14th Congress ISP - Hamburg 1980 Commission III Presented Paper

RECURSIVE METHODS FOR THE SIMULTANEOUS ADJUSTMENT OF PHOTOGRAMMETRIC AND GEODETIC OBSERVATIONS IN ANALITYCAL AEROTRIANGULATION

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ABSTRACT

A number of advanced photogrammetric numerical and computational concepts and techniques were integrated to exploit the whole potential of analytical aerotriangulation more efficiently. Starting from a set of independent observations a recursive process is presented in order to add new information to the existing set, or to eliminate some information. Two situations are presented: when the change in the number of the equations of condition does not alter the number of mathematical parameters, and when the number of parameters, and hence the order of the normal equation matrix, changes. Facilities connected to the particular structure of the matrix of the normal equation coefficients are maintained by the proposed algorithm, reducing to a minimum the disadvantages of the destruction in the regular structure of the matrix due to the geodetic observations. The technique presented above is capable of handling a virtually unlimited number of unknowns with increased efficiency. A computer program in the FORTRAN IV language has been developed and tested with fictitious and real data for both strips and blocks.

INTRODUCTION

This paper presents some basic principles and techniques concerning the sequential data processing with application in analytical aerotriangulation. These may be regarded as an extension of regularly practiced least square adjustement procedures and will play an increasing role in the adjustment methodology because of current rapid advances in on-line modes of data acquisition and the ever-increasing role of electronic computers.

By using some concepts encountered in data processing by recursive methods, a mathematical model has been developed capable of including geodetic observations in a simultaneous solution for aerotriangulation. Problems concerning geodetic and photogrammetric measurements are distinctly treated in the classical analytical aerotriangulation. On the one hand ground surveys and their processing are performed in order to obtain a unique set of coordinates for the ground control points used as input data in the photogrammetric solution and on the other hand photo-coordinate measurement and aerotriangulation computation which are performed by photogrammetrist. Thus a gap appears in the technology of performing an aerotriangulation project which leads to an unjustified delay.

The integrated approach of the problem by simultaneous adjustment of photogrammetric and geodetic observations with the sequential processing data method utilization has many advantages over the classical methods. Firstly, the accuracy requirements of all geodetic as well as photogrammetric measurements can be brought into accord with the observance of the desired accuracy of the entire aerotriangulation project. Secondly, geodetic measurements can also be used which is of particular importance for areas where there are no determined points in the terrain or when their determination by traverse or triangulation proves difficult (isolated, difficult of access zones, etc). Thridly, the disadvantages of the destruction in the regular structure of the normal equation coefficient matrix are reduced to a minimum due to geodetic observations which leads to a decrease in the number of arithmetic operations and to a reduction in the requirement of core memory.

1. SEQUENTIAL DATA PROCESSING

Some general aspects regarding the properties and application possibilities of sequential data processing methods will be discussed.

Let a set of measurement be by which r observation equations may be formed having u parameters in all. To obtain the estimate of these parameters the associate system of normal equations is formed and then solved. Let us assume that at a given moment we have new measurements at our disposal which lead to q observation equations (for example, we have an on-line data acquisition system or we wish to introduce some geodetic observations which should replace the ground control points in aerotriangulation). It is not economical to combine the new information with the former one and to take adjustment all over again especially when q is much smaller than r. Another problem related to this one refers to the elimination of some observation equations due to the rejection of certain measurements at the stage of data processing.

The most efficient manner of solving these problems is that of using sequential techniques where the contributions of measurements may be added or substracted without repeating the computations with reference to the other measurements.

Let a set of observation equations be:

$$V_f + A_f X = l_f \qquad (1)$$

and the set to be added or subtracted be:

$$V_g + A_g X = l_g \qquad (2)$$

noting that the parameter vector is identical in both sets.

