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# OBSERVATIONS OF DISPLACEMENTS OF A BRIDGE LOADED TO FAILURE, USING ANALYTICAL PHOTOGRAMMETRY 

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#### Abstract

In connection with an analytical photogrammetric measurement of a motor bridge loaded to failure, some calculations are carried out to establish the deformation in the vertical and longitudinal directions of the bridge (the socalled strains). The basis for the calculations is a stereo model corresponding to the unloaded bridge together with a series of single photographs of each of the following load steps. A state of the measuring situation is given together with an estimate of the elements which might influence the calculated deformations.


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# OBSERVATIONS OF DISPLACEMENTS OF A BRIDGE LOADED TO FAILURE, USING ANALYTICAL PHOTOGRAMMETRY 

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## Introduction

In October 1977 the National Danish Road Laboratory carried out tests to failure on a twenty-year old motor bridge, which was abandoned due to road alteration. This opportunity was found useful to investigate the application of analytical photogrammetry for vertical displacement and longitudinal strain measurements. The conservative way to determine the strain is to measure elongations over a distance which cannot exceed 300 mm due to limitations in the measuring device. This determination can be done with an accuracy of $10^{-4} \mathrm{~mm} / \mathrm{mm}$.

By using analytical photogrammetry to measure displacements of typical coordinates, the distance between measuring points used to calculate strain can be increased up to 2000 mm . The demand to accuracy will hereby be diminished.

This will mean considerably less demand for accuracy of measurements if the same accuracy strain determination is maintained. Another problem in using a conventional measurement device is that the displacements of the bridge could be so large that the measure points would move out of the observation range. This problem could also be avoided by using analytical photogrammetry.

## Description

The method is based on taking stereo exposures of the unloaded bridge followed by taking mono exposures from the right-side camera after every load step. Originally it was planned to take stereo exposures after every load step, but unfortunately the left-side camera did only work satisfactorily at the beginning of the loading. Thus the principle described above was used.

Two UMK/10 cameras were used, the left-side camera with rolling film and the right-side camera with glass plates. Both cameras were placed on supports three meters above road level and at a distance of fourteen meters from the bridge facade. The distance between the cameras was 4.5 meters. Five control points were determined by intersection. The arrangement of control points and cameras is shown in Figures 1 and 2. Due to poor exposure condition, point No. 64 was omitted. In order to establish the scale, the two points, 98 and 99, of a substance bar were furthermore intersected. Those two points were also used as control points at the absolute orientation of the stereo model. The measure-points were placed in the upper and lower sides, respectively, of the bridge in vertical sections at a distance of approximately one meter. Furthermore, a few points were placed on the two columns, see Figure 1.


The bridge just before failure

## Analysis

Measurement of the pictures was carried out in a Zeiss-Jena Stecometer with automatic recording (LOGIC). The calculation was made in the follow ing order:

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1. The stereo model corresponding to the unloaded bridge was measured and the coordinates to all points calculated.
2. Every picture from the right-side camera corresponding to the different loading steps were put into the right-side picture holder and measured together with the left-side picture from the stereo model. Every measurement made in this way was then considered a "stereo measurement" and calculated as such.
3. All the vertical parallaxes to the points at the bridge were calculated, and since the bridge facade and the picture plan were both vertical, the displacement of these parallaxes was equal to the vertical displacement of the bridge. These results could be checked by comparison with similar results achieved by levelling to a series of rods hung up under the bridge (see Figure 1).

## Calculation of the Longitudinal Strains in the Bridge Facade

Based on the principles explained above, the calculation of the longitudinal strains in the bridge will proceed in the following order:

1. Establish a system of $u-v$ coordinates with origin in the projection center of the right-side camera and with its $u$ axis parallel to the bridge facade (see Figure 2)


Figure 2
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2. Transform all $x-y$ coordinates (including control points) determined by means of the stereo model to the system mentioned under 1.
3. Convert to angles $\theta$ by means of the camera constant, all $x^{\prime \prime}$ coordinates corresponding to the control points.
4. Determine the position of the camera in the $u-v$ system by a resection to the control points using the angles calculated under 3 as observations.
5. Calculate the rotation of the camera necessary to bring the axis of the camera perpendicular to the bridge facade.
6. Correct $x^{\prime \prime}$ corresponding to the calculated rotation of the camera.

Hereafter all coordinates can be calculated. All the measured horizontal parallaxes are corrected for possible $\varphi$ and $\kappa$ rotation as well.

## Results

Vertical displacements: As can be seen from the tables, regarding the vertical displacement, there is reasonable accordance between the two methods. The reason why not all load steps are shown by the photogrammetric method is that the steps where the pressure was relieved are not included. The real failure occurred at load step 16.

