14th CONGRESS OF THE INTERNATIONAL
SOCIETY OF PHOTOGRAMMETRY - HAMBURG - 1980

COMMISSION II

Invited paper

ANALYSIS AND COMPARISON OF THE WORKING PRINCIPLES OF THE DIGITAL GEOMETRY STEREOPLOTTERS (DGSs).

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Abstract

After the definition of Digital-Geometry Stereoplotter the two available mathematical models are demonstrated. Five operational modes related to the degrees of freedom of the operator are defined: orientation mode, space constrained stereo pointing mode, surface constrained stereo pointing mode, line constrained stereo pointing mode, point constrained stereo pointing mode; for each mode the role of the "conditioned input" is discussed.
Some general considerations are made about the design of the stereocomparators and the role of the computer of the DGSs.

Definition of the Digital-Geometry Stereoplotters.

1 - In this paper a tentative has been made of dealing with the analytical stereoplotters in the most general way; the working principles, the functional and operative features and some general design characteristics will be presented and analysed trying to forget the specific and limiting notions which are peculiar to the analogue stereoplotters.
The analytical stereoplotters dealt with in this paper are identified as Digital-Geometry Stereoplotters (DGS) according to the proposal of FORREST (1977), because this name seems to be the most appropriate to define the photogrammetric stereoplotters which according to the HELAVA formulation meet this requirement:

> the fundamental photogrammetric operation of projecting image points in the object space or viceversa of projecting object points on image surfaces is performed in an analytical way and in real time using a computer.

It follows that the basic components of a DGS are a stereocomparator and a computer and that these two components must be interfaced and provided with a real time routine; the peripherals of the computer shall provide means for different outputs of the photogrammetric products of the DGS.
The use of the stereocomparator as measuring device determines for the DGS the capability of using all the kinds of image materials, because through the flexibility of the software all kind of exposure geometries can be exploited.

2 - A definition must also be given for the photogrammetric products of a DGS:

> the photogrammetric products of a DGS are those obtained through an on line procedure which processes only the data measured through the stereocomparator.

Actually this definition is redundant and obvious for a stand-alone instrument, but it is necessary to the identification of a specific instrument in all the systems where the stereocomparator, or many stereocomparators, together other measuring devices are interfaced to a central processor. In this case the DGS consists of the stereocomparator and of the shares of the central processor and output peripherals, when the photogrammetric products of the system depends only on the metric information produced by the stereocomparator.
This definition establishes that an analytical system where indirect transfer of data take place is not a DGS even if the system is able to produce the same final products of a DGS. This establishment seems appropriate because the general working organization and the operative approach are different when the on line and the off line procedures are considered.

The definition does not take into account which specific products have to be produced by a DGS; actually the minimum capability requested for a DGS is that of producing ground coordinates in real time, but these coordinates can be processed on line in many ways depending on the power of the computer, the available peripherals and the available software. The situation seems to be the same as that of the analogue plotters where the capability ranges from a sheer cartographic capability to the capabilities of the universal stereoplotters, where terrestrial, tilted, aerial photographs can be exploited, cross sections and D.T.M. can be determined, the aerialtriangulation can be performed and so on.

Since the presence of the computer the capabilities of a DGS can be larger than those of the analogue plotters, but it is necessary to establish as in the definition that the products of a DGS are those obtained through an on line procedure.

Mathematical models of the DGS.

1 - The solution of the photogrammetric operation through a computer which operates in real time can be made according to the two ways shown in the definition.
The first one can be denoted as AP system and is that of reproducing through an analytical procedure the functions of an analogue plotter. In a mechanical projection plotter a point is located in the model space through a determined position of the X Y Z carriage and the rods have the function of locating on the photographs the corresponding image points. Similarly in a AP system a point is defined by the coordinates given as an input to the processor and the processor with servomechanisms determines the collimation on the photographs of the corresponding image points. As a result of this concept the four stages of the stereoviewer must continously be controlled by the processor.
The second way can be denoted as DS system and is simply that of using a stereocomparator interfaced to a processor as a stereoplotter.
The stereocomparator sends as an input the four photocoordinates to the processor and the processor determines in real time the ground coordinates. As a result of this concept an analytical plotter can be built where the four stages of the stereocomparator are manually controlled and no control of the processor on the stages is necessary.

