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ADJUSTMENT OF OVERLAPPING AREAS IN NEIGHBOURING MODELS

## Abstract

Solutions for adjusting information related to the edges of adjacent DTM's are presented. Three classes of problems are discussed; matching feature lines,adjusting elevations of grids, establishing boundaries between models. These problems are solved with regard to the information referred to the overlapping areas formed by models of one strip and models of different strips.

The DTM's adjusted by the above procedures enable to generate contours passing smoothly and continuously from model to model and ensure the proper joining up of individually plotted areas, thus providing a means for compiling a map with a minimal amount of manual intervention.

## 1. Introduction

A problem inevitably encountered when utilizing DTM's for mapping purposes is the matching of information along the edges of neighbouring areas. Even when a DTM is extended over a very large region, one can always assume that there will be formed an adjacent DTM covering the adjoning area, which will entail the necessity to adjust the data related to the boundaries between the neighbouring DTM's.

Taking into account that fact and considering the requirement to process the data in minicomputer, it is only logical to define over the region to be mapped a series of single DTM's, each representing an area-unit corresponding to an individual photogrammetric model. Such an approach is in full agreement with the data acquisition procedure, since each photogrammetric model is also scanned separately, and provides a flexible means to compile a whole map sheet from several models or sections of models. A DTM representing a single photogrammetric model is composed of two types of information, a grid of elevations, the topographic data obtained from surveying the model is transformed into, and a data file describing geomorphological and planimetric features present in the model.

Photogrammetric models usually share common areas whether they belong to one and the same strip, or to adjacent strips. Because of the unavoidable errors accompanying the surveying procedures, topographic and other sets of information acquired in the individual models disagree with each other in the overlapping zones of the models. Thus, a neccessity arises to adjust the topographic and planimetric data related to the overlapping areas, prior to the stage of generating contour lines, in order to ensure a unique definition of contours and other lines, and a smooth transition of these lines from model to model, when the relevant parts of the models are placed on the common sheet during the plotting of the map.

Referring to the stated above one can single out three major stages in the process of adjusting the overlapping zones:

- Mathcing and merging features (topographic and planimetric) located within the overlapping areas.
- Adjusting elevations of grid corners contained by the area resulting from intersecting two neighbouring grids.
- Determining boundaries between models inside the overlapping areas to enable the separate plotting of each model on the common map sheet.

2. Matching and merging feature lines.

Assume that two models share a common area, inside of which lines describing topographic and planimetric features have been surveyed. A line is usually represented by a string of points picked up at what is regarded as being characteristic locations. Since each model is surveyed separately, a segment of a feature line, which has been recorded in one model, is represented in the overlapping zone by a sequence of points, which differ from the points constituting the string describing the same segment in the counterpart model. Even when the operator of the photogrammetric instrument would have been able to select the points along the feature lines at identical locations in both models, the disagreement between the data acquired in both models would still exist, due to the errors attached to the surveying procedures.

Several problems have to be considered in connection with the matching of lines; establishing correspondence between segments of features common to both models, deleting erroneous data, copying data from model to
model is cases where a line has been picked up in one model and omitted in the second. It seems practically impossible to devise a logical procedure based on rigid criteria for solving all these problems in a satisfactory manner. Hence, the solution accepted here is of an interactive nature, the intelligent decisions about the questions listed above are made by the user while being assited by a graphical display of the data (interactive terminal, or drawing) prepared by the computer. Adjusting two strings of points which represent one line can be defined as a procedure which merges the data of both strings and determines the best pssible single line fitting the two data sets. The term best possible has to be given a certain meaning, to permit judging the quality of the fit. Intuitively, one would suggest a "best fitting line" as being determined by points located between the two lines at equal distances from each one of them. Adoptation of that criterion would result in a considerable computational effort. Matching and merging of lines are only one component of a sizable mapping process. If that is to be expedient, one has to reduce the amount of computations, at each stage, as far as possible. For that reason, a merging procedure is applied which, while not being based on the criterion above, yields a line which approximates very closely to the "best fitting line" on one hand, and requires a limited number of computational operations on the other.

Prior to defining the single line, the two data sets to be matched are prepared for merging. That includes the following operations:

1. Identification of the shape of the line; three classes of lines are considered - elongated lines, lines having a horse-shoe shape and lines forming closed loops. That classification is mandatory, since the shape of the line has a bearing on the processing.
2. Establishment of a uni-directional pick-up sequence along both lines to be merged. For example, a line may have been surveyed in one model in a south-north directioh, whereas its counterpart has been surveyed in the second model in north-south direction. In order to carry out the merging of both lines properly, the sequence of the points along one of the. lines has to be reversed.
3. Determination of common extremeties for both lines, thus coercing the lines to start and terminate at the same points.

