APPLIED FORMULAE FOR ACCURATE CALIBRATION OF AERIAL PHOTOGRAMMETRIC CAMERAS

Prof. Wang Yuwei

Xian Research Institute of Surveying
and Mapping, China

Commission I

ABSTRACT

This paper, theoretically and practically, states a set of applied formulae of much higher accuracy for calibrating all of the applied geometric parameters of aerial photogrammetric cameras, with only the angles α or β measured with goniometer. According to the parameters calibrated by the formulae, the radial and tangential distortions of points in a photograph can be accurately corrected, and the position of image principal point can be exactly determined abiding by the definition in photogrammetry. The accuracy of photogrammetry can be enhanced if the formulae are applied to production.

Keywords: Photogrammetry, Calibration, Camera.

1. INTRODUCTION

With the development of photogrammetric science, the accuracy requirements for calibrating inner orientation and distorion of aerial photogrammetric camera, especially inthe latter case,havebeen much increased.

Now, in the field of calibration of aerial photogrammetric camera, it is easy to observe the well-known angles α or β , but it remains to be researched, how to only use them to determine all geometric applied paremeters of aerial photogrammetric cameras and how to introduce them into production. Here is given a set of calibration formulae for discussion.

2. DERIVATION OF FORMULAE FOR CALIBRATION

For convenience, it is first supposed that there is an ideal aerial photogrammetric camera without any distortion and any mistake of installation. A photograph V_0 taken with the camera is shown in fig 1.

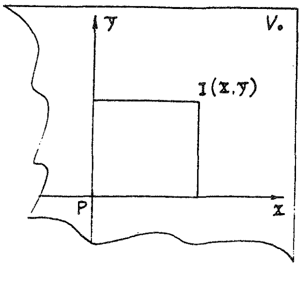

Fig 1. An ideal photograph

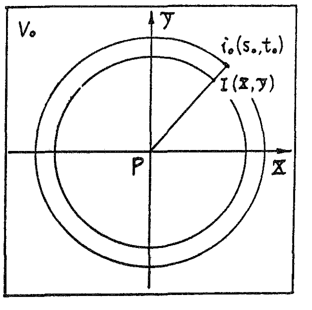

Fig 2. An optical designed photograph

The image principal point defined in photogrammetric literature is represented with P, coordinate axes are represented by X and Y with the origin point at P; arbitrary image point in the photograph could be represented with I which coordinates are (X,Y). We would name the photograph an ideal photograph.

In an optical design, however,the distortion,optical distortion, exist indeed. If the optical and mechanical parts and installation of them are made without mistakes,distortions are symmetrical with respect to point P and axes X and Y, as shown in Fig 2. In that case, the distortion-free image point I will be shifted from I to i_0 producing imege displacement Ii_0, i.e. the length of Ii_0 is the distortion of image point I and is defined as positive in Fig2. When the coordinates of i_0 are designated with (s_0 ,t_0) in the coordinate system (P-X,Y),according to the opinim of Dr H Ziemann and other scholars[Ref.5], we can write

$$Ii_0 = \sum_{i=1}^{n} a_i (Pi_0)^{2i+1}$$

For convenience of application, we can rewrite

$$Ii_0 = \sum_{i=0}^{n} c_i \left| Pi_0 \right|^{i+1}$$

i.e.

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} s_0 \\ t_0 \end{bmatrix} \left(1 - \sum_{i=0}^{n} c_i \left| (s_0^2 + t_0^2)^{\frac{i}{2}} \right| \right) \qquad (1)$$

