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THE CONCEPT OF "PHOTO-VARIANT" SELF-CALIBRATION AND ITS APPLICATION IN BLOCK ADJUSTMENT WITH BUNDLES

ABSTRACT

This paper describes the concept of "photo-variant" self-calibration, which offsets many shortcomings raised by "block-invariant" self-calibration. The photo-variant approach differs significantly in that it assigns an individual set of compensation parameters to each photograph or group of photographs (strip) rather than postulating a common set of parameters for all photographs in the block.

Although this approach requires a more laborious computational effort, it is a generalized self-calibration scheme applicable for any type of camera (metric or non-metric) and photography (aerial or close-range). Mathematical formulations and associated computational considerations, especially the means of circumventing the problem of ill-conditioning, will be discussed in detail in this paper.

INTRODUCTION

In recent years, aerotriangulation has become a suitable tool for geodetic control densification. This is primarily due to the fact that rigorous simultaneous block adjustments have provided suitable accuracies with relatively low ground control requirements. Theoretically, the fully analytical bundle approach is most accurate. With the inclusion of additional compensation parameters or other forms of "self-calibration", the bundle adjustments are now coming close to the theoretical accuracy expectations. As will be shown later, computational considerations have restricted the majority of self-calibration approaches to some kind of block-invariant formulations (i.e. calibration parameters being assumed to remain unchanged throughout the block of photographs).

Aside from the great advances in aerotriangulation techniques to date, much effort has been expended, especially in the last two decades, to broaden the application of photogrammetry into the close-range field of study, where non-metric (or amateur) cameras are widely utilized to solve specific problems. Since non-metric cameras are not designed with the acquisition of metric data in mind, their use in photogrammetry must be accompanied by a more complex calibration system than that normally associated with metric (aerial) cameras. The nature of the close-range, non-metric photography usually requires that interior orientation be recovered from exposure to exposure. Existing self-calibration approaches as such are not suitable because of their block invariant modelling.

The main objective of this study is to develop a system which contains the benefits of both, namely self-calibration scheme applicable whether the camera is metric or non-metric. More specifically, the purpose is to complement the conventional self-calibration concept with a more general approach - photo-variant self-calibration - by postulating an individual set of calibration parameters for each photograph or group of photographs in the block. Rather than limiting it to close-range situations, it is attempted to make it general so that it may be utilized to cover the main fields of photogrammetric applications ranging from aerotriangulation to close-range laboratory photogrammetry.

Although the photo-variant concept gives a more theoretical account of self-calibration than the block-invariant concept, an appreciable increase in cost of data reduction may not always be justified in the area of production aerotriangulation. Rather, it proves useful for research and scientific purposes, particularly for close-range laboratory applications. This is why this paper is presented to Commission V rather than to Commission III.

BLOCK TRIANGULATION WITH SELF-CALIBRATION

The conventional block triangulation approaches may be categorized broadly as "polynomial", "independent-model" and "bundle" approaches. The polynomial approach is rather classical although still used today. It requires more control points and yields poorer accuracies than any other approach due possibly to lack of mathematical rigour. Therefore, the word "block triangulation" is presently almost exclusively taken to mean the rigorous approaches either with independent models or with bundles.

Theoretically, the bundle approach is expected to provide higher accuracies than the independent-model approach, since the latter requires an additional procedure of relative orientation to produce independent models, which implies possible degradation of accuracies. However, comparative tests between the two approaches have shown that, contrary to theoretical expectations, there is no significant superiority of the bundle approach to the independent-model approach unless the image coordinates are free of systematic distortion[11].

The existence of systematic errors usually plays havoc with restitution of a bundle of rays, since it is only uncorrelated random errors that are commonly postulated for the least-squares principle on which the vast majority of block adjustments are based. Consequently, it is a major concern of calibration to single out systematic criteria of image distortion.

Depending upon the degree of functional sophistication, calibration techniques are commonly classified as three basic categories: "pre-calibration", "on-the-job calibration" and "self-calibration"[17]. A most rigorous calibration in this context can be accomplished with the method of self-calibration. This technique differs significantly from the others in that it relies for the determination of interior orientation on the distribution of unknown object points rather than known object control points, and on the projective geometry of the multi-stereo formation overlapped by two or more photographs.

As indicated by its designation, self-calibration was initially devoted to a complete recovery of interior orientation in the absence of absolute control information. This was successful in several experiments with the aid of the strong configurations afforded by somewhat unusual photography such as highly convergent photography, and by the specific extension of the mathematical constraints[7,20,23].

When applying self-calibration in general practice such as in aerotriangulation, the advantage of the strong configurations supported by the convergent photography is given up for the advantage of requiring the normal (vertical) case of photography. In this type of application, there are only mathematical constraints governing the positioning of the image-object rays. They are therefore much freer to move around during the solution, and require good three-dimensional geometrical configurations in the object space, plus certain amount of ground control to effect strong mathematical solutions.

