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COMPENSATION OF SYSTEMATIC ERRORS IN BUNDLE ADJUSTMENT

Abstract

Proper consideration of systematic errors of data is, without doubt, of greatest importance in improving the accuracy of aerotriangulation. This paper concentrates on the compensation of systematic errors in bundle adjustment by extending the functional model of adjustment.

The new computer program implemented in a minicomputer, the HP 21MX, includes various parameter sets for compensation, treats additional parameters (as well as other parameters) as weighted observations, and allows the solution of 12 000 simultaneous linear equations with the method of conjugate gradients. The results achieved by applying different parameter sets are discussed. The data employed in the tests originate from the recent test field photographies carried out for WG III/3 of the ISP.

1. INTRODUCTION

Simultaneous self calibration (block adjustment with additional parameters) has in many experiments, proved to be a very efficient means of compensating systematic errors, but, on the other hand, it has also turned out that some problems may arise in its application.

The primary means to improve the reliability and accuracy of simultaneous self calibration is naturally significance and correlation testing, but this involves much extra work, in particular, if an iterative algorithm is used for solving normal equations.

Another way suggested for avoiding instabilities due to overparametrization is to treat additional parameters as weighted observations. This approach, if not theoretically equal to the first approach, is attractive when considering the computational effort.

The new bundle program prepared at the Helsinki University of Technology (HUT) makes it possible to study the effect of weighting additional parameters on the accuracy and reliability of the results. This paper reports some results obtained using six different additional parameter sets with and without weighting. In addition, results of a comparison of simultaneous and a-posteriori self calibration have been given.

This work is part of the investigation of the ISP Working Group III/3 ("Compensation of systematic errors of image and model coordinates"), and the test material used originates from the Working Group.

2. THE BUNDLE PROGRAM OF THE HUT

The general features of the new bundle program prepared at the Helsinki University of Technology are the following:

- All parameters involved in the functional model (exterior orientation, object coordinates, additional parameters) can be used as weighted observations.
- The normal equations are solved iteratively by the method of conjugate gradients.
- The capacity of the program is 12 000 unknowns and 32 000 observations. The program is implemented at the HUT on a minicomputer HP 21 MX with a 32 K core memory.

With regard to the compensation of systematic image errors, the program offers rather versatile means:

- For simultaneous self calibration, six parameter sets are available (Appendix A).
- For a-posteriori self calibration, a 5th degree regressionpolynomial and linear least squares interpolation with or without filtering are available, in addition to the six parameter sets mentioned above.
- Weights can be introduced to additional parameters in three different ways:
    1) Direct introduction of weights separately to each individual parameter (based on a-priori knowledge).
    2) Based on variances, which correspond (according to error propagation) to an acceptable and/or desired image point displacement at a certain image point. So the proper choice of the magnitude of the displacement is critical.
    3) Based on a-posteriori weight estimation /2/. This method requires no a-priori knowledge, but is considerably laborious.

All the runs concerning weighting of additional parameters made in this study have been performed by using the method described in item 2.

3. DATA

The data used in these studies are part of the test material collected for the ISP Working Group III/3. The Jämijärvi test area was photographed at scale 1:4 000 with a wide-angle camera MRB and the Willunga test area at scale 1:12 000 with a wide-angle reseau camera RMK AR. More detailed information on these materials can be found in the report of the Working Group /6/. The three control point patterns used in this context are given in Figure 1.

4. RESULTS

4.1 Presentation of the results

The root mean square errors (RMSE) μ_{xy} (in planimetry) and μ_z (in height) computed at check points have been used as the quantities for measuring the effectiveness of the compensation of systematic errors. The RMSEs are given in micrometers at the image scale. In addition to the RMSEs, the improvements are indicated in percentages with respect to the so-called reference adjustment. Reference adjustment, in this connection, means an ordinary adjustment, where no special effort is made to compensate systematic errors. For it, the following steps are taken:

1° Affine transformation with 6 parameters on 4 fiducial marks.
2° Correction of mean symmetric radial distortion according to calibration report.
3° Correction of refraction according to Bertram's formula.

4.2 The effect of weighting additional parameters

The particular aim to treat additional parameters as weighted observations is to stabilize the resulting system by suppressing the correlations to a tolerable level. Thereby reliability and accuracy of the results can be improved (to some extent) without a considerable additional effort involving significance and correlation testing.

