# AN INTRODUCTION TO COMPUTER-CONTROLLED DIGITAL MAPPING SYSTEM

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#### Abstract

This paper presents a computer-controlled digital mapping system and the involved method, hardware as well as software, with emphasis on the concept of curve fitting and data reduction.

It can be said the automation of photogrammetry can be realized by the combination of photogrammetry and computer, while the application of computer seems to be developed along the way of "numerical computation-real time control-information engineering-wisdom". Currently we might be in the period of the third stage.

Practice has proved that the DTM data bank is more ideal and effective, therefore the establishment of this data bank is the main problem in this paper. The traditional way of DTM establishment is to determine the X, Y, Z coordinates of model points directly by a stereoplotter. This belongs to the concept of object space plotting. In this paper, the concept of image space plotting is adopted, that is, having determined the image coordinates x, y and the parallax p of the grid points of the image, compute the corresponding model coordinates X, Y and Z. The advantages of doing so are that, firstly, the errors of establishing model are avoided, secondly, the requirements on the computer is reduced, the only thing to do with it is to correct for the vertical parallaxes at the observed image points. The disadvantage of it is that the position of the corresponding object space points is not standard, but this can be over-

come by certain software technique. As the DTM is particularly used here for the drawing of contour lines, this disadvantage does not actually exist and gives no inconvenience.

According to the existing conditions, the system is presently made up by the following block diagram. (Fig. 1)



Fig. 1

#### I. Hardware

The hardware includes mainly photo coordinate digitizer, computercontrolled graphical plotter, computer-controlled orthoprojector, and a general purpose computer etc..

Photo coordinate digitizer. This is an instrumentation system that can register the value p of image points at the given values of x, y coordinates on the photograph. (See Fig. 2)



Fig. 2

The relationship between the parts of the instrument is shown in Fig. 3.



Fig. 3

The image points for relative and absolute orientation can be determined by an operator simply by controlling a joystick and rotating handwheel p. After that, the instrument automatically turns to the computation of relative and absolute orientation. The observation of topographic characteristic points is then made under operator's control. When this is done, the instrument starts automatically the scanning of a square grid based on the coordinates of the

left-hand photo, measures the value p of all points already scanned, and punches the tape to give the output of all the related data.

The characteristics of our photo coordinate digitizer are summarized as follows:

1. There is only one 8-bit counter used for registering the last few digits of p, this is a self-designed reversible circulation code counter with simple circuit, stable and reliable performance. The restoration of the total readings of p is done by software.

2. In the scanning, the value p of the observed points can be forecasted to reduce the labor of the observer.

3. Scanning in steps may result in higher observation accuracy. When the floating mark rests correctly on the ground, the handwheel p would cease rotating. At this moment, the computer stores the value p and begins a new scanning step to the next point. In this way, the accuracy of dynamic observation can be increased from 0.04% to about 0.02%, and the scanning speed can be matched with that of observation.

4. Completeness of administrative programm. The output data is checked step by step. In the scanning, the topographic characteristic points can be determined or the scanning points that are uncorrectly observed can be eliminated and observed again.

5. User's programs are available, so that the topographic profile or the DTM of a ground square grid can be directly produced as output for immediate use by special agencies.

Computer-controlled plotter. For off-line plotting with high accuracy, we have developed a minicomputer-controlled plotter of contact type which can make scribe directly with the accuracy and line quality required by the map standard. (Fig. 4)



Fig. 4

Except that the controlled photocoordinator should be substituted by a plotter desk and related software be changed, the structure of the system permits it to share the same hardware with the digitizer, thus increasing the application efficiency of the equipment. Following is the block diagram of this part.



The hardware is very simple without any interpolator hardware. The forward and backward pulses needed by two motors are directly processed by the computer. The programms are responsible for the homogenization of pulses as well as the progressive increase and decrease of the frequency. Various measures have been taken to reduce the interface circuit to the minimum.

Its characteristics are:

1. The paper tape is used for off-line plotting. This requires special data reduction technique.

2. The plotter commonly used controls only the speed in the directions x and y, so the linear speed in the direction  $45^{\circ}$  is  $\sqrt{2}$  times larger than that in  $0^{\circ}$ . This is unfavorable for scribing. But in this machine, it is the linear speed that is to be controlled.

3. Output of the instantaneous tangential direction, the curvature and its changing rate of the curves for control purposes are made available at any time.