In the following presentation all measurements will be assumed as independent. The coefficient matrix and the constant term vector of the normal equations obtained from the 2 sets of equations (1) and (2) are respectively:

$$N = N_f + N_g \quad ; \quad L = L_f + L_g \qquad (3)$$

in which:

$$N_f = A_f^t P_f A_f ; N_g = A_g^t P_g A_g , \; L_f = A_f^t P_f l_f , \; L_g = A_g^t P_g l_g$$

where P_f and P_g are diagonal matrices representing the weights associated to the respective measured quantites.

By using matrix algebra operations only the following relations are obtained for the computation of matrix N^{-1} and of vector X, modified:

$$N^{-1} = N_f^{-1} \mp \Delta N \quad , \qquad X = X_f \pm \Delta X \tag{4}$$

where

$$\Delta N = N_f^{-1} A_g^t (P_g^{-1} \pm A_g N_f^{-1} A_g^t)^{-1} A_g N_f^{-1}$$

$$X_f = N_f^{-1} L_f$$

$$\Delta X = N_f^{-1} A_g^t (P_g^{-1} \pm A_g N_f^{-1} A_g^t)^{-1} (l_g - A_g X_f)$$

Thus starting from N_f^{-1} and X_f known, the matrices N^{-1} and X may be computed by means of relation (4) where the upper and lower sign refers to the addition and, respectively, to the elimination of some observation equations.

In many problems such as in the analytical aerotriangulation the increase or decrease in the number of observation equations leads to changes in the number of parameters in the model. Thus if the collinearity equations are used as the basic mathematical condition, every time a pass point or tie is added, three new parameters corresponding to its coordinates will also be added. A similar situation occurs if such a point is eliminated then the number of parameters is reduced by three.

Let (1) a set of observation equations be and

$$V_g + A_g X + a_g x = l_g \tag{5}$$

a new set of equations to be added where x is a subvector of the supplimentary parameters, then:

$$N^{-1} = \begin{bmatrix} E & G \\ G^t & H \end{bmatrix} \tag{6}$$

in which:

$$E = \dot{N}^{-1} + \dot{N}^{-1} n_g (\dot{n}_g - n_g^t \dot{N}^{-1} n_g)^{-1} n_g \dot{N}^{-1}$$

$$G = - E n_g \dot{n}_g^{-1}$$

$$H = \dot{n}_g^{-1} - \dot{n}_g^{-1} n_g^t G$$

where:

$$n_g = A_g^t P_g a_g ; \; \dot{n}_g = a_g^t P_g a_g ; \; \dot{N}^{-1} = N_f^{-1} - \Delta N \;{}^{+)}$$

and:

$$\begin{bmatrix} X \\ x \end{bmatrix} = N^{-1} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \tag{7}$$

where:

$$L_1 = A_f^t P_f l_f + A_g^t P_g l_g \; ; \; L_2 = a_g^t P_g l_g$$

The relations used when eliminating information are introduced in a similar way.

The concept of data processing by recursive methods may be extended to the case of apriori determination of estimates

+) With the upper sign.

of independent parameters as well as their weights, and may be also applied to the case of indirect measurements with condition equations between the unknowns .

2. SIMULTANEOUS ADJUSTMENT OF PHOTOGRAMMETRIC AND GEODETIC OBSERVATIONS

The mathematical model of simultaneous solution for aerotriangulation uses the spatial intersection of conjugated rays and is capable of incorporating into adjustment besides conventional photogrammetric measurements, such geodetic measurements as: distances, azimuths, horizontal angles, vertical angles and height differences. The method applied is a three-dimensional simultaneous block adjustment where the bundle of rays comprising one photograph is used as basic unit of the block adjustment. The stochastic model is based on the assumption of normally distributed variated. The geodetic and photogrammetric observations are considered independent with corresponding weights function of measurement errors.