In the latter case, it is possible to introduce horizontal and vertical controls separately, and their minimum requirements are theoretically independent of the number of unknown calibration parameters being carried. However, a problem of block triangulation consists essentially in determining the absolute dimension of the object space, which makes the inclusion of control information inevitable regardless of whether self-calibration is used or not.

The extension of the conventional triangulation approaches, primarily developed for the bundle approaches, has been accomplished by incorporating additional compensation parameters as part of the unknowns in the mathematical formulation, thus permitting simultaneous recovery of these parameters at the exact instant of object photography. This approach is called bundle adjustment with additional parameters or self-calibration. The first successful application of self-calibration into conventional aerotriangulation was presented by Bauer and Müller[2] in 1972. Since then, the application of this computational procedure to many other approaches followed immediately[9,12,15,19,21,26,29].

COMPENSATION PARAMETERS IN SELF-CALIBRATION

In early approaches self-calibration was practically limited to the determination of the basic interior orientation parameters (principal point and principal distance) without carrying image distortion as unknowns[23]. Depending upon the practical considerations of introducing additional parameters, systematic image distortion has been parameterized with many variations, which can be divided into two major categories.

The first is concerned with the "decomposition" of image distortion into various components of lens distortion and film deformation. This procedure has been predicated on the assumption that all factors affecting image perturbations could be physically interpreted, thereby predicting an individual mathematical model for a specific distortion component. Such an interpretation is compatible with the conventional calibration parameters for symmetric radial lens distortion[4], decentering lens distortion[5,6, 10] and affinity film distortion[16].

A somewhat more complex situation exists when treating the sophisticated distortion characteristics that are not explicitly interpretable, and hence the "composite effects" of systematic errors are modelled as an entity regardless of the contribution from each individual source. In this case, image distortion is considered as representing collectively a population of systematic errors whose members are not explicitly specified. This forms the basis for the second category of parameterization technique.

Recently, there has been a strong tendency to rely on this technique, which is essentially connected with the attemp to introduce computational convenience characterized by an orthogonality of functions such that computational problems due to ill-condition may be better controlled. The mathematical functions in this category consist principally of two classes of functions, namely algebraic polynomials[8,9,13] and trigonometric functions [2,3,22,24]. Included with the latter is also the spherical harmonics function[14], which has recently received attention in developing the self-calibration systems.

In both of these cases the parameterization techniques may face a criticism that the actual form of the distortions may not fully conform to the prescribed distortion functions. However, because of the nature of the stochastic approximation, it is practically impossible for any function to perfectly synthesize various distortion characteristics without excluding the unpredictable components. Despite a large volume of theoretical work, the choice of parameters is still empirical, and the accuracy of self-calibration, therefore, depends largely on how well the selected parameters define the "reality".

"BLOCK-INVARIANT" AND "PHOTO-VARIANT" SELF-CALIBRATION

Although two distinctive procedures for self-calibration were conceived at the outset, much effort has been expended to depict its potential with only one of them. In reviewing numerous self-calibration approaches cited in the literature, it is perhaps understandable that the majority of investigations has been principally concerned with calibration of aerial cameras for which a set of compensation parameters is commonly forced to remain invariant throughout the block of photographs. This procedure, known as block-invariant self-calibration[8,13], forms the conceptual basis for many self-calibration approaches in use today.

The block-invariant procedure, however, has some serious drawbacks that can cause bottlenecks in production environment. Obviously, the validity of block-invariancy must be questioned in many instances where there is no uniformity of interior orientation between a series of exposures. This raises difficulties, for instsnce, in aerotriangulation where the photography employs more than one cameras with different interior orientation. Even for one camera, time-dependent variations in film and lens distortion could occur in different missions. An alternative difficulty also arises in close-range photography, which usually requires a different focus setting for each exposure. Therefore, existing block-invariant self-calibration is theoretically restricted to the homogeneous photography, however such an ideal condition is very limited in actual practice. Despite this limitation, most of the references cited thus far have uncritically adopted the block-invariant concept as accounting for self-calibration.

The shortcomings raised by block-invariant self-calibration can be offset if an independent set of compensation parameters is postulated for each individual photograph involved[26]. This procedure is termed here as photo-variant self-calibration. Although the idea of photo-variancy has been sporadically proposed in the literature[13], the successful application of this approach to block triangulation has been very limited until now.

