients for sample. Those ninety numerical values are provided in text format with Ikonos stereo products…." . Workers such as Fraser and Toutin have shown that the use of RFM is not essential for the restitution of Ikonos images. Their results are presented below.

Rational functions can also be used in the image to ground, or inverse form, (Yang, 2000), but then require an iterative solution to find the Z co-ordinate. Tao and Hu (2001c) use a forward form for 3D measurement and compared both inverse and forward forms in terms of accuracy and computation aspects. Di et al. (2001) developed, independently, a similar method (called downward form) for this computation. These methods allow one to use RFM for 3D reconstruction and stereo measurements.

Investigations into the Accuracy of Rational Functions

There have been a number of tests carried out to determine the accuracy of the RFM. These break down into those using sensors for which the model is known, such as aerial frame cameras or SPOT, and those using Ikonos data for which the sensor model is not known, and for which the tests must compare only results from the RFM, with ground control points (GCPs). The former are better indicators of the accuracy of the model, although the latter are of more interest.

Tao and his co-workers have carried out extensive tests on different formulations of the RFM, mainly on SPOT and aerial photography, Hu and Tao (2001), Tao and Hu (2001a, 2001b, 2001c). They conclude that the RFM can give very high accuracy for aerial photography and SPOT data in the terrain-independent case. The RFM with unequal denominator often gives a better result. The high order RFM is favourable sometimes, for example, for SPOT data. The normal equations are usually well conditioned. In the terrain dependent case the solution is very sensitive to the GCP distribution, and its design matrix is almost rank-deficient. Tao and Hu, (2001a) propose computational methods to improve the numerical stability of the solution.

Yang (2000) also reports on using the rational function model with aerial photography and SPOT and achieved

negligible errors 'when proper polynomial order is used' (under the terrain-independent case). In the case reported, functions lower than third order were used and the correct order can be chosen, based on an root mean square error threshold.

The rpcs provided with Ikonos imagery allow the object-to-image transformation to be performed. This gives an accuracy in the 3D object space which is consistent with the specifications for the different Ikonos products such as GEO or Precision. The rpcs for the GEO product, which are expected to produce a RMS positioning accuracy of about 25m, are derived solely from satellite ephemeris and attitude data, whereas those for Precision products are computed with the additional aid of ground control. Grodecki and Dial (2001) report that tests with 140 ground control points gave horizontal accuracy 'of the order of 1m while vertical accuracy was of the order of 2m' from a controlled stereo pair over San Diego. Hanley and Fraser (2001) tested Ikonos Geo product by first projecting the control points onto 'planes of control', to minimise the effect of terrain, and then transform the image to these points using Similarity, affine and projective transformations. The results show that 0.3-0.5m geopositioning accuracy is achievable from the Geo product without using the rational function solution. Fraser et al (2002a) have extended this work in two dimensions into three, using similar techniques. Table 1. shows the results, first from a stereo solution using only the rpc's provided with the Geo images. This only shows that the results are within specification. The true relative accuracy is shown when the same stereo pair is transformed by a translation, in the first case using 1 single ground control point (repeated with 4 single GCPs), and in the second case using 4 ground control points (repeated with four sets of 4 GCPs).

As with the planimetric test, this does not give a true test of the accuracy of the rpcs because they were computed only using the camera model and sensor position and attitude recorded by on-board GPS receivers and star trackers. It does however clearly indicate that the rpcs can give good results with Ikonos data. Similar test results were shown in Baltsavias et al (2001).

The RFM solution can be improved in the case of aerial photography and Ikonos, without using the sensor model, Hu and Tao (2001). This method uses a sequential adjust-

Image configuration	No. of GCPs	No. of check points	RMS discrepancies (m)		
			X	Y	Z
Standard stereo solution	0	40	8.2	31.5	1.7
Stereo solution with bias removed by translation	1	39	0.58-0.75	0.41-0.83	0.87-0.98
Stereo solution with bias removed by translation	4	36	0.59-0.69	0.43-0.50	0.83-0.96

Table 1: Checkpoint discrepancies from stereo and 3 ray rpc spatial intersection. (From Fraser et al, 2002a).