Let p be the total number of perspective rays in a block of n photographs and m aerotriangulation points. Then the complete set of observation equations for the photogrammetric measurements which describe the collinear condition of perspective rays through all of the control points, pass points and tie points needed for block formation may be expressed in matrix notation as follows:

$$\underset{(2p,1)}{V_f} + \underset{(2p,6n)}{\bar{B}}\;\underset{(6n,1)}{\bar{x}} + \underset{(2p,3m)}{B}\;\underset{(3m,1)}{x} = \underset{(2p,1)}{l_f} \qquad (8)$$

where:

- $\bar{x}$ - is the vector of the unknown corrections to approximate values of the exterior orientation of photographs;
- x - is the vector of unknown corrections to approximate values of the object space coordinates;
- $\bar{B}$ - is a matrix whose coefficients are partial derivates of the collinearity equation with respect to exterior orientation elements;
- B - is a matrix composed of partial derivates of the collinearity equation with respect to the object space coordinates.

The complete set of observation equations for the geodetic measurements may be represented by the following matrix equation:

$$\underset{(q,1)}{v_g} + \underset{(q,3g)}{G}\;\underset{(3g,1)}{x} = \underset{(q,1)}{l_g} \qquad (9)$$

where:

- q - is the number of geodetic measurements performed
- g - is the number of points where geodetic measurements were performed.
- G - is the coefficient matrix of observation equations for geodetic measurements.

The complete mathematical model is obtained by combining equations (8) and (9)

$$\begin{bmatrix} v_f \\ v_g \end{bmatrix} + \begin{bmatrix} \bar{B} & B \\ 0 & G \end{bmatrix} \cdot \begin{bmatrix} \bar{x} \\ x \end{bmatrix} = \begin{bmatrix} l_f \\ l_g \end{bmatrix} \qquad (10)$$

i.e.

$$v + A X = 1 \qquad (11)$$

According to the principle of least squares, the coefficient matrix of normal equations has the following composition:

$$N = A^tPA = \begin{bmatrix} \bar{B}^tP_f\bar{B} & \bar{B}^tP_fB \\ B^tP_f\bar{B} & B^tP_fB+G^tP_gG \end{bmatrix} = \begin{bmatrix} L & R \\ R^t & H_f+H_g \end{bmatrix} \qquad (12)$$

where P is the weight matrix associated to the measured quantities and has the following composition:

$$P = \begin{bmatrix} P_f & 0 \\ 0 & P_g \end{bmatrix}$$

in which P_f and P_g are diagonal matrices representing the weights of the photogrammetric measurements carried out by the stereocomparator, respectively of the geodetic measurements performed in the terrain.

In the case when geodetic observations are lacking the matrix N is reduced to:

$$N_f = \begin{bmatrix} L & R \\ R^t & H_f \end{bmatrix}$$

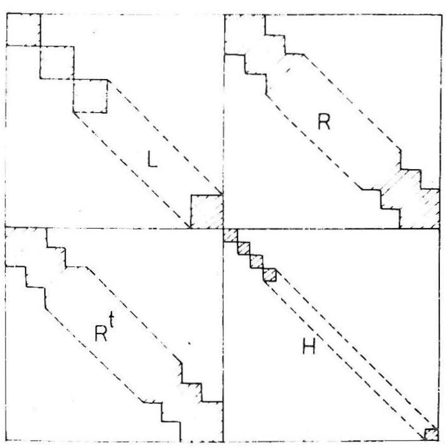

Fig.1 Structure of matrix N_f

Figure 1 presents the special structure of this matrix schematically. It can be noticed that the elements equal to zero are prevailing. Furthermore:

- L is a block diagonal matrix which has along its diagonal n square submatrices of order 6;
- R has a banded structure which has submatrices with a dimension of 6x3;
- H_f is a block diagonal matrix with m square submatrices with a dimension of 3x3;

Starting from the normal equations:

$$N X = A^tPl \qquad (13)$$

it results:

$$X = N^{-1}A^tPl \qquad (14)$$

in which:

$$N^{-1} = \begin{bmatrix} S^{-1} & -S^{-1}T^t \\ -TS^{-1} & H^{-1}+TS^{-1}T^t \end{bmatrix}$$

where:

$$H=H_f+H_g \;,\; S=L-RH^{-1}R^t \;,\; T=H^{-1}R^t$$

Hence the inversion of N is reduced to the inversion of

S and H and to the matrix multiplications indicated above.

