

## A NEW APPROACH TO PHOTOGRAMMETRIC PLOTTING

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### ABSTRACT

The vertical exaggeration, or exaggeration of depth, in stereo vision is applied for the first time to the design of a portable analogic plotter. The operator perceives, in the photographic stereomodel, a floating mark which is driven vertically by an altimetric rod, allowing the angle between vertical direction and camera ray to be simulated, and a continuous orthographic plotting to be performed. Besides elevations, other real topographic magnitudes including lengths, dips, slopes, etc., can be analogically obtained.

Key Words: Analog, Binoptical, Instrument Design, Photogrammetry, Plotter, Vertical Exaggeration, Stereoscopic, 3-D.

### INTRODUCTION

Stereo-vision has been widely used in photogrammetry, for example, as a qualitative aid for drawing contour level lines whose heights have to be quantitatively obtained by other means. In general, stereo-vision is recognized as an effective way for showing objects in three dimensions. Recently, the solution to some problems faced in the interpretation of satellite images is being obtained by means of stereoscopic images. In this respect, a contributing work on synthetic stereo imaging has been done by D. Lorenz (1983). The question is that, as to proper stereo-vision is concerned, models perceived stereoscopically lack quantitative value because no method for establishing a scale for depth impression is known. It is generally accepted that stereoscopic depth impression varies directly with parallax, but the mathematical equation connecting these variables has not been found.

The clue to the solution may lie in the phenomenon responsible for a terrain to be perceived vertically exaggerated when it is viewed stereoscopically through a pair of aerial photographs. However, this vertical exaggeration is to be entirely explained. On the basis of geometric relationships apparently existing between real object and perceptual stereomodel, different mathematical equations for vertical exaggeration have been proposed, yet with limited applicability in practice. It looks as if geometric optics had not been effective enough as a tool in the search for a solution. Beyond the classical approach, the theory of a non-Euclidean perceptual space was suggested. Collins (1981) mentions the hypothesis of a Lovachevskian perceptual space proposed by Luneburg (1957), and some attempts for representing perceptual space as a topological homeomorph of real space. Such a controversial situation has provided reasons for the perceptual stereomodel to be considered by Collins as a "mystery", and the search for determining its vertical exaggeration to be regarded as a "Quixotic effort" (Jacoumelos, 1973). Rosas (1986) refers to an atmosphere of skepticism that seems to have caused a stagnation of the investigations in this field.

Observations carried out in INGEOMINAS with the cooperation of COLCIENCIAS, Colombia, and the Institute for Photogrammetry and Engineering Surveys, University of Hannover, Germany, have led to the recognition of some photogrammetric

relationships connected with vertical exaggeration, which have allowed new photogrammetric methods to be developed.

### STEREO-VISION THROUGH AERIAL PHOTOGRAPHS

When a pair of aerial photographs is viewed stereoscopically, three space models take place :

1. The real model, represented by the terrain being photographed, or real terrain.
2. The photographic model resulting from reproducing the real model at photographic scale, or photographic terrain.
3. The perceptual model, corresponding to that formed in the mind of the observer, or perceptual terrain.

#### Real Terrain

The geometric relationships connecting real terrain with photographic stereopair (Figure 1a) are well known in photogrammetry. By triangle similarity, the following equation is obtained :

$$h = \frac{HP}{Bs + p}$$

where

- h= Real height of a point above real reference plane
- H= Height of flight
- p= Parallax difference on the photographs
- B= Camera base
- s= Photographic scale

For small parallaxes, that is, near the reference plane, we may write :

$$h = \frac{Hp}{Bs} \quad (1)$$

#### Photographic Terrain

It is also well known in photogrammetry that an exact reproduction of the real terrain at photographic scale is obtained by replacing height of flight for camera focal length, and camera base for its corresponding distance on the photographs, as shown in Figure 1b. Then, for this model we have:

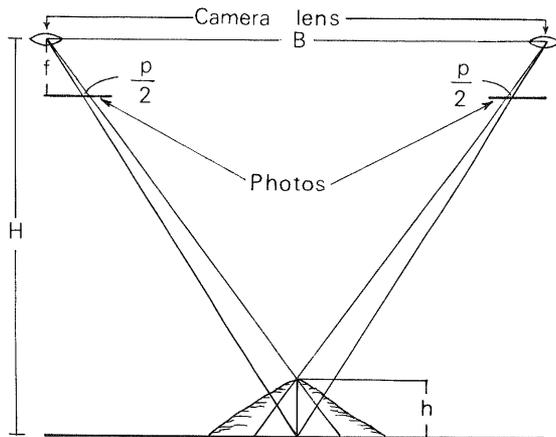
$$h' = \frac{fp}{b} \quad (2)$$

where

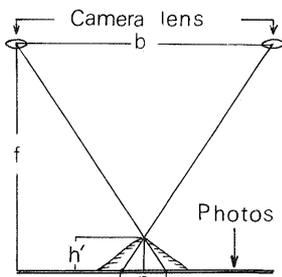
- $h'$  = Photographic height of a point, above plane of photographs.
- $f$  = Camera focal length
- $p$  = Parallax on the photographs
- $b$  = Camera base at photographic scale

and

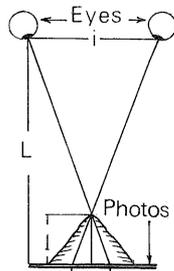
$$h' = hs$$



(a)



(b)



(c)

Fig.1. Geometry of terrain models. a) Real terrain b) Photographic terrain. c) Perceptual terrain.

### Perceptual Terrain

With regard to the perceptual terrain model, its geometry is similar to that of the real one. The observer's eyes can be considered as two cameras separated by the interpupillary distance, which capture two photographic images from a certain distance. For the real model, this distance corresponds to the height of flight, whereas for the perceptual model it becomes the perceptual viewing distance.

A very common reasoning is that the concept of convergence involved in the formation of a perceptual model by intersection of eye rays cannot be applied when the photographic stereopair is viewed with parallel or divergent eye axes. At this point, it is convenient to keep in mind that stereo-vision is obtained by fusion of two retinal images in the brain, through an internal mechanism independent of the convergence of eye axes. By geometric optics we may say that, provided the photographs remain at the same distance and in the same position relative to each of the eye axes, the mere rotation of these axes has no

effect upon the corresponding retinal images, nor upon the result of fusing them in the brain, or perceptual model. Taking advantage of this freedom of choice for ocular convergence, the geometry of the perceptual model is more easily illustrated by choosing those axes that converge at the perceptual viewing distance.

Figure 1c shows the geometry of the perceptual terrain model. In this figure, the optical axes were arbitrarily made to converge at the perceptual viewing distance ( $L$ ) and therefore, the photographs appear overlapping as in the anaglyphic vision. This overlapping is only virtual, for it is valid for any real position of eye axes, including those parallel or divergent. Then, by triangle similarity, the following equation, similar to those obtained for real and photographic terrain, is derived:

$$l = \frac{Lp}{i} \quad (3)$$

where

- $l$  = Perceptual height of a point above perceptual reference plane
- $L$  = Perceptual viewing distance
- $p$  = Parallax on the photographs
- $i$  = Interpupillary distance

### DIRECT STEREO-VISION

Stereo vision can also be performed when a real space object is viewed directly from two viewpoints, with or without the aid of lenses. Natural vision belongs to this type of situation. By the way, the hypothesis that natural binocular vision provides a correct mental representation of tridimensional objects has been advocated in most of the investigations on stereo-vision. However, this assumption has not been proved. If the object is viewed by means of a lens system, the viewpoints are determined by the position of the objective lenses. In direct stereo-vision two space models are involved:

1. The real model represented by the object that is viewed directly, or real object.
2. The perceptual model corresponding to the mental impression produced in the mind of the observer, or perceptual object.