Correction method	RMS discrepancies (m)		Maximum Errors (m)	
	X	Y	X	Y
Simple Polynomial	1.7	4.1	4.1	7.5
Rational Polynomial	2.2	5.2	5.1	10.4
Rigorous Model	1.3	1.3	3.0	3.0

Table 2: Comparison of error results with 23 independent check points and 7 GCPs using simple polynomial, rational polynomial, and rigorous model. (From Toutin and Cheng, 2000).

ment to update rpcs using additional GCPs.

Fraser et al (2002a) further show that this accuracy can be improved by post rpc transformation of the object point coordinates to control points using additional parameters, an affine projection or a Direct Linear Transformation (DLT) in ground space.

Interestingly enough, Dial and Grodecki (2002), Fraser et al. (2002b) and Tao et al (2002) all reported at the 2002 ASPRS conference that the image based transformation (bias correction) to improve the rpc accuracy using GCPs is more effective. Dial and Grodecki (2002) has further extended the rpc model for block adjustment. Their work shows that the block adjustment of Ikonos images with the rcps is also viable.

Toutin (Toutin and Cheng, 2000) has developed a physical Ikonos model using basic information from the metadata and image files. (For example, approximate sensor viewing angles can be computed using the nominal collection elevation and the nominal ground resolution in the across and along scan directions.) The model has ported into PCI OrthoEngine. Toutin compared three methods of handling Ikonos data and his results are shown in table 2. It is worth noting that these RFM tests were based on the terrain-dependant case.

More recently Toutin and Cheng (2002) have also reported on tests carried out with Quickbird data using similar methodology the work done with Ikonos data. The results are shown in table 3 and are almost identical for the rigorous model, and worse using the 1st order rational function model. This cannot be compared directly with table 2, where higher order polynomials were used.

Other Developments

A number of commercial photogrammetric systems incorporate RFMs. These include, LH Systems SOCET Set, ZI Imaging and ERDAS. Tao and Hu (2001) report the devel-

opment of Rational Mapper for RFM based orthorectification and stereo measurement [http://www.geoict.yorku.ca/project/rationalmapper/rationalmapper.htm]. PCI have adopted a generalised model of Toutin to correct the low-cost Geo products that are currently available from Space Imaging. With the use of a small number of control points and a DEM high quality ortho-images can be generated.

It is reported that DigitalGlobe will be releasing the sensor model for Quickbird, at least to commercial software developers, and maybe to the wider user community. Quickbird data will be available as Basic, Standard or Orthorectified products. All products are supplied with rpc's and the Basic product is supplied with the spacecraft telemetry and sensor model so that the users can process the data themselves.

Conclusions

This paper has reviewed developments in the use of rational function model since 2000. It has shown that the technique is likely to become a fixture in photogrammetric image processing, although some scepticism still exists over its use. A prime driving force to use RFMs is the decision of Space Imaging not to release the Ikonos sensor model, but to provide the information necessary to restitute the data in the form of rpcs. This facility is used by many organisations and software packages increasingly provide the means to use rpcs to restitute Ikonos data. However it has also been shown that there are alternative ways to achieve high accuracy without the use of rpcs. Ingenuity on the part of photogrammetrists has overcome restrictions placed on the use of the data by vendors.

Although it has been shown that RFMs are accurate and robust and useful in setting up Ikonos data, there is no evidence that they are about to replace rigorous methods for photogrammetric restitution. RFMs can be considered as an effective representation (or approximation) of the rigorous sensor models.

Correction method	RMS discrepancies (m)		Maximum Errors (m)	
	X	Y	X	Y
Rational 1st order Polynomial	4.0	2.1	9.5	4.3
Rigorous Model	1.4	1.3	2.5	2.8

Table 3: Comparison of error results with 12 independent check points using simple 1st order polynomial rational functions and Toutin's rigorous model. (From Toutin and Cheng, 2002).

Acknowledgement

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