As regards the structure of matrix S mention must be made that if there are geodetic observations this will be a complete matrix;if there are no geodetic observations or if they are not taken into account at a certain stage of the adjustment algorithm then matrix S is reduced to:

$$S_f = L - RH_f^{-1}R^t$$

and will have a banded structure which anyhow will be more easily inverted than a general matrix using for example the method of reccursive partitioning.

With respect to matrix H it can be noticed that H_g arises from geodetic observations.If the points where such observations were performed are not numbered distinctly from the other points,then H_g will be a complete matrix.The inversion of matrix $H=H_f+H^g$ will thus involve the inversion of 3mx3m matrix which in the case of large block raises many problems from the viewpoint of the requirement of memory,of computation time and of the loss of significant figures.To overcome these difficulties two manners may be used: If the geodetic observations are required to have a limited span (for example by stipulating that no geodetic measurements should span across more than two strips)then a banded structure is obtained for matrix H.A second possibility is to number the points separately,first the points where photogrammetric observations were performed and then the points where geodetic observations were performed. In this manner the largest submatrix to be inverted in computing $H^{-1}$is 3gx3g.Figure 2 illustrates the composition of matrix H under various situations:

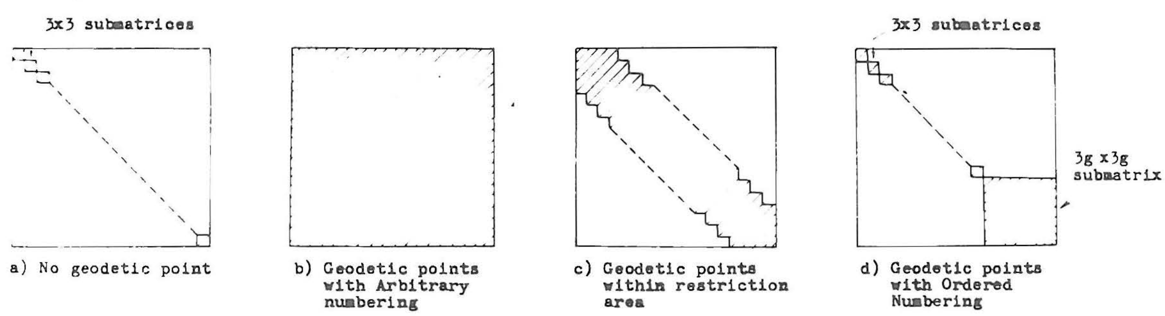

Fig.2 Structure of matrix $H=H_f+H_g$

Taking into account that the number of observation equations for geodetic measurements is smaller than the number of observation equations for photogrammetric measurements(as a rule,less than 10 percent) and that geodetic measurements may also miss(when there is a sufficient number of control points) and considering the matrix structures indicated above an algorithm has been created for obtaining the solutions of the matrix equation (13) by adding contributions to the solutions obtained in the case of inexistent geodetic observations.

If the notations are used:

$$A_f = \begin{bmatrix} \bar{B} & B \end{bmatrix}, \; A_g = \begin{bmatrix} 0 & G \end{bmatrix}, \; X^t = \begin{bmatrix} \bar{x}^t & x^t \end{bmatrix} \qquad (15)$$

then the equations (1) and (2) are identical to 8 and 9($r=2p$, $u=6n+3m$) and according to relation 4, the matrices N^{-1} and X may be calculated if the matrices N_f^{-1} and X_f are assumed as known. These relations may be also applied recursively by gradually adding new observation equations and by computing every time both the inverse and the solution, as the inverse and respectively the solution are known at the former stage.

By adding a number of q observation equations only a small part of the coefficient matrix of the normal equations is modified (namely, a matrix is added which has non-zero elements only in a submatrix of H - Fig. 2d) which determines a series of advantages to be obtained in the application of relation 4 by the submitted algorithm: The order of the matrix which has to be inverted in these relations which is equal to the number of newly-introduced observation equations is much smaller than the order of matrix N. If, however, the number of observation equations for geodetic measurements is still great (the case of large blocks) relations 4 may be applied recursively, reducing the order of the matrix which has to be inverted each time. With respect to the multiplications of matrices which appear in the computation of N^{-1} and X it should be noticed that the separate numbering of the points mentioned above leads to a matrix A_g which has non-zero elements in the last $s=3g$ columns only thus obtaining a reduction of the number of arithmetic operations (which are performed only between non-zero elements) and the decrease of the memory requirement.