### Real Object

The geometry of the real object is equivalent to that of the real terrain. It is obtained by replacing the camera stations for the viewpoints, and the height of flight for the actual viewing distance (Figure 2a.). By applying a similar procedure, the following equation is derived :

$$d = \frac{Dp}{I} \quad (4)$$

where

- $d$  = Real height of a point above real reference plane
- $D$  = Actual viewing distance
- $p$  = Parallax difference on the real reference plane
- $I$  = Viewpoint interval

### Perceptual Object

Essentially, the geometry of the perceptual

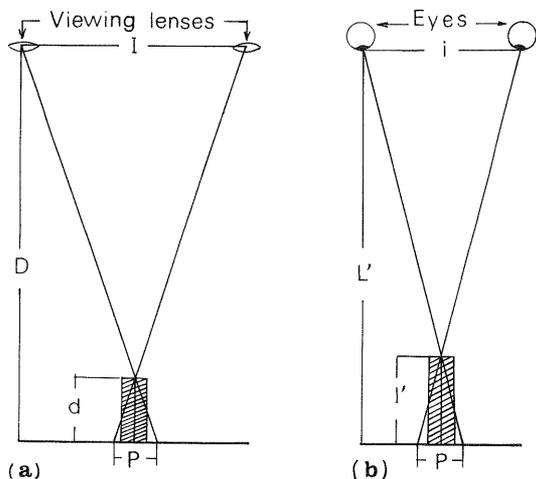


Fig.2. Geometry of object models. a) Real object. b) Perceptual object.

object (Figure 2b) is identical to that of the perceptual terrain. An accidental difference of the object model is that parallax, instead of belonging to the photographs refers to the real reference plane. On this basis, the following equation is obtained :

$$l' = \frac{L'P}{i} \quad (5)$$

where

- l' = Perceptual height of a point, above perceptual reference plane
- L = Perceptual viewing distance
- P = Parallax on the real reference plane
- i = Interpupillary distance

#### VERTICAL EXAGGERATION

Vertical exaggeration ( $E_v$ ) is defined as the ratio of vertical scale ( $S_v$ ) to horizontal scale ( $S_h$ ). That is:

$$E_v = \frac{S_v}{S_h} \quad (6)$$

In stereo-vision, the vertical exaggeration refers to that of the perceptual model relative to the real one. Because exaggeration occurs only in the vertical scale, the vertical exaggeration of the perceptual model can be referred to as perceptual exaggeration. For the perceptual terrain,  $S_v$  becomes the ratio of perceptual terrain height ( $l'$ ) to real terrain height ( $h$ ); and  $S_h$  becomes the photographic scale ( $s$ ). Then, being  $E$  the vertical exaggeration of the perceptual terrain, or perceptual terrain exaggeration, we have:

$$E = \frac{l'}{hs} \quad (7)$$

Substituting  $l'$  and  $h$  for their values obtained in Equations 3 and 1, we have :

$$E = \frac{B}{H} \cdot \frac{L}{i}$$

In this equation,  $B/H$  represents a measure of the convergence of camera rays. Therefore, this ratio will be conventionally called photographic convergence and will be represented as

$V$ . Then, the above equation will take the following expression :

$$E = \frac{V}{i} \cdot L \quad (8)$$

For the perceptual object,  $S_v$  becomes the ratio of perceptual object height ( $l'$ ) to real object height ( $d$ ); and  $S_h$  becomes the natural scale, that is, one (1). Then, being  $E'$  the perceptual object exaggeration, we have :

$$E' = \frac{l'}{d} \quad (9)$$

Substituting  $l'$  and  $d$  for their values obtained in Equations 5 and 4 :

$$E' = \frac{L'}{D} \cdot \frac{L}{i}$$

In this equation,  $L/D$  represents a measure of the convergence with which object is viewed. Therefore, this ratio will be conventionally called viewing convergence and will be represented by  $V'$ . Then, the above equation may also be expressed as :

$$E' = \frac{V'}{i} \cdot L' \quad (10)$$

Equations 8 and 10 show that vertical exaggeration depends on perceptual viewing distance ( $L$  and  $L'$ , or  $L$  in general) which continues being an interrogative, since no precise way for its accurate determination has been found. Initially it was thought that  $L$  would correspond to the viewing distance, but experience has shown this assumption to be wrong.

Authors like Raasveldt (1956), Miller (1958) and La Prade (1972) tried to develop methods for determining the perceptual viewing distance. However, their observations have not gone beyond some empiric estimations not free from subjectiveness. Others, like Goodale (1953), Treece (1955), Jacoumelos (1972) and Collins (1981) have questioned the equation itself proposing different mathematical expressions.