Indeed, the results obtained in this study are very encouraging (Table 1 and also Tables 2-4). In a more detailed way:

- The advantages of weighting in unstable systems (few control and/or tie-points) is clearly visible. If, in such a system, additional parameters are not weighted, the results may deteriorate considerably. On the other hand, it is very important to observe that even in very stable systems and with rather strict weighting hardly no unfavourable effect appears.
- Somewhat suprisingly, the best results are achieved by applying rather strict weighting, that is, weights corresponding to image point displacements of even less than five micrometers (Table 1).
- A further evidence of an improvement of the condition of the normal equations matrix, when additional parameters are used as weighted observations, is the sharp decrease of iteration steps required for the solution with the iterative conjugate gradient method (Table 5). The decrease of time, which is directly proportional to the number of iteration steps, can be as great as 50 %. This, again, is a very favourable phenomenon also from the economical point of view.

4.3 A comparison of different parameter sets

On the basis of the results presented in Table 1, it was decided to use in further computations with different parameter sets, two ways of weighting:

a) weights equal to zero (i.e. all parameters free),
b) weights, which correspond to an image point displacement of five micrometers at an image point x = y = 100 mm. This was by no means an optimal choice in all blocks, but seemed to be succesful on an average.

It turned out that the results differed greatly depending on side overlap. Therefore the results are given separately for 60 % side overlap and for 20 % side overlap.

<u>60 % side overlap</u>

- The differences between the results achieved with different parameter sets are practically negligible (Table 4).
- Whether or not the weights for additional parameters are introduced makes no difference.
- Neither did the variation of control point patterns make any difference in the performance between different parameter sets.

<u>20 % side overlap</u>

- Contrary to the blocks with 60 % side overlap, the parameter sets to be compared behaved in a wholly different manner in blocks with 20 % side overlap.
- Parameter sets A and E are those that seemed to manage fairly well both with and without weights. However, weighting improves the accuracy also when these parameter sets are applied with the weaker control patterns.

- Parameter set D is the one which weighting affects most strikingly. The poor results achieved without weighting follow obviously from two facts:
  1) The assumption of the orthogonality of additional parameters does not hold exactly for the image point distribution used (there were 15-20 irregularly distributed points against the 25 regularly distributed points required).
  2) The great number of additional parameters (44) and the modest numerical precision of the minicomputer (23 bits for mantissa) turned the system ill-conditioned.
- Also when using parameter set F, weighting has a very favourable effect. This obviously follows from the suppressing of strong correlations due to the formulation of the extended model /1/.
- Parameter set C is distinguishable from the other sets for its different behaviour in planimetry and in height. In planimetry the results are good even when the additional parameters are treated as free unknowns, but in height the results remain poorer than with the other parameter sets, even when weighting is applied. A comparison of the image deformations presented in Figure 2 points to heavy correlations between some of the parameters and exterior orientation. In fact, a separate experiment (test field calibration) proved that four of the parameters had correlations of the order of 0,95 with exterior orientation.
- In spite of the fact that the parameter set B is not designed for blocks with 20 % side overlaps, /7/, the results achieved with it are very good when weighting is applied.

4.4 A-posteriori self calibration

A-posteriori self calibration was performed by analyzing the residuals of the reference adjustment by using parameter sets A, D and F and by using a general 5th degree polynomial. No weighting was applied to the parameters. The essentials of the results are:

- According to expectations, the polynomial was slightly, but consistently, better than the parameter sets, which are specially designed for simultaneous self calibration (Table 6).
- The accuracy improvements obtained are typically 10-15 % both in planimetry and in height (Tables 6 and 7). It is worth observing that the results do not vary much and no deteriorating effects appear.
- Compared with simultaneous self calibration, a-posteriori self calibration produces considerably poorer results (Table 7).

CONCLUDING REMARKS

The danger of overparametrization seems to be real when additional parameters are introduced into poorly controlled blocks. In particular, this concerns blocks with minor side overlap. This danger can be essentially reduced by the weighting of additional parameters.

In the tests carried out, simultaneous self calibration proved to be far more efficient in compensating systematic errors than a-posteriori self calibration.