Table 1
Vertical Disclacement (Levelling)

| Loadstep <br> (No) | Lcad <br> (kN) | Pnt. |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{gathered} 03 \\ (n m) \end{gathered}$ | $\begin{gathered} 06 \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} 08 \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} 10 \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} 12 \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} 14 \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} 17 \\ (-m .) \end{gathered}$ |
| 1 | 235 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 423 | 4.5 | 7. | 8.5 | 8.5 | 8.0 | 5.5 | 2.5 |
| 3 | 523 | 7.5 | 12.5 | 14.5 | 14.0 | 13.0 | 9.5 | 4.5 |
| 4 | 849 | 11.0 | 18.0 | 21.3 | 21.5 | 19.4 | 14.5 | 7.3 |
| 5 | 812 | 18.1 | 31.0 | 36.5 | 37.5 | 32.0 | 24.0 | 11.0 |
| 6 | $3 \times$ | 28.0 | 47.5 | 56.5 | 58.0 | 4.3 | 26.5 | 17.5 |
| 7 | 1114 | 39.5 | 67.0 | 81.3 | 8.2 .0 | 6.5 | 51.5 | $24 .=$ |
| \% | 140 | 52.0 | 87.3 | 107.5 | 110.0 | 72.5 | $\therefore 8.0$ | 32.0 |
| $\checkmark$ | 2F | 6.5 | 12.0 | 15.5 | 15.5 | 14.0 | a. 5 | -. 0 |
| 10 | 124: | 54.5 | 93.0 | 1ヶ2. | 115.0 | 76.0 | 70. | 33.0 |
| 11 | 73 | 10.0 | 18.0 | 22.0 | 23.0 | 20.0 | 14.5 | 7.0 |
| 17 | 1:5x | 56.5 | 97. $=$ | 118.5 | 121.0 | 101.5 | 74.5 | 34.5 |
| 13 | 337 | 10.3 | 19.0 | 23.5 | $2^{\prime}+.0$ | 21.0 | 14.5 | 7.0 |
| 14 | 7454 | 81.0 | 141.0 | 170.5 | 175.0 | 145.0 | 105. $=$ | 48.0 |
| 15 | $3=4$ | 15.5 | 30.0 | 37.0 | 38.5 | 32.0 | 23.5 | 10.3 |
| O6 | 1464 | 85.0 | 140.0 | 180.8 | 185.5 | 153.5 | 112.0 | 50.0 |
| 17 | $1=2$ | 104.5 | 182. 5 | 2.4 .0 | 232.0 | 190.0 | 138.0 | E1.0 |
| 1 | 707 | 107.0 | 102.0 | 248.3 | 280.5 | 212.0 | 148.8 | 60.5 |

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Table 2.

Vertical Displacement (Photogrammetry).

| Loadstep <br> (No) | Load $(\mathrm{kN})$ | $\begin{gathered} 03 \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} 06 \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} \text { Pnt } \\ 08 \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} 10 \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} 12 \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} 14 \\ (\mathrm{~mm}) \end{gathered}$ | $\begin{gathered} 17 \\ (\mathrm{~mm}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 235 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 423 | 4.0 | 8.6 | 9.9 | 9.8 | 7.9 | 6.6 | 3.3 |
| 3 | 523 | 7.3 | 13.1 | 15.2 | 15.8 | 12.5 | 10.1 | 4.7 |
| 4 | 649 | 9.9 | 19.6 | 22.7 | 23.1 | 19.0 | 14.7 | 7.3 |
| 6 | 969 | 29.0 | 46.8 | 56.6 | 58.9 | 48.7 | 36.9 | 17.6 |
| 12 | 1258 | 54.2 | 94.6 | 115.2 | 120.1 | 100.8 | 74.4 | 34.5 |
| 14 | 1454 | 77.9 | 137.8 | 168.6 | 175.0 | 146.3 | 108.5 | 50.3 |
| 16 | 1464 | 86.6 | 146.2 | 179.5 | 187.7 | 156.1 | 114.4 | 52.9 |
| 17 | 1529 |  |  |  |  |  |  |  |
| Hour $15.52$ | 1464 | 126.3 | 231.2 | 203.2 | 341.1 | 263.8 | 190.4 | 82.3 |
| $\begin{aligned} & \text { Hour } \\ & 16.00 \end{aligned}$ | 1464 | 125.6 | 232.7 | 248.6 | 351.8 | 270.0 | 193.2 | 80.7 |

Strains: For clarity, only a few distances are shown.

Table 3. Comparison between strains measured directly and strains determined photogrammetrically.

| Load Step | A |  | B |  | C |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{a} o / 00$ | $\mathrm{~b} o / 00$ | $\mathrm{a} o / 00$ | $\mathrm{~b} o / 00$ | $\mathrm{a} o / 00$ | $\mathrm{~b} o / \mathrm{oo}$ |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0.04 | 0 | -0.07 | 0 | 0.04 | 0 |
| 3 | 0.09 | 0.1 | -0.14 | -0.1 | 0.08 | 0.1 |
| 4 | 0.12 | 0.1 | -0.21 | -0.2 | 0.13 | 0.1 |
| 6 | 0.27 | 0.2 | -0.43 | -0.4 | 1.37 | 1.3 |
| 12 | 1.54 | 1.4 | -0.69 | -0.7 | - | 2.3 |

A. Upper side of bridge at supports
B. Upper side of bridge between supports
C. Lower side of bridge between supports
a. Directly measured
b. Photogrammetrically determined

The disagreement between the results from the strain determination is presumably due to the uncertainty by which the angle $\alpha$ is determined. The reason for this is that the camera and the control points lie almost on the same circle, where point 64 as earlier mentioned is missing. These problems can of course be overcome by using two cameras.

A better placing of the control points in relation to the bridge level would, moreover, have been desirable, but in practice this would be difficult to arrange. Finally, one should be aware of the fact that the reaction from the force acting upon the bridge was transferred to the terrain below, where the control points were placed. This may cause also the control points to undertake displacements.

