SPACE ALLOCATION OF EMERGENCY CENTERS USING VORONOI DIAGRAMS

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ABSTRACT:

Space allocation of emergency centers which is dividing a territory into sub-areas based on specific criteria, is used for better management of emergency centers in disaster situations. Voronoi Diagram, as an influence area for each center can be used to specify service area of emergency centers. In standard Voronoi Diagram analysis, the only parameter used to define service area of each emergency center is Euclidian distance. Whereas in this paper, we develop a novel Network Voronoi diagram method to consider more parameters and improve space allocation of emergency centers. Accordingly, we consider capacity of center, service demands and network distance parameters. Also, resulting sub-areas of this method are both continuous and geospatially compact. In this method, distances are given by shortest path on the network. Dijkstra's algorithm is developed to calculate shortest paths from centers at the same time. The result of the method is the partitioning of the nodes and the arcs of the network. By using this method, when emergency centers are contacted, an ambulance is sent to the location based on position of demand center and allocated space to emergency centers. Finally, as a case study, we applied the standard and network Voronoi diagrams in Tehran, capital of Iran. The results of this case study empirically show that the improved network Voronoi diagram can more precisely represent the service areas in urban regions than the corresponding Standard Voronoi diagrams.

1. INTRODUCTION

Space allocation of emergency centers which is dividing a territory into sub-areas based on specific criteria, is used for better management of emergency centers in disaster situations. To delimit boundaries and space allocation of emergency centers, the research utilized the Voronoi diagram, a widely known method for definition of spatial proximity and bounding determination technique. Voronoi Diagram (VD) has been extensively used for a wide range of different applications including determining mutually exclusive partition of space in the social and environmental sciences (Okabe et al. 2000) and for the purpose of allocation by a large number of researchers including Vincent and Daly (1990), Boots and South (1997), and Cox and Agnew (1974). It has also been widely used for GIS applications and spatial analysis (Mostafavi 2002, Mostafavi and Gold 2004). Voronoi diagram as an influence area for each center can be used to specify service area of each emergency center. In standard Voronoi diagram analysis, the only parameter used to define service area of each center is Euclidian distance whereas in circumstances like space allocation of emergency centers, parameters like capacity of center, service demands and network must be taken into account. Missing parameters in standard Voronoi diagram makes it unsuitable to be used in real world determination of service.

In this paper we aim to use Network Voronoi Diagrams (NVD) for space allocation of urban emergency centers. It has been studied by Okabe et al., (2002, 2009). In this method, distances are given by shortest path on the network. Dijkstra's algorithm is developed to calculate shortest paths from centers at the same time. The result of the method is the partitioning of the nodes and the arcs of the network.

In this research, space allocation of Tehran municipality's district five is examined. Considering network parameter and using network Voronoi diagram, we obtained more real world results but supply and demand were imbalanced because number of ambulances in each center were not included in the calculations. To include volume of equipments and facilities of each center in calculations, we used Weighted Network Voronoi Diagram (WNVD) in which area was assigned to ambulance center based on number of ambulances it possessed as its center weight. By using this method, when emergency centers are contacted, an ambulance is sent to the location based on position of demand center and space allocation of emergency centers. Also, resulting sub-areas of this method are both continuous and geospatially compact.

The paper is organized as follows. In Section 2 a general paradigm for Voronoi diagram and some desirable properties to be met by VD algorithms are elaborated. In Section 3 and section 4, Network Voronoi diagram and weighted network Voronoi diagram are described. In Section 5 the case study is introduced and the results are illustrated. Section 6 concludes the paper.

2. VORONOI DIAGRAM

A classic way of regionalizing or allocating space around a predetermined set of points or generators is Voronoi tessellation. This method is based on finding the nearest generator for every point in the space. The resultant regionalization is known as a Voronoi diagram. In a comprehensive presentation on the subject including an extensive review of the literature, Okabe et al. (2000) present the standard VD as well many of its generalizations.

In mathematical terms, let $P \equiv \{P_1, P_2, \dots, P_m\}$ be a finite set of distinct points in the 2-dimensional Cartesian space R^2 (Voronoi diagrams are also defined in R^d, with d > 2, but our application is limited to R^2). We denote by $\|.\|$ the Euclidean norm in R^2. Let the region $V(P_i)$ be the set of locations $P \in R^2$ verifying $\|P - P_i\| \leq \|P - P_j\|$ for all j, except for j =i, which is symbolically written as (Okabe et al. 2000):

$$V(P_i) = \{P \in R^2 \quad | \ \|P - P_i\| \leq \|P - P_j\|, j \neq i, j = 1, \dots, m\}$$

The region $V(P_i)$ is called the standard Voronoi polygon associated with P_i, and the resulting set, $\{V(P_1), V(P_2), \dots, V(P_m)\}$, the planar standard Voronoi diagram generated by $P \equiv \{P_1, P_2, \dots, P_m\}$. P is a generator set, and for each index $i \in \{1,2, \dots, m\}$, P_i is a generator point. The edges of Voronoi polygons in R^2 are line segments, as shown in Figure 1. When the Voronoi diagram is constrained to a finite region, it becomes a bounded Voronoi diagram. Although Voronoi diagram generators can be points, lines, circles, or areas of diverse shapes (Okabe et al., 2000), our application deals with point generators only.