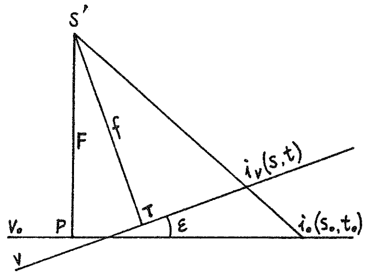

Fig 3. A show of the real image plane

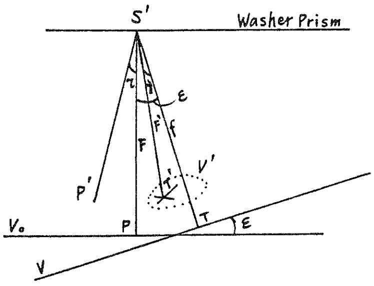

Fig 4. A show of the model of thin prism

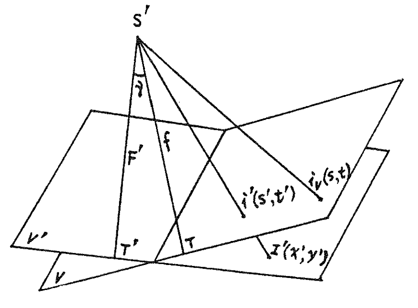

Fig 5. Geometrical state of supposed plane V'

In the process of installation, the image plane, the plane of registering frame, isnot set down into ideal position. We suppose that the real image plane V is positioned in angle ε with respect to ideal image plane V_o, so that the profile of planes V and V_o in principal tilt direction is shown in Figure 3. We suppose that an arbitrary ray $S'i_o$ through the rear nodal point S' intersects the planes V and V_o at points i_v and i_o respectively;T is the perpendicular foot from point S' to plane V, and f represents S'T; P is the perpendicular foot from point S' to plane V_o and F represents S'P. We take point T for origin of coordinates in plane V, and (s,t) for coordinates of i_v. When the planes V and V_o coincide each with other by rotating them around their intersecting axis, base on the Wang Yuwei Formula [Ref.1], we can write

$$\begin{bmatrix} s_o \\ t_o \end{bmatrix} = (1+\xi)\begin{bmatrix} s+\frac{f}{2}\sin\varepsilon_x \\ t+\frac{f}{2}\sin\varepsilon_y \end{bmatrix} + \begin{bmatrix} \frac{F}{2}\sin\varepsilon_x \\ \frac{F}{2}\sin\varepsilon_y \end{bmatrix} \qquad (2)$$

in which

$$\xi = \frac{F - f + (s+\frac{f}{2}\sin\varepsilon_x)\sin\varepsilon_x + (t+\frac{f}{2}\sin\varepsilon_y)\sin\varepsilon_y}{f - (s+\frac{f}{2}\sin\varepsilon_x)\sin\varepsilon_x - (t+\frac{f}{2}\sin\varepsilon_y)\sin\varepsilon_y}$$

ε_x and ε_y represent the tilt angles of coordinates axes in the plane V with reference to the plane V_o, and F and f represent the principal distances to the planes V_o and V respectively.

In the process of optical installation, inevitably, various errors would appear. In order to describe the effect of the errors in the geometric state, F E Washer suggested a model of thin prism [Ref.4], I would name it Washer Prism. First, we suppose that Washer Prism is set at the rear nodal point S'. Based on the model, we can consider that the errors cause a cone of rays with central ray S'P to be deviated at an angle τ , and relatively, the ray S'T to be deviated at an angle γ , as shown in Fig 4. In Fig 4, S'P' represents the deviated position of the prime principal ray S'P, τ represents its deviated angle correspondingly, S'T' represents the deviated position of ray S'T which deviates simultaneously with the ray S'P, γ represents its deviated angle. The so-called simultaneous deviation of ray S'T with ray S'P means that all of the rays in the cone with ray S'P as a centre are rotating around a line which is passing through point S' and perpendicular to plane S'PP'.

We make up a perpendicular plane V' across the ray S'T' at point T', and S'T' is represented with F'; we take the plane V' as a photograph with principal distance F'. On the plane V', we define T'as origin, and take lines which are corresponding to coordinate axes on planes V as coordinate axes,i.e. the axes on planes V' and V are through the same image points correspondingly. We can write an equation of relationship between arbitrary image point $i_v(s,t)$ on plane V and its corresponding image point I'(x',y') on plane V' as

$$\begin{bmatrix} s \\ t \end{bmatrix} = \frac{f}{F'}\begin{bmatrix} x' \\ y' \end{bmatrix} \qquad (3)$$

Along the plane S'T'T, we cut out a profile as Fig5. When the cone with the central ray S'T deviates at an angle γ causing the ray S'T to rotate from S'T to S'T', at the same time the ray $S'i_v$ in the prime cone with central ray S'T deviates from $S'i_v$ to S'i'; three points S',i' and I' are collinear.