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APPENDIX A. Parameter sets available in the bundle program of the HUT.

A. "Orthogonal model" (3 x 3 image point distribution) /1/:

$$dx = b_1x + b_2y - b_3(2x^2 - 4B^2/3) + b_4xy + b_5(y^2 - 2B^2/3) + b_7x(x^2 - 2B^2/3) + b_9(x^2 - 2B^2/3)y + b_{11}(x^2 - 2B^2/3)$$

$$dy = -b_1y + b_2x + b_3xy - b_4(2y^2 - 4B^2/3) + b_6(x^2 - 2B^2/3) + b_8(x^2 - 2B^2/3)y + b_{10}x(y^2 - 2B^2/3) + b_{12}(x^2 - 2B^2/3)(y^2 - 2B^2/3)$$

B. "Polynomial model" /7 /:

$$dx = c_3xy + c_5y^2 + c_7x^2y + c_9xy^2 + c_{11}x^2y^2 + c_{13}x^3$$

$$dy = c_1y + c_2x + c_4x^2 + c_6xy + c_8x^2y + c_{10}xy^2 + c_{12}x^2y^2 + c_{14}y^3$$

C. "Model of spherical harmonics" /4/:

$$dx = a_1x + a_2y + q\,\frac{x}{r}$$

$$dy = -a_1y + a_2x + q\,\frac{y}{r}\text{, where}$$

$$q = a_3r\cos\lambda + a_4r\sin\lambda + a_5r^2 + a_6r^2\cos 2\lambda + a_7r^2\sin 2\lambda + a_8r^3\cos\lambda + a_9r^3\sin\lambda + a_{10}r^3\cos 3\lambda + a_{11}r^3\sin 3\lambda$$

$$r = \sqrt{x^2 + y^2} \quad\text{and}\quad \lambda = \arctan\left(\frac{y}{x}\right)$$

D. "Orthogonal model" (5 x 5 image point distribution) /5/:

$$dx = a_{12}x + a_{21}y + a_{22}xy + a_{31}l - b_{22}\,\frac{10}{7}\,k + a_{14}xp + a_{23}yk + a_{32}xl + a_{41}yq + a_{15}r + a_{24}xyp + a_{33}kl + a_{42}xyq + a_{51}s + a_{25}yr + a_{34}xlp + a_{43}ykq + a_{52}xs + a_{35}lr + a_{44}xypq + a_{53}ks + a_{45}yqr + a_{54}xps + a_{55}rs$$

$$dy = -a_{12}y + a_{21}x - a_{22}\,\frac{10}{7}\,l + b_{13}k + b_{22}xy + b_{14}xp + b_{23}yk + b_{32}xl + b_{41}yq + b_{15}r + b_{24}xyp + b_{33}kl + b_{42}xyq + b_{51}s + b_{25}yr + b_{34}xlp + b_{43}ykq + b_{52}xs + b_{45}yqr + b_{54}xps + b_{55}rs\text{, where}$$

$$k = x^2 - \frac{b^2}{2};\; l = y^2 - \frac{b^2}{2};\; p = x^2 - \frac{17}{20}b^2;\; q = y^2 - \frac{17}{20}b^2;\; r = x^2\left(x^2 - \frac{31}{28}b^2\right) + \frac{9}{70}b^4;\; s = y^2\left(y^2 - \frac{31}{28}b^2\right) + \frac{9}{70}b^4$$

E. "Physical model":

$$dx = b_1x + b_2y + b_3xr^2(1-r_o/r) + b_4xr^4(1-r_o/r) + b_5xr^6(1-r_o/r) + b_6\cdot 2xy + b_7(r^2 + 2x^2)$$

$$dy = -b_1y + b_2x + b_3yr^2(1-r_o/r) + b_4yr^4(1-r_o/r) + b_5xr^6(1-r_o/r) + b_6(r^2 + 2y^2) + b_7\cdot 2xy,$$

where r_o is a given constant (first radial distance, where radial distortion is wanted to be zero).