When comparing the two procedures, it is important to bear in mind an essential requirement for computational design such as to maintain its determinacy. This requirement is usually met by making an appropriate selection of unknown parameters such as to avoid excessively high correlations between them. From this point, the block-invariant procedure is highly advantageous since the number of compensation parameters remains the same no matter how many photographs are included in the solution. Therefore, block-invariant self-calibration has always a recourse to the determinacy provided by a "limited" number of unknowns within which the unfavorable correlations can be controlled.

In contrast to the block-invariant scheme, the photo-variant formulation increases the number of unknown compensation parameters linearly with the number of photographs. This takes immediate effect by generating mutually ill-conditioned correlations among the freely increasing number of unknowns. In addition, the great increase in the number of unknowns makes a more laborious computational effort inevitable, especially in solving a large system of normal equations. This unfavorable situation has naturally caused the majority of investigators to use the block-invariant procedure without a serious risk of running into ill-conditioning problems. The widespread adoption of the block-invariant procedure, together with its computational economy, has tended to overshadow the shortcomings inherent to this procedure.

It is now necessary to complement the block-invariant concept with its counterpart now defined as photo-variant scheme. Although computer intensive, photo-variant self-calibration is a far more desirable method of calibration applicable for any type of camera (metric or non-metric) and photography (aerial or close-range), which is virtually independent of the application. It is in this context that the photo-variant scheme is regarded as the most powerful form of self-calibration, and in fact this is one major reason for this research.

"PHOTO-VARIANT" SELF-CALIBRATION PROGRAM

The self-calibration system, proposed in this paper, has been developed for application to bundle triangulation in general, and the photo-variant procedure in particular. The mathematical basis was laid down earlier in the experimental program system UNBASC1[16] for a limited number of photographs, which the author developed at the University of New Brunswick. The interim phase of this program design was presented to the previous congress of the International Society for Photogrammetry[26].

Development and refinement of the software actually spanned a number of years, during which the experimental program was extended into two different versions. The new computer programs (named UNBASC2 and CMPASC3) make explicit provision for photo-variant self-calibration with no limitations placed on the number of photographs, image points and object space controls to be handled. The two programs differ widely in accommodating the compensation parameters, however both are based on the same form of the collinearity condition equations. Since a fuller derivation of these equations can be found elsewhere, the description given herein is meant only to bring out a short summary.

The collinearity condition, including compensation parameters, can be expressed in analytical form:

$$
\begin{aligned}
F_{x_{ij}} &\equiv [(x_{ij} - x_{0_j}) + \Delta x_{ij}]\cdot M_{3_j} + c_j\cdot M_{1_j} = 0 \\
F_{y_{ij}} &\equiv [(y_{ij} - y_{0_j}) + \Delta y_{ij}]\cdot M_{3_j} + c_j\cdot M_{2_j} = 0
\end{aligned}
\qquad \text{.....(1)}
$$

where $[M_1, M_2, M_3]_j^T = [R]_j\cdot[(X_i - X_{c_j}),\ (Y_i - Y_{c_j}),\ (Z_i - Z_{c_j})]^T$

and $(X, Y, Z)_i$ are unknown ground coordinates of object point i,

$(X_c, Y_c, Z_c)_j$ are unknown ground coordinates of exposure station of photograph j,

$[R]_j$ is the 3×3 rotation matrix of photograph j in terms of rotations ω, ϕ and κ,

$(x, y)_{ij}$ are comparator coordinates of object point i measured on photograph j,

$(x_0, y_0, c)_j$ are unknown parameters of principal point and principal distance of photograph j,

$(\Delta x, \Delta y)_{ij}$ are the x and y components of image distortion of object point i on photograph j.

The object space control coordinates are incorporated into the solution separately in X, Y and Z, and therefore they may be either horizontal or vertical control in the following condition equations:

$$
\begin{aligned}
G_{x_i} &\equiv X_{G_i} - X_i = 0 \\
G_{y_i} &\equiv Y_{G_i} - Y_i = 0 \\
G_{z_i} &\equiv Z_{G_i} - Z_i = 0
\end{aligned}
\qquad \text{.....(2)}
$$

where $(X_G, Y_G, Z_G)_i$ are the known coordinates of control point i.

Consider the condition equations as a function of two sets of unknowns $\bar{X}_1$, $\bar{X}_2$, and the observed quantities L:

$$
\begin{aligned}
\bar{X}_1 &= (\Delta x, \Delta y, x_0, y_0, c, X_c, Y_c, Z_c, \omega, \phi, \kappa) \\
\bar{X}_2 &= (X, Y, Z) \\
L &= (x, y, X_G, Y_G, Z_G)
\end{aligned}
$$

then, equations (1) and (2) can be expressed in general form as:

$$
\begin{aligned}
F(\bar{X}_1, \bar{X}_2, L) &= 0 \\
G(\bar{X}_2, L) &= 0
\end{aligned}
\qquad \text{.....(3)}
$$

The system of condition equations (3) is non-linear, and any redundancy in observations lends itself to a least-squares adjustment. Therefore, the solution is based on a linearized form of the condition equations following the least-squares principle. Note that the object space control coordinates are treated as observed quantities, so that their individual accuracies may be differentiated in the least-squares adjustment.