Since peripheral generators can grow unchecked in the opposite direction of the center of gravity of the generator configuration as a whole, they become infinitely large. This problem is normally resolved by introduce an artificial outer boundary and limit the growth of the external generators to these boundaries. This is the approach used in most GIS systems.

Here we review four of the basic geometric properties of the Voronoi diagrams.

Property 1: The Voronoi diagram of a point set P, VD(P), is unique.

Property 2: The nearest generator point of pi (e.g., pj) is among the generator points whose Voronoi polygons share similar Voronoi edges with V P(pi).

Property 3: Let n and ne be the number of generator points and Voronoi edges, respectively, then ne · 3n ¡ 6.

Property 4: From property 3, and the fact that every Voronoi edge is shared by exactly two Voronoi polygons, we notice that the average number of Voronoi edges per Voronoi polygon is at most 6, i.e., 2(3n ¡ 6)=n = 6 ¡ 12=n · 6. This means that on average, each generator has 6 adjacent generators.

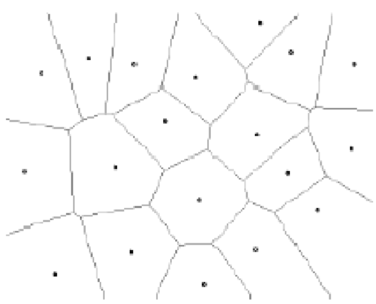

Figure 1. Standard Voronoi Diagram

3. NETWORK VORONOI DIAGRAM

A network is defined by its nodes and the arcs. In this paper the result of the Network Voronoi Diagram (NVD) is the partitioning of the nodes and the arcs of the network (The Arc Network Voronoi Diagram). In principle there are three types of Network Voronoi diagrams (Okabe et al. 2000).

• The Node Network Voronoi Diagram: It assigns each node in a network to a generator by the rule that the network distance from the node to the generator is smaller than to any other generator in the network.

• The Arc Network Voronoi Diagram: It assigns each arc in a network to single generator, or it splits arcs into two parts such that the points on one part of the arc are assigned to one generator (which is closest to these points), and the points on the other part of the arc are assigned to another generator (which is closer to them).

• The Area Network Voronoi Diagram: the partitioning of the area is done in a way such that each point is assigned to its closest generator, considered the minimal Euclidean distance to the next network point, and from that point network distances. This diagram is different to the planar Voronoi diagram described above, and it requirese a planar network.

The NVD is defined for a set of points P={p_1,…, p_n} on the network,where the distance d (p, pi) between an arbitrary point p and a generator point pi on the network is measured using the shortest-path distance, d_N (p, pi). In these terms,the NVD is defined as a set of subnetworks, (referred to as Voronoi subnetworks), V={V_1,…, V_n}, where the Voronoi subnetwork Vi is given by (Okabe et al. 2009):

$$V(p_i) = \{p \quad | d_N(p, p_i) \leq d_N(p, p_j), i \neq j, i = 1, \dots, n\}$$

For the construction of the network Voronoi diagram an algorithm is used based on the shortest path algorithm of Dijkstra (Erwig 2000; Okabe et al. 2002a; Okabe et al. 2002b). Dijkstra's algorithm calculates in a connected network the shortest path from a selected start node to any other node in the network (Dijkstra 1959). In the given context, it has to be modified to calculate the shortest paths from many generators in parallel. Each node in the network is assigned by this way the shortest distance only to the next generator (Graf and Winter 2003). Fig. 2 assumes that the weights of the arcs are identical with their Euclidean distance between their start and end nodes. Furthermore the shown network is undirected, i.e., the weights are considered to be symmetric in both directions. For the Arc Network Voronoi Diagram, the arcs with a start node assigned to a different generator than their end nodes are the critical ones. In this method the points on these arcs that have equal distance to the two generators is calculated simply, and the arc here with a node representing a Voronoi boundary is split (Okabe et al. 2000).

The construction of the Arc Network Voronoi Diagram is usually sufficient. Consider a street network: if its arcs are assigned to closest generators, then we know also the assignment of buildings along the streets to generators. The entrance point of the building is the decisive factor.