2 - The equations which determine the real time routine can be considered as the mathematical model of a DGS, and precisely those equations which establish relationships between model coordinates and the photocoordinates. In an instrument which includes a computer the transformation of the coordinates from a reference system to an other can be easely performed such as the transformation from model rectangular coordinates to ground rectangular coordinates; this transformation is a linear transformation and does not change the geometric relationship between points, lines, surfa-

ces and so on. It can be noted that the map projection coordinates or the geodetic coordinates cannot be considered as ground rectangular coordinates, but it is always possible to transform the first types into the second type.

As a consequence in the following X, Y, Z denote model coordinates; x_1, y_1, x_2, y_2 denote photocoordinates and P a whole of relative orientation parameters.

3 - The mathematical model of an AP system is

$$
\begin{aligned}
x_1 &= x_1(X,Y,Z,P)\\
y_1 &= y_1(X,Y,Z,P)\\
x_2 &= x_2(X,Y,Z,P)\\
y_2 &= y_2(X,Y,Z,P)+ \Delta y
\end{aligned}
\tag{1}
$$

three independent output x_1, y_1, x_2 correspond to the three independent input X, Y, Z so that when the parameters P are not equal to the exposures parameters a fourth input Δy is necessary to put under pointing the four photocoordinates of a specific point. When the parameters P are equal to the exposures parameters the first three equations can give X, Y, Z as functions of x_1, y_1, x_2, which put in the fourth equation give

$$
y_2 - y_2(x_1, y_1, x_2, P) = \Delta y = 0 \tag{2}
$$

since $y_2(x_1, y_1, x_2, P)$ is the value of y_2 as derived by the values of x_1, y_1, x_2 and P, and this value, when P are the true parameters, coincides with y_2.

The equation (2) is the relative orientation condition.

4 - The value of a photocoordinate and the position of the perspective center with respect to the image surface determine a projection plane; four photocoordinates define 4 planes which intersect in 6 points, but the planes relative to x_1 and x_2 must always be taken into consideration because of the specific geometry of a stereopair and the intersections to be considered are only two.

Then the basic equations for a DS analytical system can be formally expressed as in the following:

$$
\begin{aligned}
X &= X(x_1, y_1, x_2, y_2, P)\\
Y' &= Y'(x_1, y_1, x_2, y_2, P)\\
Y'' &= Y''(x_1, y_1, x_2, y_2, P)\\
Z &= Z(x_1, y_1, x_2, y_2, P)
\end{aligned}
\tag{3}
$$

These equations establish in the most general way that the four independent measured image coordinates x_1, y_1, x_2, y_2, are the input of a computation which determines, through the orientation parameters P, four coordinates X, Y', Y", Z.

In order to get from four image coordinates the three coordinates X, Y, Z of a point in the object space is necessary that

$$(4) \qquad Y' = Y'' = Y$$

and it follows that when the orientation parameters P have true values the image coordinates x_1, y_1, x_2, y_2 are not independent and that a condition equation

$$(5) \qquad r(x_1, y_1, x_2, y_2, P) = 0$$

for the four image coordinates is established. This equation is the relative orientation condition equation and can be used for the determination of one image coordinate, for example y_2, as a function of the other three; in this way the corrispondence is established between three image coordinates x_1, y_1, x_2, and three coordinates X, Y, Z of a point in the object space.

<u>Manual input, conditioned input, operational modes.</u>

The DGSs we are dealing with are not automatic instruments, that is all the orientation and plotting operations are determined by the manual control of the operator. The manual input are the values X, Y, Z, Δy for the AP, or the values of x_1, y_1, x_2, y_2 for the DS, which are determined by manual operations on the handwheels or on the footwheel; actually the operator can operate using a different number of manual controls, this number ranging from 4 to 0; one can say that the degrees of freedom of the operator range from 4 to 0 according to 5 different "operational modes" which determine 5 specific ways of performing operations with the instrument.
In an analogue stereoplotter the no operation on the X carriage means that the model coordinate X does not change when the other two coordinate change, or in a stereocomparator the no operation on the x_1 carriage means to make stereopointing on a line x_1=const..
In a DGS the case of a not operated input can correspond as in the analogue plotter to a constant value input, but in the DGS the possibility exists of controlling with the computer the not operated inputs; the case when the computer controls the not operated input taking the information from a different input device, such for example that of keeping the track of a line giving the coordinates of points along the line, is a trivial case; the operative features of a DGS can be widely enlarged when a relationship, or relationships, can be established for the inputs so that the input, or the inputs, not operated is controlled by the computer as a function of the operated manual inputs. This case is identified as "conditioned input" and is the case of one or more condition equations which can be established for the four independent input values.