So far the mathematical formulation has been confined to the condition equations with no consideration being given to the compensation parameters. As mentioned earlier, analyses of these parameters have led to the two different versions of self-calibration programs.

Self-Calibration with UNBASC2:

The UNBASC2 program utilizes the conventional distortion functions of radial-symmetric and decentering lens distortion[4,6], as well as film shrinkage and non-perpendicularity of the comparator axes[16]. By introducing the latter, the image coordinates are solely based on the comparator system, which makes the need for fiducial marks optional rather than mandatory. The compensation parameters Δx_{ij} and Δy_{ij} in equations (1) are defined as:

$$\begin{aligned}
\Delta x_{ij} &= (x_{ij} - x_{0j})(k_{1j} \cdot r_{ij}^2 + k_{2j} \cdot r_{ij}^4 + k_{3j} \cdot r_{ij}^6) \\
&\quad + p_{1j} \cdot [r_{ij}^2 + 2 \cdot (x_{ij} - x_{0j})^2] + 2 \cdot p_{2j} \cdot (x_{ij} - x_{0j})(y_{ij} - y_{0j}) \\
&\quad + A_j \cdot (y_{ij} - y_{0j}) \\
\Delta y_{ij} &= (y_{ij} - y_{0j})(k_{1j} \cdot r_{ij}^2 + k_{2j} \cdot r_{ij}^4 + k_{3j} \cdot r_{ij}^6) \\
&\quad + p_{2j} \cdot [r_{ij}^2 + 2 \cdot (y_{ij} - y_{0j})^2] + 2 \cdot p_{1j} \cdot (x_{ij} - x_{0j})(y_{ij} - y_{0j}) \\
&\quad + B_j \cdot (y_{ij} - y_{0j})
\end{aligned} \qquad \ldots\ldots(4)$$

where $r_{ij}^2 = (x_{ij} - x_{0j})^2 + (y_{ij} - y_{0j})^2$

and $(k_1, k_2, k_3)_j$ are the parameters for symmetric radial lens distortion of photograph j,

$(p_1, p_2)_j$ are the parameters for decentering asymmetric radial and tangential distortion of photograph j,

$(A, B)_j$ are the parameters for scale differences along the comparator axes and their possible non-perpendicularity on photograph j,

from which it is possible to interpret some of the distortion characteristics by the following relations:

$$P = r^2 \cdot \sqrt{[p_1^2 + p_2^2]} \qquad \theta = \tan^{-1}[-p_1/p_2]$$

$$\lambda_y = \sqrt{[A^2 + (B+1)^2]} \qquad \beta = \tan^{-1}[A/(B+1)]$$

where P is the profile of maximum decentering distortion along the axis defined by the anti-clockwise angle θ from x-axis, and λ_y is the scale factor for y-axis relative to a unit scale along x-axis, β being the angular deviation from the orthogonality between x- and y-axes[6,16].

Due to expected changes in interior orientation between a series of exposures, a set of the ten calibration parameters (x_0, y_0, c, k_1, k_2, k_3, p_1, p_2, A, B) is recovered for each photograph besides the six parameters of exterior orientation (X_c, Y_c, Z_c, ω, ϕ, κ). This leads to, at most, sixteen parameters per photograph, plus the X, Y, Z coordinates of every observed point in the object space to be carried as unknowns and solved for simultaneously in the UNBASC2 program.

The foregoing formulation, however, has the disadvantage that some of the parameters are strongly correlated. This sometimes results in an ill-conditioned system of normal equations. Therefore, the solution of the unknowns requires a special consideration such as to minimize the disturbing effects of these correlations. Basically, this is done by segmenting the set of unknown parameters into several subsets in the UNBASC2 program. This process is called "fixed" or "sequential segmentation", and will be explained in some detail later.

Self-Calibration with CMPASC3:

The CMPASC3 program has been designed to acquire better computational control over the problem of ill-condition. More specifically, the mathematical development has started with the consideration of constraining all distortion parameters to a system of harmonic functions. A notable feature of the harmonic functions is that they are orthogonal. Orthogonal functions are generally useful because any tendency of ill-conditioning can easily be counteracted. This feature has a definite bearing on selecting the distortion functions, particularly for the photo-variant approaches.