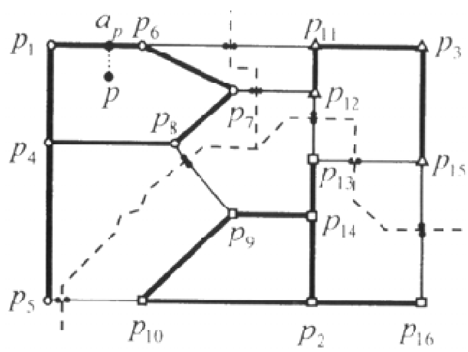

Figure2: Assignment of all nodes, arcs, and areas to the three Voronoi generators p1, p2 and p3 (Okabe et al. 2000).

4. WEIGHTED NETWORK VORONOI DIAGRAMS

Compared with the weighted VD, the weighted NVD is rarely found in the related literature(Okabe et al. 2009).
When the unit cost $\alpha_i$is considered as weight of generator i, the weighed distance (Okabe et al. 2000):

$$d_{WN}(p, p_i) = \alpha_i d_N(p, p_i)$$

is appropriate. In this term, the weighed NVD is formulated as(Okabe et al. 2009):

$$V(pi) = \{p \quad | d_{WN}(p, pi) \leq d_{WN}(p, pj\,), i \neq j, i = 1, \ldots, n\}$$

In order to assess partitioning methods an operational error metric must be selected. Although many are possible, the error metric we use throughout this paper is proposed by Reitsma et al., (2007):

$$\text{Error} = \frac{|P_i - p_i|}{P_i}$$

Where
P_i is the expected population of the districting i as proportion of total population, and
p_i is the estimated population of districting i as proportion of total population and is get from sum of population in district i

5. CASE STUDY

The proposed methodology is implemented in a space for a road district resides in municipal District 5 of Tehran. Figure 3 represents the scope of the studied area. A topographic map of Tehran at the scale of 1:2000 is used which is updated in 2006, and census data extracted from demography carried out in 2007.

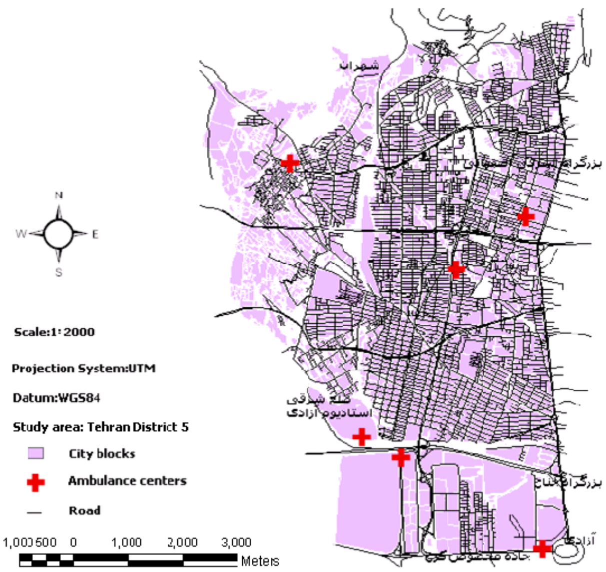

Figure 3. The studied area

Standard Voronoi diagram is utilized to create emergency centers service areas. The result for six ambulance centers is six areas that each point in each area has nearest distance to its generator. So in standard Voronoi diagram, as the only parameter used to define service area of each ambulance center is Euclidian distance, it is not a suitable method. Figures 4 shows districting, using Standard Voronoi Diagram (SVD).

In network Voronoi diagram, distances are given by shortest path on the network. Figures 5 shows partitioning of the nodes and the arcs of the network on the six emergency centers using network Voronoi diagram.
In weighted network Voronoi diagram, in addition to using network distances, each center has a weight and number of ambulances in each center is included in the calculations. Figures 6 shows partitioning of the nodes and the arcs of the network on the six emergency centers, using weighted network Voronoi diagram.

