NOISE AND OBJECT ELIMINATION FROM AUTOMATIC CORRELATION DATA USING A SPLINE FUNCTION

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ABSTRACT

A procedure for noise and object elimination from automatic correlation data, which uses cubic splines is proposed. The proposed method consists of approximating cubic splines subject to given constraints, over a set of parallel profiles generated from the automatic correlation data. This is performed by varying the values of the given constraints as a function of the residual errors generated by the approximation function adopted.

The performance of the proposed method is evaluated by comparing the results of the cubic spline algorithm and (i) the results of applying a finite element algorithm; and (ii) measurements over the real surface.

KEY WORDS: Photogrammetry, Image Correlation, Smoothing, Spline Approximation.

1. INTRODUCTION

The development of an efficient algorithm for automatic height measurements, over a photogrammetric model, has occupied several research centers during the last two decades. Several methods have been proposed but few of them implemented successfully. Only recently, with the emergence of digital photogrammetric systems, the problem was overcome. Based on image treatment, automatic processes are performed and, as a result, among others, a database input for a digital terrain model can be obtained.

The basic procedure to perform the automation is select a sufficient number of points in one image and find (automatically) the conjugate points in the other image. This task is accomplished by image matching methods, such as gray level correlation or feature matching, whence the name automatic correlation methods.

The results of these image correlation methods, however, only partially present the height of the natural terrain. Obstacles like buildings or vegetation covering the terrain and false points due to a weak correlation are included in these measurements. To obtain the true surface of the terrain, a post-processing must then be carried out to eliminate these blunders.

The paper shows that the approximation function theory may be applied successfully for blunder's elimination. The procedure consists on smoothing the data by applying an approximation function over the output correlation data combined with a method for blunder's elimination. Two generals smoothing methods have been tested: (i) a bidimensional method, by applying a spline function; and (ii) a three-dimensional method, by applying a finite element algorithm. For blunder's elimination, three methods have been tested: (i) two statistical methods based on the residual errors from the approximation function; and (ii) a deterministic method by using vectored contours from an image processing over the correlated area.

In this article we only present the bidimensional method, which uses cubic splines. The method consists on approximating cubic splines, subjected to a given constraint, over a set of parallel profiles generated from the automatic correlation data. The blunder's detection is performed by varying the values of the given constraints as a function of the residual errors generated by the approximation function adopted. The three-dimensional method is described by Da Silva, I. (1990). Next we present the relevant points of the algorithm.

2. DATA EDITING

As a result of the automatic data sampling, the resulting ground coordinates output is distributed over the correlated surface in different ways, depending on the correlation method used. At any rate, they will always represent a 3-D surface, which could be easily portioned out in a set of profiles.

The use of profiles has the advantage of allowing the application of a bidimensional approach as, for example, the smoothing theory by using cubic splines, as well as a three-dimensional approach as, for example, the finite element theory. In addition, it allows the application of a pre-filtering process over each profile to eliminate gross-errors. Hence this is the data distribution used in our algorithm.

Further, to obtain a suitable coordinate system for graphical representation, the generation of parallel profiles combined with a coordinate transformation can be useful to obtain profiles parallel to the **x**-axis. Figures 2.1 and 2.2 show an example of a 3-D output correlation data, generated by applying the "Multi-Templet Matching" program developed by the Swiss Federal Institute of Technology - Lausanne, Switzerland, and its representation in profiles.