In short one can say that when the degrees of freedom are less than four in addition to the trivial case of constant input values conditioning equations can be used in order to obtain some specific results in the outputs, the maximum number of conditioning equations is of course the complement to four of the number of degrees of freedom.
The use of the conditioned inputs renders the DGS much more flexible than the analogue plotter, since with a digital geometry many results can be obtained which are not possible with the analogue plotter. It is necessary to point out that the use of the conditioned input is useful for expanding the capabilities of an AP system whereas is essential for a DS system.

Four degrees of freedom: orientation mode.

1-When the four inputs are manually and independently operated the only operation which can be performed is the determination of the orientation parameters.

2-When the orientation parameters P are not known the pointing of the fiducial marks or the stereo pointing of two photo images, can be obtained operating on the four independent inputs x_1, y_1, x_2, y_2 for a DS system or X, Y, Z, Δy for an AP system. In both the cases the operation leads to the storage of pairs of photocoordinates and to the computation of the orientation parameters.
In an AP system the coarse movements of the stages can be controlled by the computer after the storage of suitable initial parameters which determine congruent linkages between the manual movements and the stages movements.
The same applies to a DS system when a four stages computer control of stages is available; on the contrary when only a two stages control is available the coarse movements of the stages can be performed with the aid of manually controlled engines.

Three degrees of freedom: space constrained stereo pointing mode.

1 - When three inputs are manually and independently operated the operation can be performed of determining the coordinates of points in the object space, since the stereo pointing is constrained to individuate corresponding image points.

2 - DS system - The equation (5) determines a conditioned input; the photocoordinate y_2 (or y_1) can be determined as a function of the other three photocoordinates

$$y_2 = y_2(x_1, y_1, x_2, P) \qquad (6)$$

and put in the equations (3), so that

$$\begin{aligned} X &= X(x_1, y_1, x_2, P) \\ Y &= Y(x_1, y_1, x_2, P) \\ Z &= Z(x_1, y_1, x_2, P) \end{aligned} \qquad (7)$$

The value y_2 is computed and the computer can control the stage y_2 so that the operator looks at y-parallax free stereomodel.

It is necessary to point out that the equations (7) are valid for all the values of x_1, y_1, x_2 but the solutions which correspond to a real point of the object are those where to each pair of values x_1, y_1 corresponds a specific value of x_2. Actually the finding of this specific value is entrusted to the operator or can be performed automatically through an image correlator.

An other remark can be made with reference to a DS system where no control of y_2 can be made by the computer since no computer stage control is available.

Actually since the equations (7) do not depend on y_2, the measure and the control of y_2 could be avoided and the elimination of the y-parallax could be made through a footwheel with a mechanical control of the stage; this does not means that the degrees of freedom are four because the knowledge of y_2 is not necessary for the determination of X, Y, Z.

FORREST in 1971 proposed an instrument where a specially shaped floating mark is able to hide the y-parallax.

- AP system. The equations for the real-time routine are

$$
(8) \qquad
\begin{aligned}
x_1 &= x_1(X,\ Y,\ Z,\ P) \\
y_1 &= y_1(X,\ Y,\ Z,\ P) \\
x_2 &= x_2(X,\ Y,\ Z,\ P) \\
y_2 &= y_2(X,\ Y,\ Z,\ P)
\end{aligned}
$$

These equations which actually are the collinearity equations contain the condition (2) since the values of P are the true ones. The determination of y_2 could be avoided and the y_2 stage computer control could be substituted by a footwheel control, but this should be a nonsense in an AP system where 4 stages computer controlled are always available.

<u>Two degrees of freedom: surface constrained stereo pointing mode.</u>

1 - When two inputs are manually and independently operated the stereo pointing is constrained on a specific surface and the operation can be performed of finding the line intersection of the object and this specific surface, or specific relationships between them.