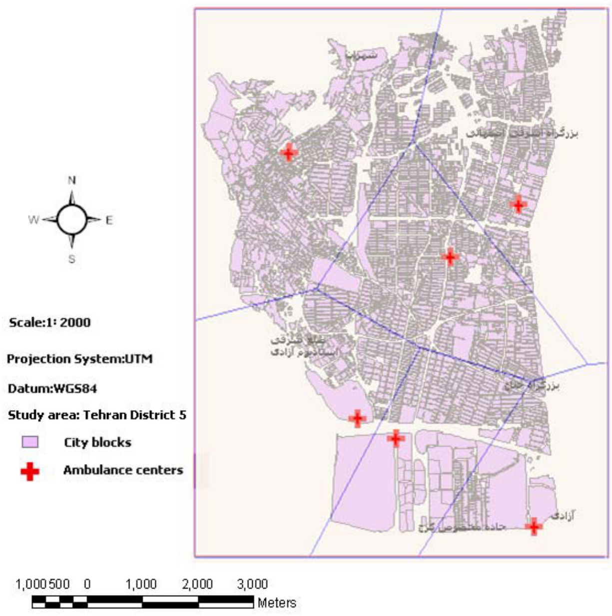

Figure 4. Standard Voronoi Diagram on emergency centers

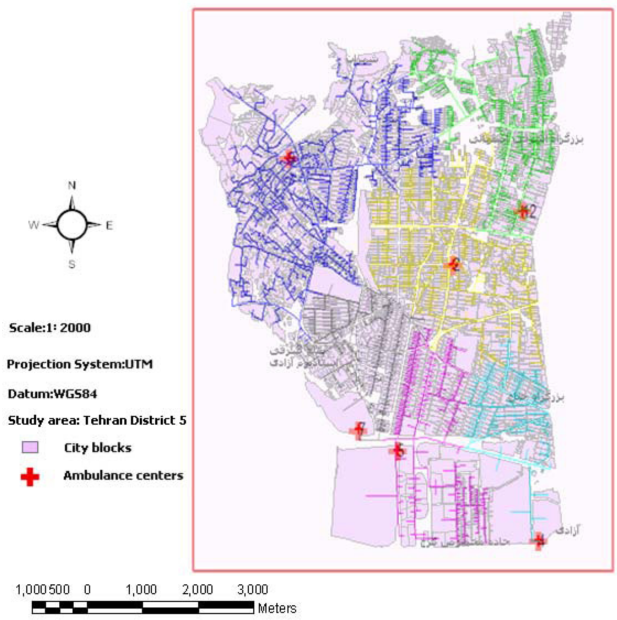

Figure 5. Network Voronoi Diagram on emergency centers

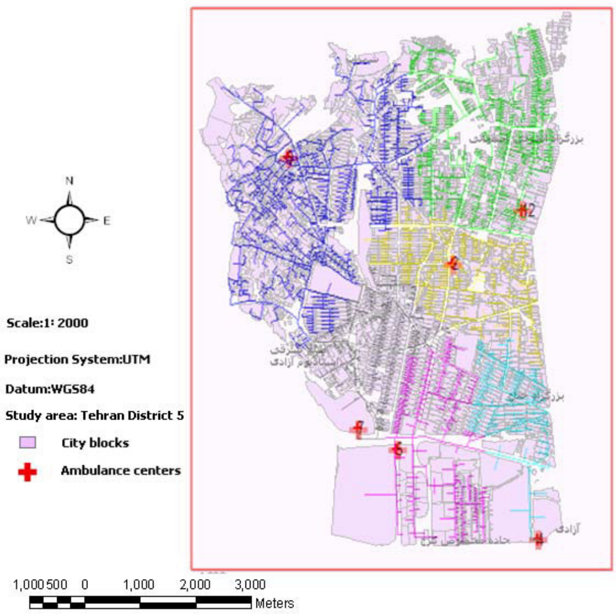

Figure 6. Weighted Network Voronoi Diagram on emergency centers

Table 1 shows three types of error: minimum, average and maximum error for standard Voronoi diagram, network Voronoi diagram and weighted network Voronoi diagram. They are obtained through error equation which is introduced in section 3. As mentioned there, error is difference of estimated and expected population in each district. Whatever this difference is less, districting is done better.

	SVD	NVD	WNVD
Min.Error	0.07	0.24	0.29
Mean Error	1.22	0.87	0.48
Max.Error	4.75	2.93	0.73

Table 1. Performance of SVD, NVD and WNVD data

Table 2 shows a brief comparison of three methods, we used in this paper.

SVD	Euclidian distance	$d_{SVD} = \|P - P_i\|$
NVD	Network distance	$d_{NVD}(p, p_i) = d_N(p, p_i)$
WNVD	Network distance and weight of centers	$d_{WNVD}(p, p_i) = \alpha_i d_N(p, p_i)$

Table 2. Comparison of SVD, NVD and WNVD

6. CONCLUSION

In this paper space allocation of emergency centers is performed to reduce ambulance arrival time to disaster centers.
The results presented, show that the standard Voronoi diagram is not useful for this application. In network Voronoi diagram, distances are given by shortest path on the network. But as capacity of emergency centers is not considered, average difference of estimated and expected population is not reduced enough.
Considering the network and weights for emergency centers, creates a much more realistic districting using weighted network Voronoi diagram. As presented the average error is become less than what is resulted for standard Voronoi diagram and network voronoi diagram.
By using this method, when emergency centers are contacted, an ambulance is sent to the location based on position of demand center and allocated space to emergency centers, according to number of ambulances in each emergency center and the network.

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