If we suppose plane V' as a vertical photograph with principal distance F' and plane V as an obligue photograph with principal distance f, based on the Wang Yuwei Formula again, we can write

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = (1+\xi')\begin{bmatrix} s'+\frac{f}{2}\sin\gamma_x \\ t'+\frac{f}{2}\sin\gamma_y \end{bmatrix} + \begin{bmatrix} \frac{F'}{2}\sin\gamma_x \\ \frac{F'}{2}\sin\gamma_y \end{bmatrix} \qquad (4)$$

where

$$\xi' = \frac{F' - f + (s'+\frac{f}{2}\sin\gamma_x)\sin\gamma_x + (t'+\frac{f}{2}\sin\gamma_y)\sin\gamma_y}{f - (s'+\frac{f}{2}\sin\gamma_x)\sin\gamma_x - (t'+\frac{f}{2}\sin\gamma_y)\sin\gamma_y}$$

γ_x and γ_y represent the tilt angles of the coordinate axes on real image plane V with reference to the supposed image plane V'.

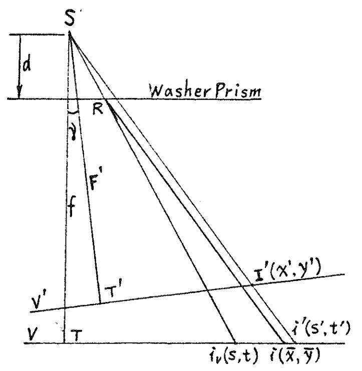

Fig 6. Effection of Washer Prism

In the above text, we have supposed Washer Prism as positioned at the rear nodal position. When Washer Prism is at some position in space as shown in Fig 6, the distance from rear nodal point S'to Washer Prism can be represented with d which direction shown in Fig 6 is positive. When d equals zero, the ray $S'i_v$ must be refracted from $S'i_v$ to S'i'; when d does not equal zero,the ray $S'i_v$ must be not refracted at segment S'R and must be refracted from $Ri_v$to Ri. Ray Ri intersects plane V at i, a real image poiint, and $(\bar{x},\bar{y})$ are its coordinates with origin at point T.

Obviously, both S'i' and Ri are become from the same ray $S'i_v$ refracted by a same thin prism, Ri run parallel to S'i', so we can write

$$Ti' = Ti + ii' = Ti + \frac{d}{f}(Ti' - Ti_v)$$

with

$$\rho = \frac{d}{f} \tag{5}$$

we can write

$$Ti' = \frac{Ti - \rho Ti_v}{1-\rho}$$

or

$$\begin{bmatrix} s' \\ t' \end{bmatrix} = \frac{1}{1-\rho}\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} - \frac{\rho}{1-\rho}\begin{bmatrix} s \\ t \end{bmatrix} \tag{6}$$

in coordinate form. When we replace (x',y') in Eq.3 by (x',y') in Eq.4, and (s,t) in Eq.6 by (s,t) in Eq.3, the mathematical relationship betwcen (s',t') and $(\bar{x},\bar{y})$ can be completely obtained. In consideration of performing iteration calculations on computer, the Eq.6 is very convenient. Therefore, other forms of mathematical relationship wouldbe no longer deduced.

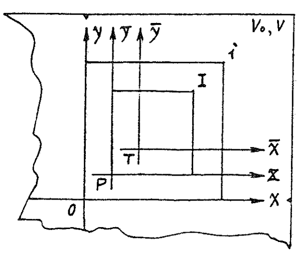

Fig 7. A show of coincident photograph

When we make the planes V and V_0inFig3 coincide each with other by rotating them around their intersecting axis, a coincided photograph can be formed as Fig 7. In the photograph, we only mark the point I in Fig.1 and thepoint i in Fig6. V_0 in Figs 1 and 2 represents ideal photograph, and V in Figure 6 represents real photograph or image plane with (o-x,y) as fiducial coordinate system. Primarily, the point I is in the plane V_0, and the point i is in the plane V. If the fiducial coordinaes of point T is expressed with (x_T,y_T), then the relationship between (x,y) in (o-x,y) and $(\bar{x},\bar{y})$ in $(T\text{-}\bar{x},\bar{y})$ for the same image point i must be written as

$$\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x_T \\ y_T \end{bmatrix} \tag{7}$$