When the preparatory operations are completed the merging itself is effected as follows (see fig. 1.). Assume that the line (labelled I) relating to one model is represented by a sring of $n$ points, and its


Figure 1. Merging Procedure

## 176.

counterpart (labelled II) refering to the second model is given by a sequence of $k$ points the numbers $n, k$ satisfying the inequality $n>k$. Each one of the lines is subdivided into $2 n$ equal parts, by $2 n+1$ points. Each pair of corresponding points, points with identical sequential numbers, forms a short line segment. The common line is now established by concatenating the points which bisect all these segments.

The coordinates $X, Y, Z$ of the points on the common line are determined by interpolating the data related to the two merged lines.

The procedure presented above yields a line which is represented by an extensive number of points. To avoid an unnecessary increase of the data referred to the models, an elimination procedure is applied for deleting points along the line which can be regarded as being insignificant.

The final line replaces the original pick-up data in the respective DTM (model) so when the models are plotted on the common map sheet bordering each other, the features will appear on the plot as lines passing from model to model continuously without interruptions or displacements.

## 3. Adjusting elevations of grid corners

Generally, two grids representing two overlapping models are shifted and rotated relatively to each other. As a consequence, a corner of one grid is usually positioned inside a mesh of the other grid. Due to errors inherent in the data acquisition processes, the elevation of such a grid corner disagrees with the elevations of the corners of the mesh within which it is located. The goal of the adjustment is to eliminate such discrepancies, correcting the elevations of both grids associated with the overlapping area until they conform to each other.

The adjustment procedure applied is of an iterative nature, nevertheless, the results yielded by the process are identical to those which would have been obtained if the elevations were adjusted simultaneously. The choice of the iterative method was made with a view to effect an adjustment of a considerable volume ofdata in a mini-environment.

To carry out the adjustment properly, a distinction has to be made between grid corners located in regular meshes, and corners situated in "special" meshes, meshes through which feature lines are passing, or meshes which contain spot heights, (see fig. 2.). The distinction between these two cases leads to different schemes for computing the parameters required for the adjustment procedure.


Figure 2a. Regular mesh


Figure 2b. Special mesh
177.

The adjustment is effected by a least squares algorithm based on condition equations which are formulated as follows: an elevation of a corner C of one grid must agree with the elevation computed from the data of the counterpart grid, at a point in that grid, the location of which coincides with the corner C. These conditions apply to both grids. Denoting by $H$ elevations related to grid I and by $Z$ elevations referred to grid.II, the condition equations for a particular corner and its corresponding mesh can be written as follows :
$H_{c}+v_{c}=F\left(Z_{1}+\delta_{1}, Z_{2}+\delta_{2}, Z_{3}+\delta_{3}, Z_{4}+\delta_{4}, P_{1}, P_{2}, \ldots\right)$
$Z_{c}+\delta_{c}=F\left(H_{1}+v_{1}, H_{2}+v_{2}, H_{3}+v_{3}, H_{4}+v_{4}, P_{1}, P_{2}, \ldots\right)$
$v, \delta$ - corrections to the elevations.
P, p - points along features, or spot heights.
Equations (1) imposes a condition for adjusting the elevation of a corner of grid I to the elevations of grid II, and equation (2) states the requirement to be fulfilled by the elevation of a corner of grid II while being adjusted to the elevations of grid I. F is a function representing the method by which an elevation of a point located inside a mesh and the coefficients of the condition equations are being computed, and varies with the type of the mesh.