The most important harmonic functions are those of spherical harmonics. The idea of applying spherical harmonics to self-calibration is not new. Brown [9] was probably one of the earliest to advocate the use of spherical harmonics for modelling the film plane as being a three-dimensional surface; yet his discussion was brief and was strictly prediction. More recently, the spherical harmonics model was adopted by Elhakim and Faig in their treatment of block-invariant self-calibration [14]. Although deviated from theoretical properties of spherical harmonics, their practical formulation employs the "anamorphic" model of two-dimensional surface harmonics such that:

$$\begin{aligned}\Delta r = \; & A_{00}\cdot r + A_{20}\cdot r^2 \\ & + A_{11}\cdot r\cdot\cos\lambda + A_{22}\cdot r^2\cdot\cos 2\lambda + A_{31}\cdot r^3\cdot\cos\lambda + A_{33}\cdot r^3\cdot\cos 3\lambda \\ & + B_{11}\cdot r\cdot\sin\lambda + B_{22}\cdot r^2\cdot\sin 2\lambda + B_{31}\cdot r^3\cdot\sin\lambda + B_{33}\cdot r^3\cdot\sin 3\lambda\end{aligned} \quad \text{.....(5)}$$

where $r = \sqrt{[(x - x_0)^2 + (y - y_0)^2]}$

$\lambda = \tan^{-1}[(y - y_0)/(x - x_0)]$

and Δr is the radial component of image distortion,

A_{ij} and B_{ij} are unknown coefficients.

Analogous to the two-dimensional transformation in the previous model, the CMPASC3 program makes direct use of Legendre's (associated) functions, after certain modifications, such that:

$$\Delta r = C_{10}\cdot r\cdot\cos\theta + C_{11}\cdot r\cdot\sin\theta + C_{20}\cdot r^2\cdot(\cos 2\theta + \frac{1}{3})$$
$$+ C_{21}\cdot r^2\cdot\sin\theta\cdot\cos\theta + C_{22}\cdot r^2\cdot\sin^2\theta + C_{30}\cdot r^3\cdot(\cos 3\theta + \frac{3}{5}\cos\theta)$$
$$+ C_{31}\cdot r^3\cdot\sin\theta\cdot(\cos^2\theta - \frac{1}{5}) + C_{32}\cdot r^3\cdot\sin^2\theta\cdot\cos\theta + C_{33}\cdot r^3\cdot\sin^3\theta \quad .....(6)$$

where $\theta = \tan^{-1}[(x - x_0)/(y - y_0)]$

and C_{ij} are unknown coefficients, all other notations are the same as before.

Recently, the CMPASC3 program has been entirely revised to accommodate all three kinds of distortion functions as defined by equations (4), (5) and (6). It has been written in such a general manner that the user can choose all or part of the unknown parameters in any function desired, and perform the calibration accordingly. Initial tests have indicated a significant accuracy improvement (about 40%) with the harmonic functions when compared to the calibration with the function (4). The same conclusion has been arrived at by Elhakim and Faig[14,15] from their comparison between the functions (4) and (5) in the block-invariant mode. This statement, however, is based on a limited number of tests only, and more thorough tests are presently being conducted.

STABILITY CONTROL - FIXED OR SEQUENTIAL SEGMENTATION

The solution of a mathematical model for self-calibration is governed largely by correlations between the unknown parameters. Generally, if any two parameters are mutually correlated to a significant extent, both tend to perform the same function and the entire system of normal equations becomes unstable, ill-conditioned or singular, depending upon the strength of correlations. A knowledge of these correlatios, therefore, is the key to stability control which in turn improves the efficiency of a calibration system. This is particularly important for the photo-variant procedure, where the amount of unknown parameters increases rapidly with the number of photographs.

Problems of ill-conditioned correlations are usually controlled by treating all unknown parameters as weighted observations with "apriori" knowledge for weights in terms of variances-covariances. This concept has been successfully utilized for several block-invariant self-calibration programs[8,13,15]. A possible disadvantage of this method is that it requires the inversion of the normal equation matrix, because the variance-covariance matrix of the adjusted parameters has to be evaluated each time new apriori constraints are estimated. This requirement presents computational inefficiency in the calibration programs where the normal equations are solved direcly without using the inversion technique such as in the UNBASC2/CMPASC3 system. In this case, the inclusion of the additional inversion of the normal equation matrix is no longer a simple matter.

Without resorting to the variance-covariance constraints, the computational determinacy is relatively easily maintained by an appropriate disposition of unknown parameters in the mathematical model. In this approach, the set of parameters under consideration is segmented into several uncorrelated subsets. The actual evaluation of the mathematical model is then carried out with different combinations of the subsets in order to avoid ill-conditioning. Only one subset of the parameters is carried as unknowns at a time while the others are kept fixed. The unknown and fixed subsets are alternatively interchanged in each successive iteration step until proper convergence is reached. This process is called "fixed segmentation" which was utilized in the interim phase of the UNBASC2 programming.