2 - DS system. Let us consider a mathematical surface such as

$$
(9) \qquad s_1\ (X,Y,Z) = 0
$$

Introducing the three equations (7) in the equation (9) a condition equation can be found for the three image coordinates

$$(10) \qquad g(x_1, y_1, x_2, P) = 0$$

thus determining a conditioned input.
From this condition equation the value of a whatsoever photocoordinate can be obtained as a function of the other two photocoordinates and the corresponding stage can be controlled by the computer; as an example

$$(11) \qquad x_2 = x_2(x_1, y_1, P)$$

The processor can compute the value x_2 and can control the stage x_2 so that the stereoscopic pointing is constrained on the surface $s_1(X, Y, Z) = 0$.
The most common use of this operational mode is that of determining a contour line. The surface is the simple

$$Z = \text{constant}$$

and the operator can determine the contour line using the manual movements x_1, y_1 and of course maintaining the stereoscopic floating mark on the terrain surface.
If the equation (9) is

$$Z = mX + nY + t$$

the operator can determine the intersection of a tilted plane with the terrain. If the equation (9) is

$$(12) \qquad Y-f(X) = 0$$

where f(X) is a polinomial in X, the operator can determine the intersection of the ground surface with a vertical cylinder defined by the equation (12); as a particular case the equation (12) could be

$$Y - pX - q = 0$$

and the operator can determine the intersection of the terrain with a vertical plane (cross sections); particular surfaces are the planes

$$X = \text{const.}$$

or

$$Y = \text{const.}$$

Only a problem of a major complexity in the formulas is the dealing with a second degree surface such for example a sphere

(13) $$X^2 + Y^2 + Z^2 - R^2 = 0$$

where the equation (10) becomes complex and some mathematical artifices should be adopted.

It is necessary to make some remarks about the photocoordinate which has to be controlled depending on the orientation and position of the surface on which the stereo pointing is constrained.

In a DS system the variation of the heights are obtained through the manual movement of the x_2 or x_1 stage; it is then obvious that the maintaining of the floating mark on a vertical plane perpendicular to the base and near to the right perspective center needs the computer control of x_2 the manual control of x_1 for the height variation, and the y_1 manual control for the translations of the floating mark; viceversa the computer control of x_1 and the free movement of x_2 and y_1 are needed for a vertical plane near the left perspective center and perpendicular to the base.

The computer control of y_2, together to the control of y_1 for the y-parallax clearing, and the free manual control of x_2 and x_1, is convenient for surfaces which are not much tilted with respect to the base.

The determining in practice of the intersection line terrain-surface with two degrees of freedom, i.e like the way of following a contour line, is made easier by the use of a pantograph which allows the intuitive correlation of the two free movements.

3 - AP system. The equation (9) made explicit to respect to Z is

$$Z = Z(X,Y)$$

which put in the equation (8) leads to

(14) $$\begin{aligned} x_1 &= x_1(X,Y,P) \\ y_1 &= y_1(X,Y,P) \\ x_2 &= x_2(X,Y,P) \\ y_2 &= y_2(X,Y,P) \end{aligned}$$

As an example if from (13)

$$Z = \pm (X^2 + Y^2 - R^2)^{1/2}$$

is obtained, the four photocoordinates given by (14) define the stereo pointing on a sphere; the operator using the two free manual input X, Y can find the intersection object-sphere.

Of course in the practical cases the equation (13) contains also linear terms and the obtaining of Z is more complex.

One degree of freedom: line constrained stereo pointing mode.

1 - When only one input is manually and independently operated the stereo pointing is constrained on a line and the operation can be made of finding the intersection point object-line

2 - DS system. Let us consider an other mathematical surface such as

$$s_2(X, Y, Z) = 0 \qquad (15)$$

Introducing the equations (7) in(15)an other condition equation can be found such as

$$g_2(x_1, y_1, x_2, P) = 0 \qquad (16)$$

If this equation is used together with equation (10) two photocoordinates can be determined as a function of the third one, for example both the photocoordinates x_2 and y_2 can be determined as a function of x_1

$$\begin{aligned} x_2 &= x_2(x_1, P) \\ y_1 &= y_1(y_1, P) \end{aligned} \qquad (17)$$