Thus,according to the all of above equations, replacing one by one, we can obtain the coordinates (X,Y) of the ideal image point I, which are free of distortion, by use of fiducial coordinates (x,y) of the real image point i. The mathematical form can be written as

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} F_x(x,y;f,x_T,y_T,c_0,c_1,c_2\cdots,c_n,\varepsilon_x,\varepsilon_y,\gamma_x,\gamma_y,\rho) \\ F_y(x,y;f,x_T,y_T,c_0,c_1,c_2\cdots,c_n,\varepsilon_x,\varepsilon_y,\gamma_x,\gamma_y,\rho) \end{bmatrix}$$

and, the distortion of image point i is (8)

$$\vec{Ii} = \begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix} = \begin{bmatrix} (x-x_P)-X \\ (y-y_P)-Y \end{bmatrix} \tag{9}$$

All of above parameters are (n+13) in number as listed below:

Distortion coefficients $C_0, C_1, C_2, \cdots C_n$;
Obligue angles of image plane ε_x, ε_y;
Deviating angle of optical axes γ_x, γ_y;
Washer coefficient ρ ;
Used principal distances F, F', f;
coordinates of perpendicular point x_T, y_T; or
coordinates of principal point x_p, y_p.

Those parameters can be determined with only the measurements of all angles α or β, but the F, F' and f can be properly chosen [Ref.3], or supposed to be equal to any calibrated principal distance. For the convenience of representation of camera's geometrical state, the F and F' would be defined by Eq.11, the f by Eq.15, the (x_T,y_T) by Eq.16, and the (x_p,y_p) by Eq.13 later.

3. THE GEOMETRICAL STATE OF CAMERA

With parameters ε_x, ε_y, γ_x and γ_y, the oblique angle and its principal oblique line direction can be written as [Ref.1]

$$\begin{aligned} \varepsilon &= \arcsin\left|\sqrt{\sin^2\varepsilon_x + \sin^2\varepsilon_y}\right| \\ \chi_\varepsilon &= \arctan\frac{\sin\varepsilon_x}{\sin\varepsilon_y} + (1-\operatorname{sgn}\varepsilon_y)\frac{\pi}{2} \\ \gamma &= \arcsin\left|\sqrt{\sin^2\gamma_x + \sin^2\gamma_y}\right| \\ \chi_\gamma &= \arctan\frac{\sin\gamma_x}{\sin\gamma_y} + (1-\operatorname{sgn}\gamma_y)\frac{\pi}{2} \end{aligned} \tag{10}$$

Obviously, ε and γ are defined as positive. They represent respectively the oblique angles that the image plane V deviates from its ideal position plane V and its supposed plane V'; on the plane V, respectively, both principal oblique line directions are defined as those of PT and P'P or T'T,which tilt towards the nodal points and intersect with the axis y making angles χ_ε and χ_γ or $\chi_{\gamma'}$, clockwise direction as positive.

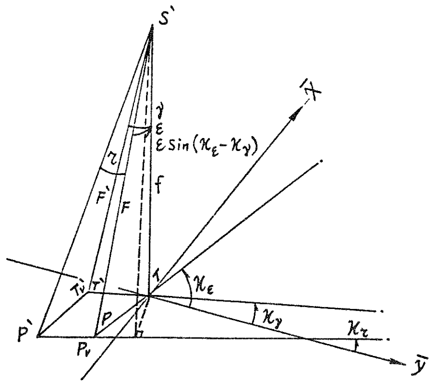

Fig 8. A show for planes V' and V_0 coincided with plane V

As shown in Fig 4, points P_V, P' and T'_V represent the points where the straight lines S'P, S'P' and S'T'intersect with plane V respectively. We take the plane V' to coincide with plane V by rotating them around their intersecting line, and do same to the plane V_0 with plane V, then the coincided image plane of the three planes V', V_0 and V can be made as shown in Fig 8. Given