Evidences show that Equations 8 and 10 provide true mathematical expressions for vertical exaggeration ; and that the real question does not concern the validity of the equation but the lack of a reliable procedure for determining the perceptual viewing distance.

Nevertheless, it will be explained that the vertical exaggeration in stereo vision, despite its unknown variable ( $L$ ), provides sufficient elements for the development of new photogrammetric methods.

#### A NEW APPROACH

In principle, vertical exaggeration implies that a vertical scale for the perceptual model can be established. In such case, perceptual dimensions could be used for measuring real dimensions. In fact, if the perceptual model could be mathematically measured, it would provide a correct representation of the terrain, allowing accurate terrain maps to be traced from perceptual models.

The problem is that absolute vertical exaggeration cannot be mathematically calculated because of its unknown variable  $L$  (perceptual

viewing distance). And even in the event that L could be mathematically known, it is very probable that this variable would be dependent on certain parameters characteristic of each person, which would affect its objectivity.

However, the above mentioned difficulties in the application of the vertical exaggeration equation, particularly as to the unknown variable L is concerned, can be overcome by using a relative vertical exaggeration. This entails a relation between two vertical exaggerations, with the possibility for the unknown variable L to be eliminated in the division, in case of being equal in both terms. Evidently, this approach requires the handling of two different models, as will be shown in the next section.

### PHOTOGRAMMETRIC APPLICATIONS

For photogrammetric applications of the vertical exaggeration phenomenon, optical instruments must be designed in such a way that two different types of stereomodels be simultaneously perceived: (1) the model perceived through the photographic stereopair (terrain model) and (2) the model perceived by viewing directly a real object (object model) which can be used for measuring the terrain model. Although these two models originate in different places and by and different means, optical arrangements allow them to be perceived in superposition one on the other.

The object model may have any desired shape, and therefore it can be designed for measuring different geometric elements in space. In this paper, an object for measuring terrain elevations will be considered. Such an object may consist of a small micrometric rod (rod model) perpendicular to the terrain model, with a measuring mark on its top, which will be called altimetric rod. In the optical superposition of models, if the rod mark matches the terrain point being measured, the micrometer reading will give a measure of the real point height. The vertical relationships between the models optically superimposed (terrain and rod) are outlined in Figure 3.

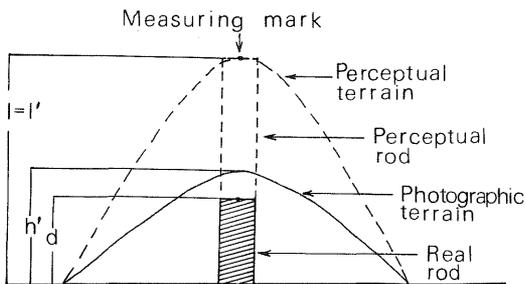


Fig.3. Vertical relationships between terrain and rod.

Since the altimetric rod represents a real object perceived by direct stereovision, the same equations and symbols used for object models will be used for the rod model. Then, let e be the ratio of rod exaggeration to terrain exaggeration, or relative exaggeration of rod. We have.

$$e = \frac{E'}{E} \quad (11)$$

Substituting E' and E for their values obtained in Equations 9 and 8 :

$$e = \frac{V'}{V} \cdot \frac{L'}{L} \quad (12)$$

Although no mathematical expression for vertical exaggeration has been developed, empirical observations have shown that the higher is the vertical exaggeration, the higher becomes the perceptual viewing distance. The same direct relationship occurs with regard to the actual viewing distance. This feature suggests some sort of positive correlation between perceptual viewing distance and actual viewing distance. At least, it can be assumed that one value of the perceptual viewing distance corresponds to only one value of the actual viewing distance. It implies that, in Equation 12, the perceptual viewing distances L and L' can be made equal by equalizing the corresponding actual viewing distances in the instrument. In such case, L/L' becomes one (1) and Equation 12 takes the following form:

$$e = \frac{V'}{V} \quad (13)$$

In this equation, the unknown variables L and L' have been eliminated. The remaining variables deal with instrumental conditions feasible to be controlled. Variable V was defined as the convergence with which photographs are taken and V' becomes the convergence with which the measuring mark is viewed.