4. A special contour annotation program is available. The program has met the following targets: With equal distance of the curves, the center of the letters lies on the contour line. The lateral axis direction of the letters is in coincidence with the tangent line of the contours at the center of the letters, with the heads of the letters pointing to the top of the mountain. This controling software is not available in usual general purpose plotters.

Computer-controlled orthoprojector. The instrument operates off-line with the control of a computer. Rectification is done by using linear elements which can give better image quality and higher accuracy than direct projection. The instrument is also capable of producing color orthophotos.

#### II. Method and Software

The paper tape from the digitizer is transported, with one map sheet as a unit, into DJS-6 computer for computation. Through data processing, photogrammetric computation, DTM interpolation, contour tracing, curve fitting, and data reduction etc., a 8-unit paper tape is finally obtained as the output for the control of curve plotting

in the plotter. The details are given seperately in the following.

Data processing. Only increments of p of the scanned points are recorded onto the tape, with each point recorded in 9-bit. The data processing program firstly directs this tape of special format into the memory, then recovers the complete form of p and matches it with the corresponding x, y coordinate values for use afterwards.

Program for photogrammetric computation

1. Formula for the transformation of coordinates of image points. The photogrammetric computation is accomplished by using the method of transforming into ideal photo coordinates of the photo pair, the twelve exterior elements of orientation (relative and absolute orientation) are solved simultaneously.

The 12 elements of orientation are:

 $X_{s}, Y_{s}, Z_{s},$  $B_{x}, T(\frac{B_{y}}{B_{x}}), V(\frac{B_{z}}{B_{x}}),$  $\Psi, W, X,$  $\Psi', W', X'.$ 

2. Solution of the exterior elements of orientation. To obtain the 12 exterior elements of orientation, there should generally be 4 known control points in a photo pair. For each point three equations can be formed:

$$F_{x} = X_{D} + X_{s} + \frac{B_{x}}{p^{o}} X^{o} = 0$$

$$F_{y} = -Y_{D} + Y_{s} + \frac{B_{x}}{p^{o}} Y^{o} = 0$$

$$F_{z} = -Z_{D} + Z_{s} + \frac{B_{x}}{p^{o}} (-f) = 0$$
(1)

For 4 points, a total of 12 equations can be set up.

Apart from this, a Q equation can also be set up, that is, the transformed coordinate  $Y^0$  for each pair of image points should be equal to  $Y^0$ ':

$$F_Q = Y^O - Y^{O'} = 0 \tag{2}$$

With a Q equation for one point, 8 equations can be formed for the 8 orientation points.

After linearization, 20 error equations would be obtained and solved by the method of least squares.

3. Special forward intersection. Apart from the observation of the eight orientation points, we have also observed thousands of grid points, how to solve their geodetic coordinates  $X_D$ ,  $Y_D$ ,  $Z_D$  based upon their respective image coordinates X, Y, X' of those grid points, after having solved the 12 elements of orientation? The answer here is to use the technique of a special forward intersection. The formula is derived as follows.

For the convenience of discussion, let's take the right projection center 5 as the origin, so its coordinates would be (0, 0, 0), and the coordinates of the left projection center 4 would be  $(-B_x, -B_y, -B_2)$ . The coordinates of the left image point 3 is now  $(X_3-B_x, Y_3-B_y, Z_3-B_2)$ . (See Fig. 6)



Fig. 6

Now we make two auxiliary points 1, 2 on the right photo, the coordinates of the image point 1 being  $X'_1=X'_3$ ,  $Y'_1=+100$  mm., the coordinates of the image point 2 being  $X'_2=X'_3$ ,  $Y'_2=-100$  mm. The space coordinates of the two points are  $(X_1, Y_1, Z_1)$  and  $(X_2, Y_2, Z_2)$ . The equation of the plane formed by the three points 5, 1, 2 is:

Equation of the straight line formed by points 4 and 3 is:

$$\frac{X + B_{x}}{X_{3}} = \frac{Y + B_{y}}{Y_{3}} = \frac{Z + B_{2}}{Z_{3}} = k_{3}$$

then

$$X = kX_3 - B_x$$

$$Y = kY_3 - B_y$$

$$Z = kZ_3 - B_z$$

$$(4)$$

It is not difficult to see that the intersection point of the line with the plane is just the model point  $(X_D, Y_D, Z_D)$  to be solved.