$$\left.\begin{aligned} F &= f \sec \varepsilon \\ F' &= f \sec \gamma \end{aligned}\right\} \tag{11}$$

point P must be coincided with point P_V , and point T'with point T'_V ; and then, the geometrical state of angles ε and γ or τ in the space are not changeable when the three planes rotate for coinciding. According to the process generating angles τ and γ , we know that the locus of point P' is a straigt line and, the locus of point T' is a quadratic line. With the consideration of that the angles τ and γ are very small,the angle between P'P and T'T can be deemed as

$$\angle(P'P, T'T) = \frac{1}{2} \arcsin\left(\sin\varepsilon \sin\gamma \sin(\kappa_\varepsilon - \kappa_\gamma)\right)$$

According to the relationship between an oblique angle and its coordinate oblique angles [Ref.1], we can write

$$\begin{aligned} \tau &= \arcsin \frac{\sin\gamma}{\cos(\varepsilon \sin(\kappa_\varepsilon - \kappa_\gamma))} \\ \kappa_\tau &= \kappa_\gamma + \frac{1}{2} \arcsin\left(\sin\varepsilon \sin\gamma \sin(\kappa_\varepsilon - \kappa_\gamma)\right) \\ \tau_x &= \arcsin\left(\sin\tau \sin\kappa_\tau\right) \\ \tau_y &= \arcsin\left(\sin\tau \cos\kappa_\tau\right) \end{aligned} \tag{12}$$

The geometrical relationship among the angles κ_τ, κ_ε and κ_γ , as shown in Fig 9, the bottom view of Fig8,are those among the principal oblique direction of angles τ , ε and γ . Then, the fiducial coordinates of points P , T' and P' can be written as

$$\begin{aligned} \begin{bmatrix} x_P \\ y_P \end{bmatrix} &= \begin{bmatrix} x_T \\ y_T \end{bmatrix} - \begin{bmatrix} f \tan\varepsilon_x \\ f \tan\varepsilon_y \end{bmatrix} \\ \begin{bmatrix} x_{P'} \\ y_{P'} \end{bmatrix} &= \begin{bmatrix} x_P \\ y_P \end{bmatrix} - \begin{bmatrix} F \tan\tau_x \\ F \tan\tau_y \end{bmatrix} \\ \begin{bmatrix} x_{T'} \\ y_{T'} \end{bmatrix} &= \begin{bmatrix} x_T \\ y_T \end{bmatrix} - \begin{bmatrix} f \tan\gamma_x \\ f \tan\gamma_y \end{bmatrix} \end{aligned} \tag{13}$$

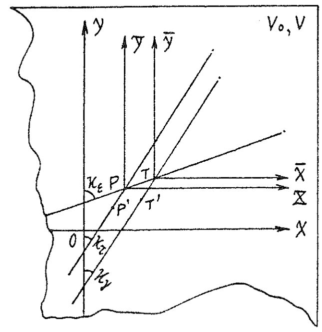

Fig 9. Relationship among the angles τ , ε and γ

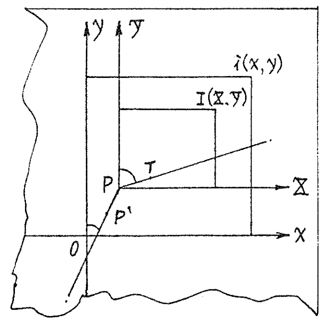

Fig 10. A coincident photograph

and the length of three segments among them

$$\begin{aligned} PT &= f \tan\varepsilon \\ PP' &= F \tan\tau \\ TT' &= f \tan\gamma \end{aligned} \tag{14}$$

In Figure 9, the point P represents the first across point where the primary principal optical ray intersect with ideal image plane, i.e. the perpendicular foot from rear nodal point to that plane. When the image plane, the plane of registering frame, rotates at an angle ε around point P along the direction PT towards rear nodal point, the perpendicular foot shifts from the point P to the point T. When the primary principal ray gets a deviation caused by errors of installation,the ray passing through point P deviates from S'P to S'P', i.e. the point P where the primary principal ray intersects with the image plane shifts from P to P'; at the same time, the ray S'T deviates from S'T to S'T'.