Dividing Equation 9 by Equation 7 :

$$\frac{E'}{E} = \frac{l'}{d} \cdot \frac{hs}{l} \quad (14)$$

According to Figure 6, the perceptual rod is supposed to match the perceptual terrain relief. That is:  $l' = l$ , or  $l'/l=1$ . Substituting this value in Equation 14, we have :

$$\frac{E'}{E} = \frac{hs}{d} \quad (15)$$

Equalizing Equation 15 to Equation 11 :

$$e = \frac{hs}{d} \quad \text{or :} \quad h = eds \quad (16)$$

Substituting h for its value given in Equation 2 :

$$h' = ed \quad (17)$$

This is the formula for calculating the photographic terrain height (h') in function of the relative exaggeration (e) and the real rod height (d). As mentioned above, e is given by the instrumental parameters used for photography and for viewing; and d is read in the rod micrometer. The real terrain height (h) is expressed in Equation 16.

On the other hand, the availability of a real altimetric rod which is viewed superimposed on the photographic terrain permits vertical magnitudes to be specially reconstructed, and terrain points to be plotted orthographically.

The above mentioned possibilities for applying the vertical exaggeration effect to photogrammetric plotting can be implemented by means of the binoptical plotter whose basic characteris-

tics will be described in the next section.

#### BINOPTICAL PLOTTER

The possibility for applying the vertical exaggeration effect to the development of new photogrammetric methods have lead the author to the desing of a portable plotter. For the present, the problem concerning orthogonal projection of photographic points is treated independendly of the external orientation of photographs. This instrument was called binoptical plotter for the reason that it permits the handling of the double optics created by perceiving simultaneously two different stereo-models: (1) a virtual photographic model of the terrain being measured, and (2) a real space model represented by the measuring object. Although several types of measuring objects can be used depending on the geometric element to be measured, attention will be centered in the altimetric rod.

The technical bases of the instrument will be firstly illustrated with the following hypothetical consideration: Since the altimetric rod represents a real physical object like that of the real terrain, it can be taken as a reference for reconstructing the geometry of camera rays, by making the eye rays to play the roll of camera rays. In such conditions, if the altimetric rod remains normal to the photographic reference plane, and its measuring mark is kept on the model relief, the rod X-Y movements will trace an orthogonal projection of the terrain. For this purpose, a movable viewing system should be designed. Its optical axes would need to be rotated in order to follow the rod movements all over the terrain model. The simulation of the photographic conditions by using a movable optical system is illustrated in Figure 4. It is important to note that for the rod to remain normal to the reference plane, it has to move parallel to line OC which connects the center of perspective (O) with the center of photography (C). Angle  $\alpha$  between eye ray and rod is  $0^\circ$  at C, and increases with the rod distance from C.

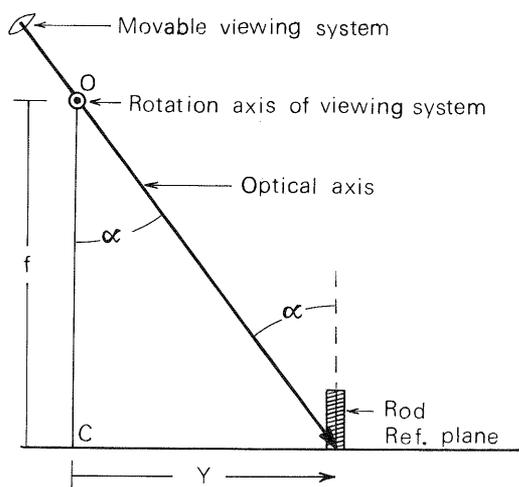


Fig.4. Reconstruction of photographic conditions with movable viewing system.

Another option (Figure 5) consists of fixing the viewing system and allowing the photographs to move on a scanning stage. In this instance, the sense of movements is inverted; the rod remains static with reference to X-Y axes, and

line OC inclines as C is moved off the optic axis. Angle  $\alpha$  can be simulated by making the rod to rotate parallel to OC. For practical reasons, this last option was adopted for building the plotter.