From the simultaneous solution of equations (3) and (4), we have projection coefficient:

$$\mathbf{k} = \frac{\begin{vmatrix} B_{\mathbf{x}} & B_{\mathbf{y}} & B_{2} \\ X_{1} & Y_{1} & Z_{1} \\ X_{2} & Y_{2} & Z_{2} \end{vmatrix}}{\begin{vmatrix} X_{2} & Y_{2} & Z_{2} \\ \hline X_{3} & Y_{3} & Z_{3} \\ X_{1} & Y_{1} & Z_{1} \\ X_{2} & Y_{2} & Z_{2} \end{vmatrix}}$$
(5)

Finally, by transforming the origin of the coordinate to the left projection center, we have:

$$\left. \begin{array}{c} X_{\mathrm{D}} = kX_{3} + X_{\mathrm{s}} \\ Y_{\mathrm{D}} = kY_{3} + Y_{\mathrm{s}} \\ Z_{\mathrm{D}} = kZ_{3} + Z_{\mathrm{s}} \end{array} \right\}$$
(6)

The interpolation of DTM. Although the interpolation of DTM in larger areas by collocation method of least squares is ideal theoretically, yet the computation takes quite a long time, moreover, it is difficult to estimate the covariance matrix accurately. To increase accuracy through additional observation of the feature points at the key points (line) or by proper densification of scanning points is in a sense more effective. That is why the interpolation by use of quadric surfaces (small area) with feature points is used here in place of the collocation method. Should the interpolation happen to bring about certain errors, they could be, within certain extent, compensated in the subsequent fitting of contour lines.

The technique for interpolation by use of quadric surfaces is very simple. In order to increase the number of feature points, a densification is firstly carried out along the topographical structure lines. Then by using the data points around a certain basic point (scanning point, feature point), a small quadric surface is set up. After the calculation of the necessary parameters by the method of least squares, the elevations of all points to be interpolated in that small area can be calculated.

Finding the point succession of contour lines. The interval between the densified grid points is about 0.3 mm. (on photo). In the direction of X, there are N big meshes (by big mesh we mean observed points) and 3N small meshes. In the direction of Y, there are M big meshes and 3M small meshes.

The elevations of these several ten thousand points are stored Each big mesh is equipped with a logic in memory in the form of byte. scale for control purpose. The process of finding the curves is devided into four circulations. The most inside of which is the core and is responsible for tracing a curve through to the end. As long as one starting print of a contour line is found, the program would carry it to the boundary of the sheet area or back to the starting The outside of the program, which is the third circulation. point. is responsible for finding out the starting points of all the contour lines within a big mesh. For example, after a curve from a big mesh is traced by the fourth circulation, it is necessary to go back to the mesh and see if there is any starting points of other contour lines, until no more starting point left. All of the MXN meshes should undergo the same process. The first and the second circulations take the responsibility to introduce the MXN meshes into the process, which can be called "searching circulations". It can be seen from Fig. 7, how the contour lines are searched and traced.



Fig. 7

Generally, the tracing of contour lines on the basis of distance and direction is not excellent, since it calculates slowly and has

other disadvantages. The method used here is a "multi-chain" one, which is a software technique. The whole line tracing program became simple due to the chain configuration. The chain is not connected in the simple way of heads with tails, but is complicately interwoven. That is why it is called "multi-chain". On this basis, the searching in the direction of tangent line is reserved in order to increase the speed and to reduce the number of searching. The special case that contour points having equal elevation exist at the four sides of the same mesh should be given a special processing. If a contour line goes right through a grid point, it is necessary to increase the elevation of this point arbitrarily by 0.00001 M. in order to keep it from degeneration.

Curve fitting. The original point succession of contour lines should go through the computation process of curve fitting and smoothing before they can meet the publication requirements for a contour line map.

The key point with curve fitting is the choice of curve to use. For example, one may use a curve of second or third degree, a circle or a parameter equation of second degree with the arc length as its parameter, or tension spline etc.. Due to the randomness of a contour trend, however, it has no symmetric center, no symmetric axis. Although each of the points along the contour lines can be expressed in rectangular coordinates, it is rather difficult to describe a random curve in a rectangular coordinate system. The above mentioned modes to present the shap of contour lines are not so satisfactory. Therefore, a new special curve that can be well suited to the randomness of contour line has been found, which is called "S curve". The equation of the curve is:

$$\alpha = \alpha_0 + KS + JS^2 + IS^3 + \dots$$
 (7)

It is defined neither in the Cartesian coordinate system nor in the polar coordinate system, but in a "local direction coordinate system". Practice has proved that this curve under the condition of determined parameters corresponds to a solely determined shape. It is very interesting that each of the parameters has a definite geometric meaning. Begining from the zero power, they are in turn the rotation coefficient, the sliding coefficient, the pantograph coefficient, and the coefficient of affine transformation ....