For the convenience of comprehension and application, summarizing all above Figures and keeping their explicit important elements, we can make a general show for results of camera calibration as Fig 10.

Note again,the Fig 10 canbe considered as a fivefold geometric state, describing negative, positive,transparent positive, image plane and calibration plane. If we take it as image plane or calibration plane,it must be with the rear nodal point S' down or scale lines down; if we take it as positive, transparent positive and negative, the latter must be with emulsions down, and others with emulsions up. No matter which geometrical state does Fig10 express,the coordinate relationships of mathematics in it are not changeable and, PT is always the direction of its oblique angle ε which rotates around point P towards the nodal points S' and S, as well as PP'is always a locus of intersected point where the primary principal ray deviated at an angle τ from ideal position to real position intersects with it.

4. ERROR EQUATIONS FOR DETERMINATION OF CAMERA PARAMETERS

In Eq.8, the first three from parameters of $f, x_T, y_T \cdots$ and ρ should be calculated first, then the others. According to the angles α or β measured on the goniometer, β represents minus α, as shown in Fig 11. The parameters (x_T, y_T and f) can be directly determine [Ref.2] by

$$f=\frac{\sum_{i=1}^{n_x}(\sin 2\alpha_i)^2 f_x+\sum_{j=1}^{n_y}(\sin 2\alpha_j)^2 f_y}{\sum_{i=1}^{n_x}(\sin 2\alpha_i)^2+\sum_{j=1}^{n_y}(\sin 2\alpha_j)^2} \tag{15}$$

$$\begin{bmatrix} x_T \\ y_T \end{bmatrix}=\begin{bmatrix} \sum_{i=1}^{n_x}\sin^4\alpha_i\cdot OT_i/\sum_{i=1}^{n_x}\sin^4\alpha_i \\ \sum_{j=1}^{n_y}\sin^4\alpha_j\cdot OT_j/\sum_{j=1}^{n_y}\sin^4\alpha_j \end{bmatrix} \tag{16}$$

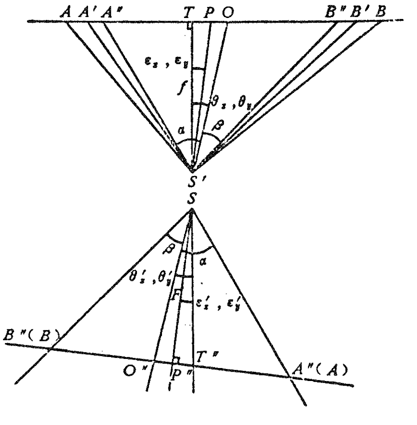

Fig 11. Angles α or β measured on goniometer

In Fig 11, P"S is a incident ray of S'P. Now we make a perpendicular plane B"P"A" at the point P"; given P"S=F, then P"A" could be deemed as X or Y in Eq.1, and P"B" as negative direction of P"A". The incident rays O"S and T"S of S'O and S'T intersect with plane B"P"A" at O" and T" respectively. Obviously, every angle α or β corresponds to its X and Y, which can be directly calculated with angles ε_x', ε_y', θ_x' and θ_y' by

$$\begin{aligned} X_i&=F\tan(\alpha_i-\theta_x'+\varepsilon_x') \\ Y_j&=F\tan(\alpha_j-\theta_y'+\varepsilon_y') \\ &(i=1,2,3,\cdots,2n_x) \\ &(j=1,2,3,\cdots,2n_y) \end{aligned} \tag{17}$$

where $2n_x$ and $2n_y$ represent the numbers of measured angles at axes x and y respectively, i.e. double band numbers of measured angles; and

$$\begin{bmatrix} \varepsilon_x' \\ \varepsilon_y' \end{bmatrix}=\begin{bmatrix} \arctan(X_T/F) \\ \arctan(Y_T/F) \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} \theta_x' \\ \theta_y' \end{bmatrix}=\begin{bmatrix} \arctan[(X_T-X_0)/F] \\ \arctan[(Y_T-Y_0)/F] \end{bmatrix} \tag{19}$$

For an arbitrary image point (x_i, o) at axis x of calibration plate, we can obtain the corresponding $(X,Y)_i$ with Eq.8, i.e.