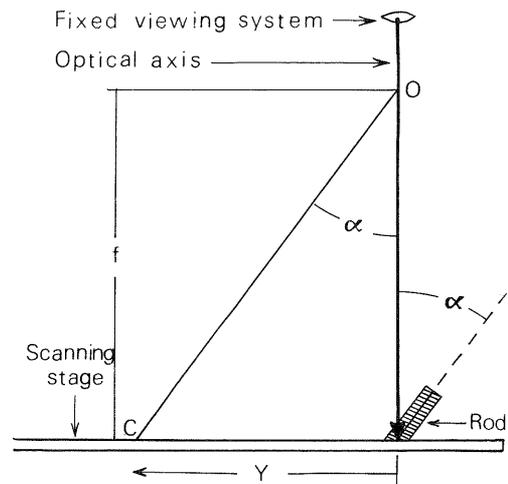


Fig.5. Reconstruction of photographic conditions with fixed viewing system.

The binoptical plotter (Figure 6) consists of a mirror stereoscope which remains fixed over an X-Y scanning stage where the photographic stereopair is placed and oriented. By optical reflexion and refraction through semitransparent mirrors, the eye axes are driven simultaneously to the photographs and to a central altimetric rod. Viewing distance is the same for the photographs and for the rod.

The altimetric rod can swing on an axis parallel to the base line, and is provided with a measuring mark at its top. A micrometer indicates the mark displacements upward and downward of the swinging axis of the rod. Because of its real nature, the altimetric rod can be replaced by objects having measuring marks different from a point, such as lines or planes for measuring slopes, bearings, dips, strikes, etc.

Behind the altimetric rod, and parallel to it, swings an extendible arm simulating the directions of the camera rays. The lower end of this arm is attached by means of a hinge to a car which runs back and forth on a rail suspended over the scanning stage. The car is mechanically allowed to follow the Y movements of the stage, and to remain static when the stage is moved along X. This feature permits the angle between viewing direction and altimetric rod ( $\alpha$ ) to be reproduced, and horizontal displacements caused by relief to be removed in the Y direction. In the X direction, parallel to the base line, displacements are automatically corrected by optical triangulation when a pair of homologous points are viewed stereoscopically.

The extendible arm achieves its minimum length at its vertical position. For an exact reconstruction of the photographic conditions, it can be mathematically proved that this minimum length ( $m$ ) is given by the ratio of camera focal length ( $f$ ) over relative exaggeration of rod ( $e$ ). That is :