For the convenience of computation, we only carry to quadratic term:

$$\alpha = \lambda_{0} + KS + JS^{2}$$
(8)

Where S is the arc length;  $\partial_0$ , K, J are the parameters of the equation;  $\partial_i$  is the value of arc in the direction of the tangent line at the point with arc length being S (reckoning from north point of the central meridian clockwise as positive.) The meaning of this equation is: it will be a straight line, when K = J = 0; it will be a circle, if J = 0 and  $K \neq 0$ ; this circle will be a direct one, when K > 0, and a reverse one, when K < 0; when  $J \neq 0$  and  $K \neq 0$ , it will be a circle with curvature varying in the direction of arc length, such a circle may be called "circle with varying curvature". (See fig. 8) It is similar to a curved ruler with which we fit the point succession of contour lines.

$$\bigcirc$$

Fig. 8

The presentation of contour lines is made right through the connection of all sections on about several thousand such analytical curved rulers with first order smoothness.

To fit the contour lines with such curves, the following problems should be solved:

1. Given all parameters of a curve: the coordinate of initial point  $X_0$ ,  $Y_0$ , the direction  $\lambda_0$  of tangent line of the initial point, the initial curvature K, the changing rate of curvature 2J, to find the rectangular coordinates of point S, with given arc length.





Given: X<sub>0</sub>, Y<sub>0</sub>, 2<sub>0</sub>, K, J; to find: X, Y.

It is easy to prove:

$$X_{1} = X_{0} + \int_{0}^{S_{1}} \cos(\lambda_{0} + KS + JS^{2}) dS$$
  

$$Y_{1} = Y_{0} + \int_{0}^{S_{1}} \sin(\lambda_{0} + KS + JS^{2}) dS$$
(12)

It is very difficult to make further simplification of the above equations (12) so as to be suitable for calculation with a computer.

2. Given all the parameters of a curve and the rectangular coordinates of a fitted point, to find the discrepancy normal to the

curve and the arc length of this point.



Fig. 10

Given:  $X_0, Y_0, \lambda_0, K, J, X_1, Y_1$ to find:  $D_1, S_1$ 

3. Given all the parameters of a curve and the discrepancies normal to the curve of some fitted points, to find the initial curvature K and the expected value of the changing rate of curvature 2J, so that to make the new curve pass through these points based upon the principle of collocation method of least squares. The key to solve this problem is how to find the error equation of each point.



Fig. 11

Given:  $X_0, Y_0, \lambda_0, K, J, D_1 \dots D_n, S_1 \dots Sn,$ to find:  $\Delta K$ ,  $\Delta J$  in such a way as to make [PDD] — min. It is difficult to solve this problem mathematically, but we can perform "direct simulation" on a computer. This method, however, is very slow in computation speed, so we developed a computing method for rapid fitting. The concept of the method is "to make the computer have experience" -- data retrieving. Firstly, the computer should be allowed much time on making a reconnaisance of the measure quantity and differential quality of every curve of each family in our curve resource and on "establishing a bank" for the reconnaisance result. A reduction is then made of the bank without loss of the deserved accuracy (surface fitting or code reduction) into a "table" which would be stored in the computer. In the subsequent computation of the productive curve fitting, everything will go all right as long an references to the table are often made.



### Fig. 12

Fig. 12 shows one part of the whole "curve resource".

Fig. 13 can be obtained when standing at the initial point, facing the direction of tangent line  $2_0$  with curve S as feeler.



Fig. 13

The three problems mentioned above have been solved with the aid of computer and very high fitting speed has been obtained.

Since it is the random curve that is to be fitted, more random variables should be introduced into the program. These variables include interval between fitted points, residual discrepancies in the normal direction, length of curve of each section, and weight of the points participating the fitting etc. .

All of these random variables are the functions of the primitive random variables such as the slope of ground and the torsion of curve.

The advantage of using such surves lies in the realisation of the allocation in the normal direction with unequal weights. The fitting (every point should be passed through) by any method other than that of least squares is unreasonable, since the contours themselves contain errors. It is also clearly unreasonable to allocate the discripancies of fitting by decomposing in two directions  $\Delta X$  and  $\Delta Y$ . The allocation in the normal direction with equal weights is not advisable either, for the smaller the ground slope, the lower the positional accuracy of contour should be. The normal allocation with unequal weights also brings about the result of "generalization". In addition, since with this curve to fit contour can the amount of information be reduced substantially, only a reel of paper tape is needed here, whereas for a general plotter, it has to use a reel of magnetic tape for a map sheet. Moreover, it brings about many advantages for controlled plotting: there are at any time the direction of tangent line, curvature, instantaneous value of changing rate of curvature to be used as control of the tangent direction, and parameters for controlling linear speed.