$$\begin{bmatrix} X \\ Y \end{bmatrix}_i=\begin{bmatrix} F_x(x_i,0;f,x_T,y_T,\cdots,\rho) \\ F_y(x_i,0;f,x_T,y_T,\cdots,\rho) \end{bmatrix}$$

where the X is just the same geometrical value as X_i in Eq.17, so one of error equations can be made as

$$-\Delta_i=F_x(x_i,\ 0;\ f,\ x_T,\ \cdots,\ \rho)-F\tan(\alpha_i-\theta_x'+\varepsilon_x')$$

for arbitrary image point(o, y_j) at axis y of calibration plate, in the same way, we can write

$$\begin{bmatrix} X \\ Y \end{bmatrix}_j=\begin{bmatrix} F_x(0,y_j;f,x_T,\cdots,\rho) \\ F_y(0,y_j;f,x_T,\cdots,\rho) \end{bmatrix}$$

and

$$-\Delta_j=F_y(0,y_j;f,x_T,\cdots,\rho)-F\tan(\alpha_j-\theta_y'+\varepsilon_y')$$

With their weights

$$P_i=\cos^4\alpha_i,\quad P_j=\cos^4\alpha_j$$

the correction equations can be written as

$$\begin{aligned} V_i&=F_x(x_i,0;f,x_T,\cdots,\rho)-F\tan(\alpha_i-\theta_x'+\varepsilon_x') \\ V_j&=F_y(0,y_j;f,x_T,\cdots,\rho)-F\tan(\alpha_j-\theta_y'+\varepsilon_y') \\ &(i=1,2,3,\cdots,2n_x) \\ &(j=1,2,3,\cdots,2n_y) \end{aligned} \tag{20}$$

Based on the theory of adjustment, Eq.20 can be treated and the values of parameters $C_0, C_1, C_2 \ldots$ and ρ can be accuratly worked out.

According to the all solved parameters, the distortion-free coordinates (X,Y) of arbitrary image can be obtained with Eq.8, and their rms errors would be estimated by

$$\begin{bmatrix} m_X \\ m_Y \end{bmatrix}=\begin{bmatrix} 1+(x/F)^2 \\ 1+(y/F)^2 \end{bmatrix}\cdot\mu \tag{21}$$

Because the principal distance f can be properly chosen and the theoritical value of parameter C_0 is zero, the number of real parameters to be calulated would be (n+2+5) and, corresponding to unit weight, the mean error μ would be estimated by

$$\mu=\pm\sqrt{\frac{\sum_{i=1}^{2n_x}V_i^2\cos^4\alpha_i+\sum_{j=1}^{2n_y}V_j^2\cos^4\alpha_j}{2n_x+2n_y-(n+5+2)}} \tag{22}$$

By the way, when the ρ does not equal zero (see Fig6), the rays of central projection refracted by Washer Prism are no longer of central projection; but when the ρ equals zero, the rays are still of central projection. That is, when the ρ equals zero, all refracted rays must rotate around the rear nodal point S', so the geometrical relationships among the rays are unchangeable, and the deviation angle τ of the cone of rays with reference to the image plane canbe reversely deemed as a deviation angle of the image plane with reference to the cone of rays, as shown in Fig4. Thus, the position of the perpendicular S'T can be deemed as formed by twice rotation of the image plane from S'P' indirectly to S'T with going through S'P, or by single rotation of the image plane from S'P' directly to S'T without going through S'P. In the latter case, S'P can be deemed as S'P', and we can obtain a conclusion of that angles τ and γ are equal to zero. That is, the ρ is zero, the γ is zero; we must pay attention to that case during the process of calculation.

5. THE CALCULATION OF CAMERA PARAMETERS

For the set of error equations in Eg.20, the extraordinary nonlinear functions, it is a difficult problem to solve it during adjustment. But, based on the principles in aerial photogrammetric literature, and by making a concrete analysis of concrete conditions carefully, the problem has been solved. Because of limited space, the calculated steps of the solving method of the problem would be only expressed in principle here, and the steps of computer program:

1) Input the primary measurements;
2) determine the initial values of parameters;
3) perform the adjustmant;
4) find out the iterated values of parameters in (i)th iteration;
5) find out $(\bar{x},\bar{y})$, (s',t'), (x',y') and (s,t);
6) find out (s_o, t_o), (X,Y), X_i and Y_i;
7) find out all of V_i and V_j, and to analyse their situation;
8) compare the results of iteration in (i)th and (i+1)th;
9) print the final results.