This concept has been further extended such that the unknown parameters can be segmented in any iteration step according to the user's choice. A small number of unknowns is solved for first, the values obtained being the approximations used in the next iteration, when a few more unknowns are included. This process is continued until the improved approximations to the unknowns become stable, and the complete solution can then be made by iterating with all the unknowns included. Thus, this approach is termed here "sequential segmentation" and is successfully devised in the latest version of the UNBASC2/CMPASC3 program system.

The correlations are very sensitive to geometrical configurations in the object space, and may vary for different types of photography. However, a number of practical tests have indicated that, although the absolute values of the correlation coefficients differ considerably for each photography, their correlation patterns remain basically the same for the normal case of photography. Typical segmentation required of the normal case of photography is as follows:

$$(X_C, Y_C, \kappa, X, Y)$$
$$(X_C, \phi, X, Z)$$
$$(Y_C, \omega, Y, Z)$$
$$(X_C, Y_C, Z_C, \omega, \phi, \kappa, X, Y, Z)$$
$$(X_C, Y_C, Z_C, \omega, \phi, \kappa, X, Y, Z, A, B)$$
$$(X_C, Y_C, Z_C, \omega, \phi, \kappa, X, Y, Z, A, B, x_0, y_0, c)$$
$$(X_C, Y_C, Z_C, \omega, \phi, \kappa, X, Y, Z, A, B, x_0, y_0, c, k_1, k_2, k_3)$$
$$(X_C, Y_C, Z_C, \omega, \phi, \kappa, X, Y, Z, A, B, x_0, y_0, c, k_1, k_2, k_3, p_1, p_2)$$

where the notations are the same as in equations (1) and (4).

In the program CMPASC3, the mathematical model has been made closely orthogonal between the distortion parameters. Therefore, problems of correlations are essentially non-existent within the distortion parameters, although their correlations to other unknown parameters of interior and exterior orientation are not negligible. For this reason, sequential segmentation is still needed for the orthogonal model of CMPASC3, however the amount of correlations is usually very small compared to those produced by the UNBASC2 model.

The real item of interest in segmenting the parameters is that problems of ill-conditioning play a role only in the early steps of the iterative solution. This can be proven by the fact that the computational determinacy can still be maintained even when all parameters are carried as unknowns simultaneously in the later iteration steps. This phenomenon naturally leads to the supposition that the majority of the ill-conditioning problems arising in photo-variant self-calibration can be attributed to some interaction among the unstable initial approximation values for the unknowns.

There are some other cases where ill-conditioned systems are generated, most of them arise due to weak geometrical configurations in the object space. There has been no firmly established basis for defining the strength of geometrical configurations. However it is empirically known that the height variation in the terrain profile should agree with its recommended ratio to the projection distance (camera height), usually at least 10% being required for most photography. If this condition is not met, the two subsets of parameters (x_0, y_0, c) and (X_C,Y_C,Z_C) are almost perfectly correlated. This type of correlation cannot be counteracted unless additional restraints are enforced, which means that, if (X_C, Y_C, Z_C) are carried as unknowns, (x_0, y_0, c) will have to be held fixed, and vice versa. The most critical parameter appears to be c which is normally strongly correlated with most other parameters. Therefore, c is often held fixed if its value is known to a high accuracy.

PRACTICAL APPLICATIONS AND TESTS

Before concluding this article, it is necessary to illustrate the power of photo-variant self-calibration by presenting some of the results available to date. The photo-variant system developed in this research comprises two programs named UNBASC2 and CMPASC3. The latter program is an extension of the former, and includes two orthogonal models for calibration parameters. Initial tests have yielded very good results with more thorough tests presently being conducted. Unfortunately, analysis of the results from the CMPASC3 solution is not complete at the time of preparing this manuscript, which means that the results reported here are based on the UNBASC2 program only.

To judge the UNBASC2 performance under different operational conditions, several simulations have been carried out by the author and his associates, and are documented in references[25,27,28]. The specific simulations selected for presentation here are those for two types of applications, namely close-range laboratory photogrammetry and aerotriangulation. These represent extremely different situations and encompass the field of photogrammetry quite well.

Case 1. Application in Close-Range Photogrammetry

A laboratory model of a pre-stressed cable net, together with 75 horizontal and 80 vertical control points, was photographed with different focussing distances. The camera was hand held (non-metric ASAHI-PENTAX, f ≃ 25 mm.), and a series of four photographs were taken at an approximate photo-scale of 1:18 such that all of them overlapped each other. The control frame consists of a precision grid as well as metal bolts with known but different lengths which represent the elevations when fixed perpendicularly to the grid plate. The control frame was precisely coordinated to an accuracy of 0.035 mm. in planimetry and 0.009 mm. in height.