F. "Mixed model" /1/:

$$dx = a_1x + a_2y + a_3xy + a_4y^2 + a_5x^2y + a_6xy^2 + a_7x^2y^2 + \frac{x}{c}(a_{13}(x^2 - y^2) + a_{14}x^2y^2 + a_{15}(x^4 - y^4)) + x(a_{16}(x^2 + y^2) + a_{17}(x^2 + y^2)^2 + a_{18}(x^2 + y^2)^3)$$

$$dy = a_8xy + a_9x^2 + a_{10}x^2y + a_{11}xy^2 + a_{12}x^2y^2 + \frac{y}{c}(a_{13}(x^2 - y^2) + a_{14}x^2y^2 + a_{15}(x^4 - y^4)) + y(a_{16}(x^2 + y^2) + a_{17}(x^2 + y^2)^2 + a_{18}(x^2 + y^2)^3)$$

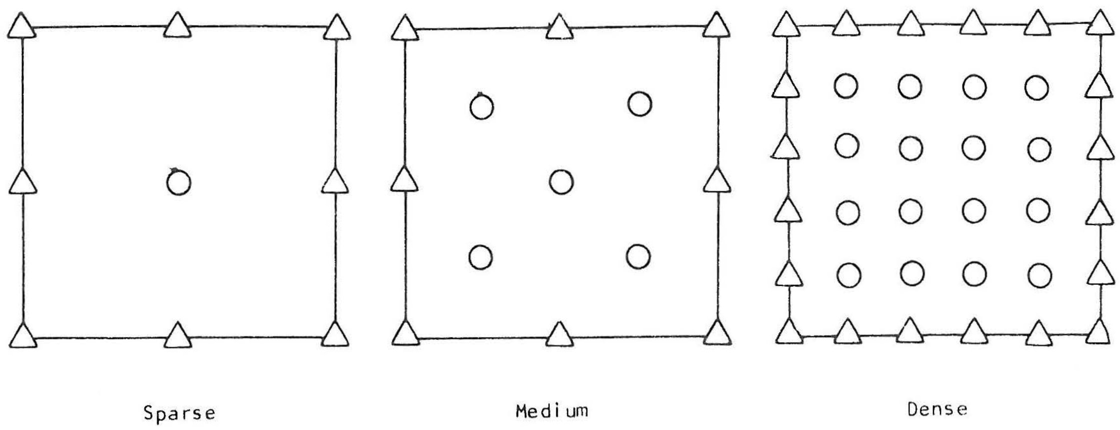

Figure 1. Control point patterns.