Practice has shown that the fitting program is basically competent also under the condition of thick contours (Fig. 14).



Data reduction. Due to the limitation of the existing condition, we have to use "paper tape file". This brings about a progress in data reduction.

The method of data reduction can be devided into two types; the method type and code type. Consideration is taken into the sectioned "S" curve of first order smoothness where each section has the need for recording the coordinates of its initial point. tangent line, curvature and its changing rate. As there are common tangent line direction and common tangent point at the border between the first and second sections of the curve, the second section does no longer need coordinates of initial point and tangent line of its own. If it is second order smoothness, even the initial curvature is not necessary either. All the parameters of the curve should go through the process of quantification and are stored and transferred in the form of integer type magnitude. A reasonable unit of quantification can reduce data without loss of accuracy. Further data reduction can be carried out, if certain parameters are quantified in non-linear Since the curve is random, the parameters representing it are way. also normally distributed random quanties. A still further reduction of data is done, if in the new coding, the one with high probability

corresponds to be code with "small bit number". To reduce data to the minimum is a very complicated technical problem. It is generally advisable to use simpler reduction technique. At present, we can use 9 bits for the recording of each scanned grid point and 7 bits for a curve length of 1 mm.

Editing. All the data obtained by now is the contour line data in a single photo pair. An editing should be done before the output of curve can be given in the form of a complete map sheet. The editing program currently used has the function of connecting contour lines at the border between photo pairs and cutting lines at the sides of a map.

## III. Preliminary Application of the Mapping System and the Results of Test

DTM is widely used. It shows clearly its superiority when used in connection with the planning and designing works by such departments as geologic exploration, water conservancy, electric power, railway, highway, agriculture, and forestry.

### For example:

1. The interpolation of nodal point elevation for gravimetric topographic correction. Based upon DTM, the interpolation of grid nodal point elevations is done by the revised digital controlled mapping program on a DJS-6 computer. Output of elevation data can be given either by narrow line printer or by broad line printer, while nodal point elevation data can be written automatically by X Y plotter according to the point position. The position of nodal point is presented by the decimal point.

Test with the data from the Exploring Team of Geological Survey gave:

Mean error of elevation  $m_h = \pm 1.14$  M.

The same data were measured on Topocart B, which gave a mean error  $\pm$  1.85 M.

2. Profile elevations of high voltage power transmission line. The profile elevations in the direction of high voltage power transmission line and the profile elevations at the places  $\pm$  16 M apart from the transmission line on both sides can be obtained by direct interpolation with DTM. The accuracy of elevation obtained is  $\pm$  0.3 M which has met the design requirements.

3. Test on mathematical basis for automatic scribing of topographic map by computer-controlled plotter. The plotter would automatically perform the plotting of inside and outside map frame lines and all grid lines (kilometer), the plotting of points and the writting of the point elevations as well as a few Chinese characters.



Fig. 15

4. Test on automatic scribing of curves and lettering of index contour elevations on computer-controlled plotter. With a 8-unit

controlled paper tape from DJS-6 computer and the piece-wise inputs of the coordinates of the curve starting point and also the curve parameters by a rapid stop and start tape reader, the curves can be directly scribed out on the scribing coating by the ruby scribing needle of the plotter. A great amount of manual work of fair drawing is thus eliminated (Tig. 16).



5. Test on the determination of coefficients of generalization. The realisation of generalization can be accomplished by the method of revising the interval between fitted points, which can be called the "method of distance generalization". The generalization can also be realized by limiting the torsion of the curve which can be called "method of direction generalization". The coefficient of generalization  $\mathcal{E}$  referred here means the tolerance of fitted discrepancies in the normal direction. Fig. 17 shows the drawing results when the fitted discrepancies are specified as 1/18 and 1/6 of the horizontal interval between contour lines. But test shows that it would be better to take  $1/5 \sim 1/7$  .



 $\mathcal{E} = \frac{1}{6} \left( \text{and } \frac{1}{18} \right)$  data quantity 2KW

Fig. 17





 $\mathcal{E} = \frac{1}{20}$  data quantity 3.8 KW Fig. 19