Tables 1 and 2 summarize the results of this simulation. Table 1 shows that the residual parallaxes at the object points are reduced significantly when calibration parameters are included. This means that a better intersection is achieved. The power of self-calibration becomes evident when one compares reduced control with the number of unknowns in the solution as shown in Table 2. This is the control reduction which is likely to be most advantageous, as rigorous calibration can be performed with little control, much less than for any other calibration approach. Unfortunately, the advantage of requiring fewer control is in most cases balanced by the practical disadvantage that the solution converges very slowly. Also evident from Table 2 is the general trend of improved check point accuracy with increasing calibration parameters and control.

Case 2. Application in Aerotriangulation

Although three aerotriangulation blocks are selected for presentation here, it is obvious that complete self-calibration can only be tested with one of them. The P.E.I. and Gloucester blocks are photographed from a high altitude, which, due to rather uniform terrain, acts in a two-dimensional fashion. It is therefore necessary to fix the principal distances (c) in the solution. The purpose is rather a thorough check of the basic self-calibration method without carrying the principal distances as unknowns. This is most likely to occur in production environment. The Sudbury block, although small with only 25 photographs, covers a more varied terrain, and represents an ideal three-dimensional situation because of lower flying altitude.

The test results are shown in Tables 3 and 4. In all three block adjustments, the residuals at the both object and control check points, as computed with self-calibration, are consistently smaller than those obtained from the adjustment without self-calibration. It is readily seen that treating the principal distances as known and equal to the precalibrated focal lengths does not lead to appreciable loss of accuracy. This fact indicates that the basic self-calibration approach underlying the system is properly working. The accuracy improvement, when using self-calibration, is not as remarkable as might be desired. This can be explained by the fact that the ground control was designed and surveyed to the mapping standard only. This could, of course, have been taken into account if accuracy information were available for each individual control point. Unfortunately, only a general indication by means of standard deviation for all points was given, which does not permit any differentiation in weighting.

Table 4 shows the speed of the solution achieved by the UNBASC2 program. The example in this table is taken from the Gloucester block adjustment. This block was also adjusted by the "independent-model" aerotriangulation program PATM43, and its time requirements are also shown in the same table for comparison purposes. It is worth mentioning that the UNBASC2/CMPASC3 program system uses a highly optimized technique for the solution of a large system of normal equations. It is based on the Cholesky's block factorization algorithm, and makes full use of external storage if there is not enough space available in the central memory, which permits the solution of an indefinite number of unknowns. The technique of block factorization, coupled with the aforementioned sequential segmentation, ensures further optimization for the photo-variant self-calibration. This is evident from the comparison with the PATM43 results.

CONCLUDING REMARKS

The practical tests have shown that the photo-variant self-calibration system works properly. It is general in nature, and can accommodate most practical situations. The photo-variant approach permits the use of different cameras for the same block without restrictions. As indicated by all simulation tests, savings in the number of control points are reduced by increased computational efforts. This is particularly the case in this photo-variant approach where the more general mathematical modelling leads to a sizable increase in the number of unknown parameters. However, an efficient solution algorithm was developed which keeps the computation time within reasonable range, as shown in Table 4. It is important to note, however, that drastic reduction in object space control requires more iterations with each one adding more computing time. It may therefore not always be the most economical way to proceed with fewer control.

Another difficulty with the photo-variant approach was the treatment of correlations between the unknown parameters. While in the block-invariant approach where apriori variances can be assigned to the unknown parameters by treating them as weighted observations[8,13], it is impractical to follow such a procedure in the photo-variant approach because of the increasing number of unknowns. This difficulty has been successfully overcome by introducing sequential segmentation of the mathematical model. Having passed these obstacles, a workable photo-variant system was obtained, which models the physical situation more precisely both from theoretical and practical poits of view than the block-invariant procedures.

It has to be emphasized that no self-calibration approach can properly function for a flat object or terrain unless additional restraint are introduced. The photo-variant self-calibration method is no exception. Other than that, the system is not restricted to a certain project type, and can effectively handle close-range laboratory and terrestrial projects as well as aerial block triangulation. This is a big advantage over the block-invariant approaches, because for non-metric photography, invariancy of interior orientation parameters between exposures cannot be assumed.