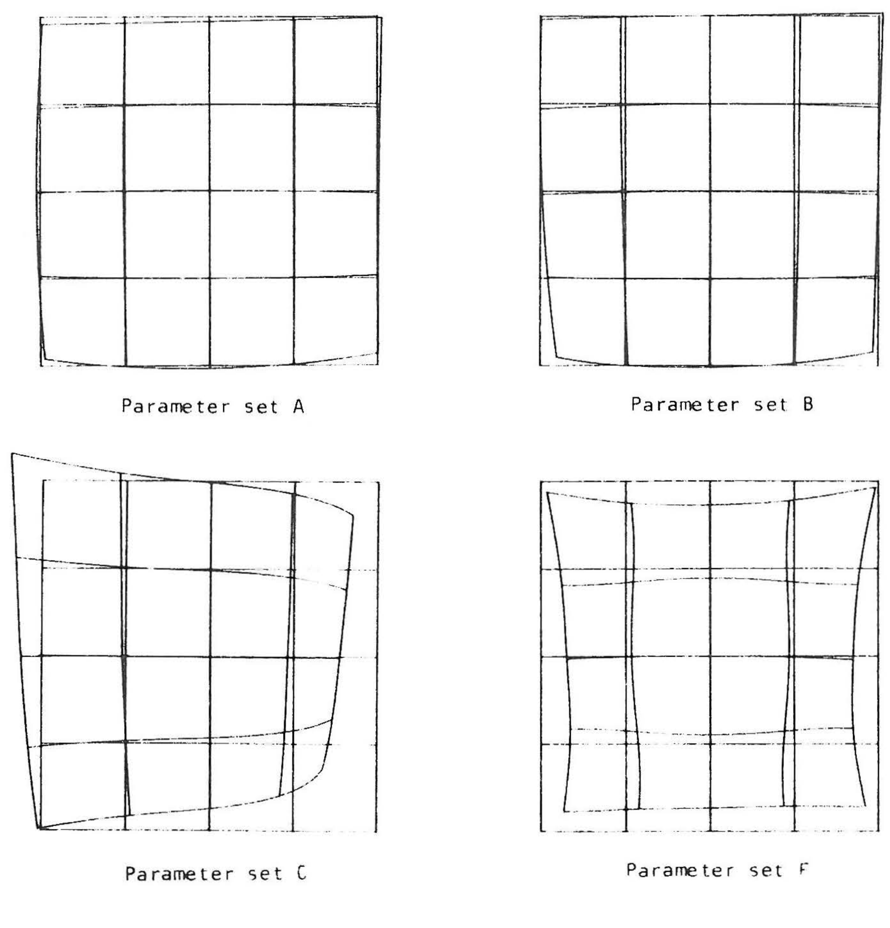

Figure 2. Example of the image deformations obtained by four different parameter sets.

Side lap [%]	Weight	Sparse control				Medium control				Dense control			
		RMSE [μm]		Impr. [%]		RMSE [μm]		Impr. [%]		RMSE [μm]		Impr. [%]	
		μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z
20	Ref.case	5,5	20,3			5,5	15,1			4,6	12,2		
(odd	all free	9,4	45,6	-71	-125	10,8	9,7	-96	+35	4,0	8,9	+13	+27
strips)	100 μm/100 mm	9,3	35,5	-69	- 60	10,5	9,5	-91	+37	4,2	8,9	+ 9	+27
	30 μm/100 mm	8,0	26,0	-45	- 28	9,9	9,4	-80	+38	4,2	8,8	+ 9	+28
	10 μm/100 mm	7,2	17,7	-31	+ 13	6,3	9,1	-15	+39	4,1	8,7	+11	+29
	5 μm/100 mm	5,6	17,6	- 2	+ 13	5,1	8,9	+ 7	+41	4,1	8,4	+11	+31
	1 μm/100 mm	4,5	17,4	+18	+ 15	4,4	10,0	+20	+34	4,2	8,2	+ 9	+33
20	Ref.case	7,1	35,5			6,2	16,5			4,2	9,7		
(even	all free	6,8	46,1	+ 4	-30	6,2	17,3	0	- 5	3,6	10,9	+14	-12
strips)	100 μm/100 mm	6,6	43,6	+ 7	-23	5,9	16,6	+ 5	- 1	3,4	10,6	+19	- 9
	30 μm/100 mm	6,6	22,3	+ 7	+34	5,5	12,6	+11	+24	3,4	9,0	+19	+ 7
	10 μm/100 mm	5,8	21,4	+18	+40	5,5	9,1	+11	+45	3,4	7,2	+19	+26
	5 μm/100 mm	5,5	17,8	+23	+50	5,0	8,2	+19	+50	3,4	6,8	+19	+30
	1 μm/100 mm	5,3	19,2	+25	+46	4,5	11,0	+27	+33	3,5	6,5	+17	+33
60	Ref.case	4,8	17,1			4,5	8,7			3,2	5,3		
	all free	2,9	7,8	+40	+54	2,7	5,3	+40	+39	2,6	5,0	+19	+ 6
	100 μm/100 mm	2,8	7,8	+42	+54	2,7	5,3	+40	+39	2,5	4,9	+22	+ 8
	30 μm/100 mm	2,8	7,7	+42	+55	2,7	5,3	+40	+39	2,5	4,9	+22	+ 8
	10 μm/100 mm	2,8	7,7	+42	+55	2,7	5,3	+40	+39	2,6	5,0	+19	+ 6
	5 μm/100 mm	2,8	7,6	+42	+56	2,7	5,3	+40	+39	2,6	5,1	+19	+ 4
	1 μm/100 mm					2,8	5,4	+38	+38	2,6	4,7	+19	+11

Table 1. Effect on the accuracy obtained by self calibration. Parameter set F, Willunga data.