As already stated, the adjustment is carried out in an iterative way. At each step data related only to one grid corner of one model and one mesh of the other model are processed. According to the case in question the coefficients $a_{i}$ and misclosure $w$ of the condition equation and corrections to the elevations are determined :
$v_{c}-a_{1} \delta_{1}-a_{2} \delta_{2}-a_{3} \delta_{3}-a_{4} \delta_{4}+w=0$
$k=-w /\left(1+\Sigma a^{2}\right)$
$v_{i}=-k$
$\delta_{j}=a_{j} k$
In case of a regular mesh the coefficients $a_{j}$ are computed from the coordinates $x$, $y$ of the point $C$ (see fig. 2.) :
$a_{1}=(1-x)(1-y)$
$a_{2}=y(1-x)$
$a_{3}=x(1-y)$
$a_{4}=x y$
$0<x<1, \quad 0<y<1$
In case of a special mesh the coefficients $a_{i}$ are determined as normalized reciprocal values of the distances $d_{i}$ (figure 2) :
$\mathrm{a}_{\mathrm{i}}=\left(1 / \mathrm{d}_{\mathrm{i}}\right) /\left(\Sigma 1 / \mathrm{d}_{\mathrm{i}}\right)$
Here, a situation may be encountered where some of the mesh corners do not particiapte in the computation since they are located beyond the feature line present in the mesh, as seen from the point $C$ whose
elevation is being calculated (e.g. corner 3 on figure 2). A seocnd fact to be stressed is, that the data related to the features having been adjusted previously are regarded as fixed quantities and remain unchanged during the entire adjustment process.

The discrepancy of the condition equation is in both cases the difference between the grid value $H$ and the computed value $\Sigma a_{i} Z_{j}$.

The corrections $v, \delta$ derived from the condition equations provide corrected values $H+v, Z+\delta$, which replace the data existing prior to the execution of the current adjustment step; thus each corrected elevation takes part in the adjustment of the following elevations of the grids.

All elevations of the grids in the overlapping area are corrected, point by point, in the manner described above, the order being adjustment of all points of the grid of model I with regard to model II, thereafter, adjustment of the heights of model II with regards to model I. One iteration cycle is completed after all eelvations of the corners of both grids have been corrected. Termination of the process is regulated by the criterion

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|w|_{\max }<0.02 \Delta H
$$

where $|w|$ max. is the extreme value of a misclosure in a condition equation detected during execution of a cmplete iteration cycle, and $\Delta H$ is the contour interval of the map being prepared.

The iteration process converges rapidly and enables to adjust elevations of a large number of grid corners in a short period of time. Since grid elevations are the basis of the procedure for generating contours, the contours in the overlapping areas, as defined in each of the grids, when plotted on the common map sheet pass smoothly from one model to the other without disagreement.

## 4. Determining boundaries between models

The aim of the procedure is to delimitate the areas of the individual models. Firstly, the type of the overlap has to be defined, i.e., whether the overlapping area is shared by two models of one strip, or by models of neighbouring strips. Determination of the type of the overlap is carried out automatically considering the positions of the areas of the models in relation to each other.

For models belonging to one strip the boundary is established as a line passing through the common area, dividing it into two parts, each part being associated with its model (figure 3.a). The boundary line is defined by two points which bisect the segments of lines formed by pairs of corresponding model corners. That line intersects the boundaries of each model at the points a,b,c,d, Thus, model I is bounded at the right by the line $b-d$, when plotted on the common map, and the other model II is delimitated at the left by the line a-c.

When the models lie in two strips, the overlapping area is associated in its entirety with one of the mode1s, with that which contributes a larger area to the map, while in the other model, the region corresponding to the overlapping zone is being declared as a void area (figure 3-b). The void area is not displayed when the respective model is plotted on the map.

The void area is encompassed by a polygon, the vertices of which originate from three sources (fig. 3-b) :

- Corners of model I which 1 ie in the surveyed area of model II eg.2;
- Corners of model II situated within the area of model I eg. 4,
- Points of intersection between corresponding lines which circumscribe
the scanned areas of both models e.g.s and $t$.
These vertices are focated by sortings and intersections, usually in a random order. An appropriate procedure is devised to rearrange the sequence of these points until they form a convex polygon.


Figure 3a.


Figure 3b.

The information on the boundaries so established is added to the data files of the respective DTM's, hence, when compiling a map the relevant parts of each model are placed on the sheet at the proper positions with regard to each other, without forming gaps between models and without plotting any type of information more than once.
5. Example

Examples illustrating the matching procedures are given below. Figure 4 represents segments of three models $A, B, C$ sharing common areas. The mode1s A \& B belong to one strip and C to another. The areas displayed on figure 4 have been cut out from models processed separately without applying any of the procedures above, they show clearly the disagreements between the various lines caused by the errors in the data of the respective DTM's.

Figure 5 depicts the overlapping area common to the three model segments after matching the information. The adjustment has been carried out in the order described above, matching lines, adjusting elevations and establishing boundaries. On the basis of the adjusted data, contours have been generated in each model and plotted on the common sheet bordering each other. The agreement between the contours and feature lines on the map is nearly perfect; displacements between lines, if any, are of the order of magnitude of the plotting accuracy.


Figure 4.




Figure 5.