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TABLE 1. SELF-CALIBRATION WITH DIFFERENT PARAMETERS† (CLOSE-RANGE STRUCTURE MODEL)

CALIBRATION PARAMETERS	RESIDUALS AT IMAGE POINTS [RMS in mm.]		RESIDUAL PARALLAXES AT OBJECT POINTS [RMS in mm.]			NUMBER OF UNKNOWNS
	(x)	(y)	(X)	(Y)	(Z)	
x_0, y_0, c	.013	.011	.120	.101	.044	276
x_0, y_0, c k_1, k_2, k_3	.012	.011	.059	.046	.020	288
x_0, y_0, c p_1, p_2	.012	.010	.057	.048	.019	284
x_0, y_0, c A, B	.011	.009	.054	.043	.018	284
x_0, y_0, c k_1, k_2, k_3 p_1, p_2	.012	.010	.056	.043	.019	300
x_0, y_0, c p_1, p_2 A, B	.011	.009	.053	.041	.017	282
x_0, y_0, c k_1, k_2, k_3 A, B	.011	.008	.053	.037	.016	296
x_0, y_0, c k_1, k_2, k_3 p_1, p_2, A, B	.004	.003	.035	.024	.010	304

† based on 75 horizontal and 80 vertical control points.

TABLE 2. SELF-CALIBRATION WITH DIFFERENT PARAMETERS AND CONTROL (CLOSE-RANGE STRUCTURE MODEL)

NUMBER OF CONTROL POINTS (USED) H-V	NUMBER OF CONTROL POINTS (CHECK) H-V	TYPE† OF CALIBRATION	RESIDUALS AT IMAGE POINTS [RMS in mm.] (x)	(y)	RESIDUAL PARALLAXES AT OBJECT POINTS [RMS in mm.] (X)	(Y)	(Z)	RESIDUALS AT CHECK POINTS [RMS in mm.] (X)	(Y)	(Z)	NUMBER OF UNKNOWNS	NUMBER OF ITERATIONS
75-80	00-00	1	.013	.011	.120	.101	.044	---	---	---	276	10
		2	.004	.003	.035	.024	.010	---	---	---	304	12
30-80	45-00	1	.013	.011	.119	.105	.042	.131	.126	---	276	12
		2	.005	.003	.040	.035	.012	.038	.029	---	304	13
10-10	65-70	1	.011	.012	.140	.111	.051	.153	.147	.149	276	15
		2	.006	.008	.055	.061	.019	.043	.038	.051	304	18
4-10	71-70	1	.012	.010	.138	.109	.065	.188	.173	.205	276	19
		2	.006	007	.053	.052	.023	.057	.048	.097	304	23

† TYPE 1: self-calibration parameters x_0, y_0, c
† TYPE 2: self-calibration parameters x_0, y_0, c, k_1, k_2, k_3, p_1, p_2, A, B

TABLE 3. EFFECT OF SELF-CALIBRATION IN AEROTRIANGULATION

BLOCK	PHOTO SCALE	NUMBER OF CONTROL POINTS (USED) H-V	NUMBER OF CONTROL POINTS (CHECK) H-V	TYPE† OF CALIBRATION	RESIDUAL PARALLAXES AT OBJECT POINTS [RMS in metre] (X)	(Y)	(Z)	RESIDUALS AT CHECK POINTS [RMS in metre] (X)	(Y)	(Z)
SUDBURY (25 photos)	1: 4000	12-14	63-30	1	.013	.015	.008	.074	.075	.149
				2	.009	.010	.004	.067	.057	.135
P.E.I. (39 photos)	1:24000	10-21	47-47	1	.084	.079	.042	.123	.180	.228
				2*	.064	.072	.034	.068	.102	.140
GLOUCESTER (85 photos)	1:35000	6-20	39-25	1	.053	.180	.062	.244	.229	.481
				2*	.036	.088	.055	.176	.183	.264

† TYPE 1: without self-calibration parameters
† TYPE 2: with self-calibration parameters (x_0, y_0, c, k_1, k_2, k_3, p_1, p_2, A, B)
* with principal distances (c) fixed

TABLE 4. COMPUTING TIME WITH DIFFERENT NUMBER OF CONTROL POINTS

NUMBER OF CONTROL POINTS (USED) H-V	(CHECK) H-V	NUMBER OF UNKNOWNS	NUMBER OF ITERATIONS	TOTAL CPU SEC.	CPU SEC. PER ITERATION	PROGRAM
45-45	00-00	2919	4	577	144	UNBASC2
30-45	10-00	2919	4	543	136	UNBASC2
22-45	23-00	2919	5	632	126	UNBASC2
6-20	39-25	2919	7	746	107	UNBASC2
6-10	39-35	2919	10	1048	105	UNBASC2
45-45	00-00	1889	4	541†	135	PATM43
6-10	39-35	1889	5	654†	131	PATM43

† including the CPU time for image refinement and formation of independent models.

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Abbreviations used:

A.S.P. - American Society of Photogrammetry
BuL - Bildmessung und Luftbildwesen
I.S.P. - International Society for Photogrammetry
PE - Photogrammetric Engineering (and Remote Sensing)