Control / Set & weighting		Dense				Medium				Sparse			
		RMSE [μm]		Impr. [%]		RMSE [μm]		Impr. [%]		RMSE [μm]		Impr. [%]	
		μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z
Ref.case		4,6	12,2			5,5	15,1			5,5	20,3		
A	a	4,2	9,0	+ 9	+26	6,0	9,2	- 9	+39	7,9	26,9	-44	-33
	b	4,4	9,1	+ 4	+25	5,2	9,2	+ 5	+39	5,7	11,0	- 4	+46
B	a	4,0	9,4	+13	+23	10,9	10,0	-98	+34	7,6	24,5	-38	-21
	b	4,1	8,5	+11	+30	5,3	9,3	+ 4	+38	5,9	15,1	- 1	+26
C	a	4,5	11,5	+ 2	+ 6	5,0	10,0	+ 9	+34	4,5	79,8	+18	-293
	b	4,6	9,6	0	+21	4,7	11,3	+15	+25	4,6	22,8	+16	-12
D	a	6,5	12,6	-42	- 3	12,5	14,1	-127	+ 7	10,2	72,8	-85	-259
	b	4,1	8,5	+11	+30	5,8	9,0	- 5	+40	5,9	11,6	- 7	+43
E	a	3,9	8,6	+15	+29	4,2	10,6	+24	+30	4,3	24,1	+22	-19
	b	4,0	8,4	+13	+31	4,3	9,6	+22	+36	4,3	10,7	+22	+47
F	a	4,0	8,9	+13	+27	10,8	9,7	-96	+35	9,4	45,6	-71	-125
	b	4,1	8,4	+11	+31	5,1	8,9	+ 7	+41	5,6	17,6	- 2	+13

a = weights equal to zero.
b = weights corresponding to an image point displacement of five micrometers.

Table 2. Effect of weighting with different parameter sets. Willunga block, 20 % side overlap (odd numbered strips).

Control / Set & weighting		Dense				Medium				Sparse			
		RMSE [μm]		Impr. [%]		RMSE [μm]		Impr. [%]		RMSE [μm]		Impr. [%]	
		μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z
Ref.case		4,2	9,7			6,2	16,5			7,1	35,5		
A	a	3,3	5,9	+21	+39	6,5	9,1	- 5	+45	7,0	28,2	+ 1	+19
	b	3,3	5,9	+21	+39	5,6	8,5	+10	+48	6,4	21,6	+10	+38
B	a	3,4	7,9	+19	+19	5,6	9,2	+10	+44	6,9	28,9	+ 3	+17
	b	3,4	6,9	+19	+29	4,8	7,9	+23	+52	5,5	17,0	+23	+51
C	a	3,8	13,5	+10	-39	5,0	12,8	+19	+22	5,4	114,7	+24	-228
	b	3,6	9,3	+14	+ 4	4,7	14,1	+24	+15	5,6	25,5	+21	+27
D	a	4,2	10,6	0	- 9	7,7	28,0	-24	-70	11,7	148,3	-65	-324
	b	3,4	6,9	+19	+29	5,3	8,5	+15	+48	6,2	22,2	+13	+37
E	a	3,6	6,7	+14	+31	4,2	9,4	+32	+43	4,6	26,8	+35	+23
	b	3,4	6,7	+19	+31	4,1	8,7	+34	+47	4,6	30,1	+35	+15
F	a	3,6	10,9	+14	-12	6,2	17,3	0	- 4	6,8	46,1	+ 4	-32
	b	3,4	6,8	+19	+30	5,0	8,2	+19	+50	5,5	17,8	+23	+50

a = weights equal to zero.
b = weights corresponding to an image point displacement of five micrometers.

Table 3. Effect of weighting with different parameter sets. Willunga block, 20 % side overlap (even numbered strips).