The processor can compute the three values y_1, x_2, y_2 and can control the three stages: y_1, x_2, y_2.
It is immediate to establish what these three controls mean. Because of the control of x_2 the collimation is constrained on the surface s_1; because of the control y_1 the stereo pointing is constrained on the surface s_2; then the stereo pointing is constrained on the line intersection of the two surfaces.
This line in the object space can be expressed in parametric equations, and the parameter could be Z: in other words the line on which the collimation is contrained can be expressed as

$$\begin{aligned} X &= X(Z) \\ Y &= Y(Z) \\ Z &= Z \end{aligned} \qquad (18)$$

The operating the handwheel (or footwheel) x_1, determines the variation of Z and according to the equations (18) the variations of X and Y. It is difficult to image what kind of plotting problem can be solved with this general capability, but it is easier to determine an use of it when the surfaces are X = const. and Y = const. that is are two planes perpendicular to the axes X and Y. In this case the function of the control of y_1, x_2, y_2 is that of maintaining the collimation on a vertical line; this is a normal operation in the analogue stereoplotters, and of course in the AP systems, where the moving of the Z carriage does not displace the X and Y carriages. This capability can be used for an easier following of the intersection of a vertical

surface with the terrain surface.
The values of X and Y along the trace which is to be followed are determined by an input or by a program; with these two values and the present value of Z the values and the computer control of y_1, x_2, y_2 are determined so that the variation of x_1 does not affect the computed values of X and Y.
It is necessary to point out that this operational procedure gives the same results of that obtainable with two degrees of freedom, but the following of a line on the ground in a whatever direction is performed in an easier way when only the height must be manually controlled.

3 - AP system. From the equations (18) put in the equations (8) is obtained

$$\begin{aligned} x_1 &= x_1(Z,P) \\ y_1 &= y_1(Z,P) \\ x_2 &= x_2(Z,P) \\ y_2 &= y_2(Z,P) \end{aligned} \qquad (19)$$

The four photocoordinates are determined only by the value of Z, by the relative orientation condition and by the conditions expressed by the first two equations (18).
The use of this operational mode has been above discussed.

No degree of freedom: point constrained stereo pointing mode.

1 - When no degree of freedom is available the only operation which can be performed is that of determining the stereo pointing of a point whose coordinates X, Y, Z are an input to the computer or in general are computed as an intersection of three surfaces, or the intersection of a line and a surface, or the intersection of two lines.

2 - This operational mode cannot be compared with the three degrees of freedom operational mode because the X, Y, Z must be known before the stereo pointing.
This operation is immediate in the concept of the AP system, but requests the fourth stage control in the DS system.
When in the DS system four simultaneously stages control are provided no difference can be found, even in the contents of the software of this specific operational mode, between the AP and the DS system.

The stereocomparator of a DGS

1 - The difference between a normal stereocomparator and the stereocomparator of a DGS is only that in addition to the measuring capability the last one needs servomechanisms for the control of the stages made by the computer.

The problem of controlling moving parts of machinery is a typical problem of the mechanical and electrical engineering; a field near to the DGS stereocomparator is that of the tool machinery where plates holding a tool or a workpiece must be moved according to specific programs. The state of the art of the mechanical and electronic control is such as to provide the best solutions for a DGS stereocomparator; the control has to provide for a fast, smooth, precise and not pendular movement of the stages which compared with the moving parts of the tool machinery are not heavy and do not present difficult problems for the inertia.
Of course the computer control of fast, large and precise movements of the stages requests more sophisticated devices than those necessary for the computer control of slow, small, precise movement of the stages; in this respect the differential movements of the stages need less complex servomechanisms.

2 - The accuracy of the stereocomparator determines the overall accuracy of the DGS since using the mentioned strict mathematical models it depends only on the accuracy of the measured stage coordinates.
It can be already observed in the existing eight DGSs (see Symposium on Analytical Plotters - Reston, USA, 1980) that the accuracy of the measured stage coordinates is no more restricted in the minimum cost-effective attainable of a few microns and that an accuracy up to a maximum error of 10 microns has been proposed. Actually it can be foreseen that also for the DGSs two different levels of accuracy shall be taken into consideration as it happened for the analogue stereoplotters; obviously the highest accuracy should be utilized for the aerial-triangulation, for the very large scale mapping of urban aereas or for small scale mapping using very high flights, whereas for all the other cartographic applications the less accurate stereocomparator could be adopted. These considerations on the accuracy should of course have a relationship with the cost of the DGS, but the fact that less accurate stereocomparators are proposed for the DGSs means that the tendence of the DGSs is that of substituting the analogue plotters in all the fields.