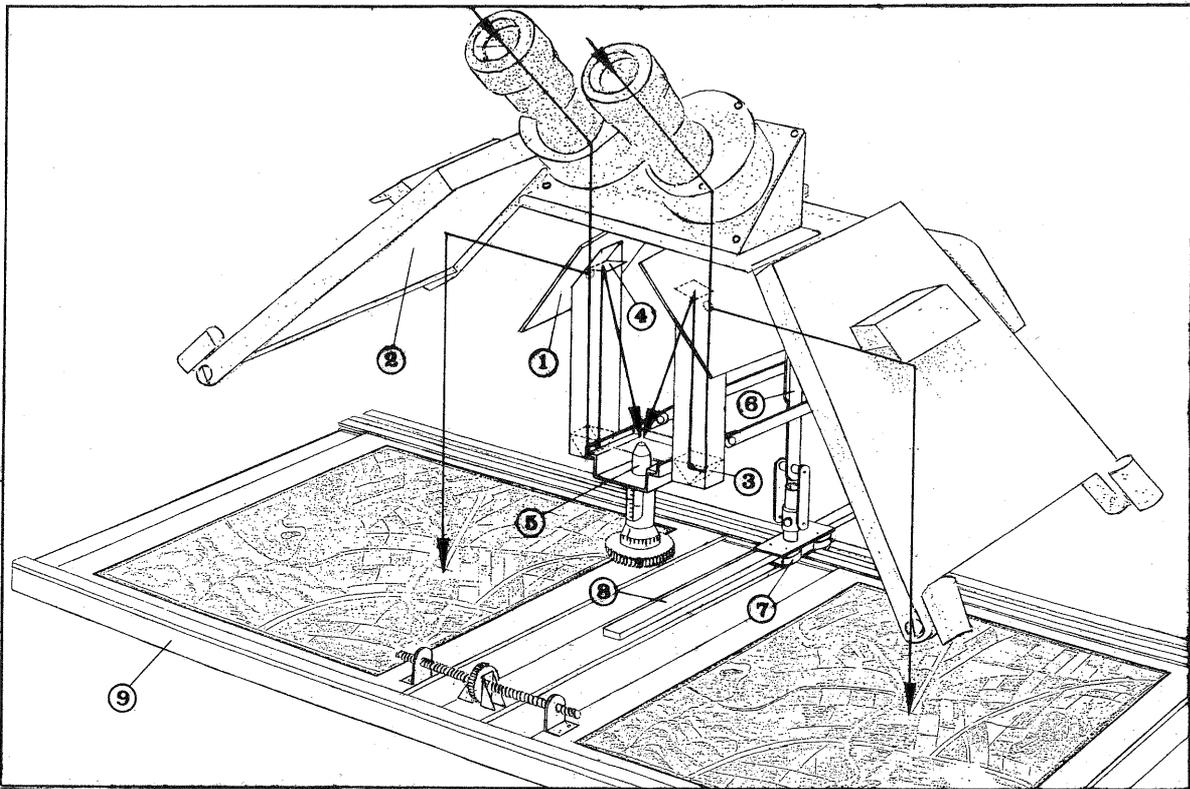


Fig. 6. Binoptical plotter. 1) Semitransparent mirror. 2) Large mirror. 3) Prism. 4) Small mirror. 5) Altimetric rod. 6) Extendible arm. 7) Car. 8) Rail. 9) Scanning stage.

$$m = \frac{f}{e}$$

for technical developments in photogrammetry.

According to Equation 13,  $e$  is the ratio of the instrumental parameter  $V'$ , over the photographic parameter  $V$ . The plotter has been designed with  $V' = 0.6$  corresponding to the standard convergence of camera rays ( $V$ ) used in conventional aerial photographs. Therefore,  $e$  normally can be regarded as equal to 1 with a considerable level of accuracy. In that case :

$$m = f$$

However, depending on the desired accuracy, mechanical provisions for  $m$  adjustments have been included. In this respect it is also important to point out that, according to Equation 16, if  $e = 1$ , then  $d' = h$ . It means that the micrometer reading indicates the terrain height at photographic scale.

The operator perceives the measuring mark as a floating mark in the terrain stereomodel. When this mark matches the model relief, the elevation of this point at photographic scale is expressed in Equation 17. The orthographic position of the point is given by the tracing device which is attached to the scanning stage.

For building this plotter, a mirror stereoscope Sokkisha MS27 was chosen; an important reason for this choice being to deal with a fix separation of the objective lenses. For supporting the stereoscope over a scanning stage a Wild APT1 equipment was used.

#### CONCLUSIONS

- The phenomenon of vertical exaggeration in stereo-vision, that had never been used in photogrammetry, offers new possibilities

- In principle, the phenomenon of vertical exaggeration in stereo-vision is influenced by certain parameters characteristic of each particular person, affecting its objectivity for practical applications. However, the concept of relative vertical exaggeration allows the binoptical plotter to be designed in such a way that subjective variables are eliminated for the sake of accuracy.
- The majority of the present stereoplotters use parallax measurements for determining terrain heights. If the altimetric rod is used, the measuring mark which is read in the micrometer (photographic scale) becomes close to twice the value of the corresponding parallax in the photographs. Evidently, the fact that altimetric rod measurements represent longer intervals than those of parallax, implies a higher level of precision for the altimetric rod method.
- For analytical treatment, the binoptical plotter allows space magnitudes (heights, slopes, strikes, dips, etc) to be directly digitized as primary data.
- The use of the vertical exaggeration effect may broaden the field of photogrammetric applications. One of them could be the development of a methodology for producing stereopairs which allow space images to be perceived at a certain vertical and horizontal scale, for measuring purposes. The parallax required for obtaining a desired depth impression, or perceptual height, could be established.

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