Control / Set & weighting		Dense				Medium				Sparse			
		RMSE [μm]		Impr. [%]		RMSE [μm]		Impr. [%]		RMSE [μm]		Impr. [%]	
		μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z
Ref.case		3,2	5,3			4,8	8,5			4,8	17,1		
A	a	2,6	4,0	+19	+25	2,7	5,4	+40	+36	2,8	8,4	+42	+51
	b	2,5	4,0	+22	+25	2,7	5,4	+40	+36	2,8	8,8	+42	+49
B	a	2,6	4,8	+19	+ 9	2,7	5,1	+40	+40	2,8	7,3	+42	+57
	b	2,6	4,6	+19	+13	2,7	5,1	+40	+40	2,8	7,3	+42	+57
C	a	2,7	4,7	+16	+11	3,0	4,8	+33	+44	3,1	7,6	+35	+56
	b	2,6	4,5	+19	+15	2,9	4,8	+36	+44	3,0	8,2	+38	+52
D	a	2,6	4,7	+19	+11	2,8	5,0	+38	+41	2,9	7,4	+40	+57
	b	2,5	5,0	+22	+ 6	2,7	5,2	+40	+39	2,8	7,6	+42	+56
E	a	2,5	5,1	+22	+ 4	2,7	5,9	+40	+31	2,8	8,5	+42	+50
	b	2,5	5,4	+22	- 2	2,7	5,9	+40	+31	2,9	8,9	+40	+48
F	a	2,6	5,0	+19	+ 6	2,7	5,3	+40	+38	2,9	7,8	+40	+54
	b	2,6	5,1	+19	+ 4	2,7	5,3	+40	+38	2,8	7,6	+42	+56

a = weights equal to zero
b = weights corresponding to an image point displacement of five micrometers.

Table 4. Effect of weighting with different parameter sets. Willunga block, 60 % side overlap.

Side overlap	Parameter set	No. of iteration rounds: Without weighting	No. of iteration rounds: With weighting	Decrease of time [%]
20 %	A	383	347	9
	B	707	438	38
	C	627	415	33
	D	950	488	49
	E	290	263	9
	F	692	447	35
60 %	A	215	205	5
	B	547	240	56
	C	498	356	29
	D	612	282	54
	E	221	203	9
	F	612	258	59

Table 5. The dependence of the number of iteration steps required to solve the normal equations by the method of conjugate gradients on weighting of additional parameters. Willunga block, control medium.

SIDE LAP	PARAMETER SET	CONTROL: Dense				Medium				Sparse			
		RMSE [μm]		Impr. %		RMSE [μm]		Impr. %		RMSE [μm]		Impr. %	
		μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z	μ_{xy}	μ_z
20 %	Ref.case	3,8	9,1			5,5	10,9			4,9	12,3		
	A	3,5	9,0	+ 8	+ 1	5,2	10,6	+ 5	+ 3	4,5	12,2	+ 8	+ 1
	D	3,6	8,8	+ 5	+ 3	5,1	10,3	+ 7	+ 6	4,4	11,3	+10	+ 8
	F	3,6	8,7	+ 5	+ 4	4,9	10,2	+11	+ 6	4,3	12,1	+12	+ 2
	G[1)]	3,5	8,7	+ 8	+ 4	5,0	9,9	+ 9	+ 9	4,3	11,0	+12	+11
60 %	Ref.case	3,4	6,6			4,3	8,7			4,3	10,3		
	A	3,1	6,3	+ 9	+ 5	3,8	8,0	+12	+ 8	3,8	9,9	+12	+ 4
	D	3,2	6,3	+ 6	+ 5	3,8	7,9	+12	+ 9	3,8	9,9	+12	+ 4
	F	3,1	5,8	+ 9	+12	3,9	7,6	+ 9	+13	3,9	9,2	+ 9	+11
	G[1)]	3,0	5,6	+13	+18	3,7	6,8	+14	+22	3,7	8,4	+14	+18

1) A general fifth degree polynomial.

Table 6. Comparison of the performance of the parameter sets A, D and F and a 5th degree polynomial in a-posteriori self calibration. Jämijärvi data.

Side lap	Control	Improvements[1)] compared with: Reference adj.		Self calibration	
		μ_{xy}	μ_z	μ_{xy}	μ_z
20 % (odd strips)	Dense	+ 4	+ 7	-15	-45
	Medium	+ 9	+15	-28	-57
	Sparse	+ 9	+26	-28	-41
20 % (even strips)	Dense	+ 7	+ 6	-19	-58
	Medium	+16	+11	-27	-69
	Sparse	+13	- 3	-13	-113
60 %	Dense	+16	+11	- 4	-18
	Medium	+20	+13	-33	-45
	Sparse	+21	+23	-36	-81

1) In per centages.

Table 7. Comparison of a-posteriori self calibration with reference adjustment and simultaneous self-calibration. Willunga block.