3 - The essential movement in the stereocomparator is the relative movement optic-stage; in the existing DGSs all the solutions for this relative motion have been adopted.

a) Each stage moves in x and y and the optical train is fixed; in this solution a simple optical train can be designed and good optical results can be obtained in a less difficult way. A disadvantage of this solution is that each stage should consist in an independent primary carriage and a secondary carriage which request more care to reach high accuracy; but to avoid these two superimposed carriages well conceived mechanical settings have been designed where the stage moves in x and y along two independent slides.

b) The optic moves in x and y and the stage is fixed; this solution is good to reduce to a minimum the size of the stereocomparator; actually the translations of a stage requests two times the lenght of the photograph in x and y, whereas the translation of the optic can be contained into the format of the photograph; the alineament and the adjustment of the optical train need more care.
c) The optic moves in x and the stage moves in y or viceversa; the independency of the two movements determines a setting easier for meeting the mechanical adjustment conditions; the optical train has a complexity in between the other two cases.

4 - Theoretically an AP system could be built where all the stage movements are differential movements, thus determining a less sophisticated way of controlling the stages; upon a carriage moving in x and y two stages moving in x and y should be mounted; since to day no problem exists for the full control of the stages this solution which defines a very complex mechanical setting can be mentioned only for sake of completeness.
In a DS system where the stages are directly moved the differential movements can be performed only for one stage with respect to the other; then only two differential movements can be considered in a DS system and the less sophisticated control can be taken into account as the only controlling device only if the controls of the stages are not more than two.

The computer of a DGS.

It can be said that the electronic component of a DGS and specifically the computer is in the same time the strong point and the weak point of a DGS.
It is the strong point because the analytical approach allows the building of the most accurate, the most versatile, the most easy to use etc... stereoplotter, whereas it is the weak point because it is necessary to recognize that in a stand-alone instrument the computer, the interface and the basic software substitute the two simple space rods of an analogue plotter; in other words this substitution is not still cost-effective when normal cartographic mapping is to be made in a stand alone instrument.
The above trivial observations are useful to outline the ways of avoiding the weak point which have been already adopted by the DGSs builders.
a) The use of a microprocessor for the interface and an EPROM for maintaining the real time routine; the orientation operations and other functions are performed by an host computer which can be used also for other computing works.

b) The interfacing of many stereocomparators to a single central processor which is programmed in time sharing in order to take care of all these specific peripherals; this solution requests a very fast central processor when the real-time routines must run fifty time a second but can be carried out by a normal minicomputer when it is possible to slow down the real time routine to five time a second, as it is possible in a DS system; actually in the existing systems where many APs are interfaced to a central processor each of them uses a microprocessor for maintaining an independent real time routine.

c) The providing the DGSs of an extensive application software so that a stand alone instrument with a dedicated computer becomes cost-effective when all the available capabilities are taken into consideration.
As an example the following application software can improve the cost-effectiveness of a DGS.
1) Computer aided graphic plotting
2) Digitized plotting with TV terminal display
3) On line editing capabilities of digitized features
4) D.T.M. measurements and specific D.T.M. exploitations
5) On line aerial triangulation
6) On line strip adjustment
7) On line block adjustment
8) Digitized profiling with specific conditioned input for industry works
9) Digitized countouring with specific conditioned input for landslide and terrain manipulations.

References.

U.V.Helava - Trends in design of digital equipment for photogrammetric instruments.
International Symposium - The role of digital components in photogrammetric instrumentation.
Turin - Italy - 1974

R.B.Forrest - Image space plotter - Photo. Eng.
August 1971

R.B.Forrest - Image data analysis centre
International Symposium - Current trends in the design and use of analytical plotters.
Brisbane, Australia, October 1977

Z.Jacsic - Notes on evaluation criteria and performance test for analytical plotters.
International Symposium - Equipment for analytical photogrammetry and remote sensing.
Paris - France - 1978.