6. EXAMPLE

According to the above formulae, with use of the calibration plate made in China and calibrated by Metrological Research Institute of China, the actual calibrated measurements and calculation have been performed for the Wild camera RC-10 in Xian, China. All measurements are obtained at diagonals and the calculated results are as follows:

No.of camera /Format: Wild RC-10 No. 2149/23x23cm
Goniometer: Made by Reseach Institute No.303, in Beijing, China.
Rms error of obseved angles: $m_\alpha = m_\beta = m = \pm 0.''8$
Num. of bands: $n_x = n_y = 15$ (Bandwidth 10cm)
Calibrated principal distance: F = 87.99684 mm

Coordinates of image principal point P:

x_p=-0.01497 mm $\quad$ y_p=+0.01732 mm

Distortion coefficients:

f =87.99684 mm
x_T=-0.01272 mm $\quad$ y_T=+0.01539 mm
C_0=-0.0
$C_1=-0.261689 \cdot 10^{-5}$ $\quad$ $C_4=-0.712027 \cdot 10^{-10}$
$C_2=-0.309024 \cdot 10^{-6}$ $\quad$ $C_5=+0.100630 \cdot 10^{-12}$
$C_3=+0.928586 \cdot 10^{-8}$ $\quad$ $C_6=-0.359739 \cdot 10^{-15}$

Oblique angles of image plane:

$\varepsilon_x = +5.''3$ $\quad$ $\kappa_\varepsilon = 130^\circ 32' 40.''8$
$\varepsilon_y = -4.5$ $\quad$ $\varepsilon = 6.9$

the deviation of optical axis:

$\tau_x = \gamma_x = -0.''1$ $\quad$ $\kappa_\gamma = 181^\circ 20' 33.''8$
$\tau_y = \gamma_y = -2.6$ $\quad$ $\gamma = 2.6$

Washer coefficient:

ρ =+0.019375 (d=+1.70474 mm)

Rms error of unit weight:

μ = 1.60 μm

Rms error of coordinates (X,Y)

$$\begin{bmatrix} m_X \\ m_Y \end{bmatrix} = \pm 1.60 \begin{bmatrix} 1 + (X/F)^2 \\ 1 + (Y/F)^2 \end{bmatrix} \mu m$$

7. CONCLUSIONS

The formulae discussed in this paper have following characteristics:

a) The obtained principal point P(x ,y)and principal distance F in keeping with what have been defined in photogrammetric literature.

b) The corrected coordinates (X,Y) of arbitrary image point are truly needed photogrammetric coordinates with the origin at image principal point defined by photogrammetric literature and, free of radial and tangential distortions; obvioully, the photogrammetric accuracy would be benefitted by application of the coordinates.

c) The obtained parameters reflect respectively the quality of photogrammetric camera with respect to optical design, optical installation and mechanical installation. The parameters, therefore, can be taken as the important indexes of the geometric quality of an aerophotogrammetric camera. For the quantitative analyses of the stability of geometric quality, it is beneficial to repeatedly calibrate parameters on one camera for a long time.

By the way, in view of that the influence of distortion on coordinates of image point have been exactly solved by means of calbration in this paper, distortion would be no longer a troublesome problem. Therefore, we suggest that aerial photogrammetric cameras, especially to the cameras for use at high altitude and stellar cameras, would be from now on designed with less or even without requirements for distortional tolerance; and using thus saved "design power" designer would consider some other quality of items, such as resolution, clarity and illuminance, especially, the latter would be possessed of very more important significance.

Thanks to Ms. Gu Xiaoling for writing program and performing calculation; without her work to help, it was impossible to complete this paper.

(Note: In this paper the explanation of concrete method for solving nonlinear error equations Eq.20 is not detailed, it would be carefully stated in my next paper)

8. REFERENCES

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