3. TESTS WITH FICTITIOUS AND REAL DATA

A program using FORTRAN language has been set up to perform computations by the submitted algorithm using a unit of magnetic disks as auxiliary memory. In order to test the accuracy of the analytical solution and the computation performances various test models with fictitious data have been created both for strips and for blocks as follows: On a map an even plane surface was chosen and then divided by means of a regular network representing the forward and side overlap. Aerotriangulation pass or tie points as well as the exposure stations were assigned coordinates in the object space in comparison with the corners of the regular network. The same procedure was used with a number of points (limited in number) designated to be control points. Orientation angles were also assigned for the photography axis as well as a known set of geodetic observations between the various points of the network. By means of these coordinates in the object space and orientation angles, a set of image coordinates was computed for all control, pass or tie points using the colinearity equations. Then random errors ranging from 0 to 10m were introduced into the terrain coordinates of the pass and tie points thus obtaining a set of approximate coordinates. For geodetic observations random errors were introduced in agreement with the measurement errors encountered in current practice.

The strip and block adjustment was performed by using as input data the computed image coordinates, the approximate terrain coordinates of pass or tie points, the terrain coordinates assigned to control points as well as geodetic observations in various configurations.

The test results show that the solution obtained by the submitted algorithm is rightly formulated from the mathematical point of view and that the computation accuracy is good. At the same time the sensitivity of the solution was noticed with respect to various configurations of geodetic observations as well as the accuracy of the measured data.

A number of strips and real blocks of various sizes were also adjusted. Unlike the tests with fictitious data where, because the accurate terrain coordinates are known the accuracy of the solution could directly be estimated by comparing the computed coordinated with the corresponding known values, here a more thorough study of error propagation has been achieved using the elements on the main diagonal of N^{-1} to determine root mean square errors in pass or tie points.

Two groups of cases have been considered. One in which 8 planimetric control points are used being placed at the block corners and in the middle of exterior sides while the altimetric control points are placed in the interior of the blocks so as each strip should rest on 6 altimetric points. In the second group of cases the planimetric control points are placed along the block perimeter at an interval of 2-3 photography bases. The number of altimetric control points being greater they are placed inside the blocks as well. The adjusted blocks of different sizes have been constrained only on the control points or by using geodetic observations as well (whose number and lay-out is in agreement with the distribution of the planimetric and altimetric control points mentioned above). The difference in accuracy between the two constraint ways was insignificant. At the same time, as within the first group of cases the errors increase significantly with the magnitude of the block sizes, the accuracy requirements for works on large scales cannot be satisfied with the photography scales conventionally used only in the case of small blocks. The results obtained with the second group of cases, where accuracy is independent of the block magnitude, prove that the simultaneous processing of photogrammetric and geodetic observations gives good results and can be used to determine photogrammetrically the points necessary for accurate cadastre works as well as for the geodetic triangulation of lower order.

CONCLUSIONS

All the advantages related to the properties of the utilized matrices that is of being positively definite, symmetrical or having a banded or block diagonal structure are used through the submitted algorithm. It is capable to incorporate the geodetic measurements in a simultaneous solution for aerotriangulation and the disadvantages related to the destruction of the regular structure due to the presence of geodetic observations are reduced to a minimum. The number of arithmetic operations to obtain the solution is:

$$usq + uq^2 + s^2q + q^2s + 2q^3 + uq + sq$$

being at any rate smaller than any of the classical methods where this number is proportional to u^3.

In the set up program the facilities of third generation computers are used with regard to operations work in direct access aiming at the increase in speed and efficiency of the computer. Remarkable results have been obtained with all tests performed concerning the analytical solution and the performances of the computation program.

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