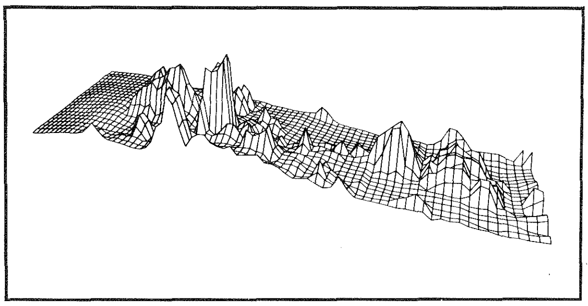


Fig. 2.1- Example of a 3-D correlation output data representation

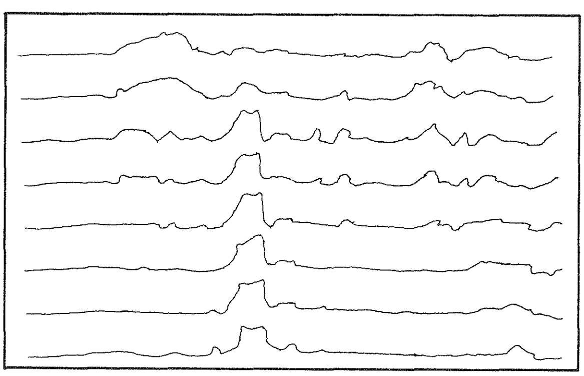


Fig. 2.2- Example of the output correlation data portioned out in profiles

3. PRE-FILTERING

We apply the pre-filtering process to eliminate the following points of the profiles: (i) coincidental points; (ii) points belonging to an acute positive angle; and (iii) points belonging to an acute negative angle. Hence, for

$$\theta_1 = \mathrm{arctg}\ \frac{z_p\ (i,j) - z_p\ (i,j-1)}{x_p\ (i,1) - x_p\ (i,j-1)}$$

and

$$\theta_2 = \mathrm{arctg}\ \frac{z_p\ (i,j+1) - z_p\ (i,j)}{x_p\ (i,j+1) - x_p\ (i,j)}$$

where xp and zp are ground coordinates.

An acute positive angle is found when

for $\theta_1 > 0$ and $\theta_2 < 0$

$$\theta_T = \theta_1 - \theta_2 \geq 85$$

An acute negative angle is found when

for $\theta_1 < 0$ and $\theta_2 > 0$

$$\theta_T = \theta_2 - \theta_1 \geq 135$$

The pre-filtering process is applied in the beginning of the first iteration.

4. APPROXIMATION THEORY - BASIC CONCEPTS

As the interpolation function theory, the approximation function theory is used to describe the behavior of an experiment for which only discrete points were measured. The main difference between them is that an interpolation function is a curve that passes through the data points and an approximation function does not.

By definition, an approximation function is the function that gives for a set of measured points (x_i, y_i), the values $(x_i, P(x_i))$, where $P(x_i) = y_i + v_i$ and v_i is the difference between the measured value and the approximation function.

As for interpolation functions, we use polynomials as function approximators and as curves for data fitting, since they and their ratios are the only functions that can be reduced to elementary arithmetic. There are, however, situations in which the approximation polynomials do not describe efficiently the experiment. In that case, as emphasized by Yakowitz, S. and Szidarovszky, F. (1986), "Instead of trying to approximate a function over the entire interval by one polynomial of high degree, one may approximate the function by piecewise polynomial function, where the degree of the polynomial piecewise associated with each subinterval is small. Splines are piecewise polynomials that prevent such erratic profiles by imposing derivate constraints at the common boundary points of the polynomial pieces".

In our algorithm we use the cubic splines approximation theory developed by Schoemberg and Reinsch (1967). A summary of that theory follows.

Let $x_1, x_2, \ldots, x_n$, and $y_1, y_2, \ldots, y_n$ be a set of measured points for a given problem and for which we seek an approximation cubic spline function f(x).

In agreement with the approximation function theory, it can be stated that for each point y_i there will be an expression of the type

$$y_i = f(x_i) + v_i \qquad (1)$$

where $f(x_i)$ is the approximation function and v_i is the difference between the measured value and the approximation function at the point x_i.

We wish a function $f(x_i)$ that could be approximated to the measured points by an appropriate constraint and that, at same time, be sufficiently smooth for not having acute angles.

Schoemberg and Reinsch propose the following conditions for the approximation function $f(x_i)$:

Smoothing condition

$$\int_{x1}^{x2} g''(x)^2 dx \qquad (2)$$

Approximation condition

$$\sum_{i=1}^{n} \left(\frac{g(xi) - yi}{\delta yi}\right)^2 \leq S \qquad (3)$$

where $g(x) \in C^2\,[x1, xn]$.

Here, $\delta yi > 0$ for $i=1,2,\ldots,n$ and $S>0$, are given numbers. The constant S is redundant and is introduced only for convenience.

The solution of the problem is obtained by introducing the Lagrangian parameter λ and an auxiliary value z. We now look for the minimum of

$$\int_{x1}^{xn} g''(x)^2 dx + \lambda \left\{ \sum_{i=1}^{n} \left(\frac{g(xi) - yi}{\delta yi}\right)^2 + z^2 - S \right\} \rightarrow \text{minimum}$$

We also want a function f(x) composed of cubic splines of the type

$$s_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

having the following characteristics:

$$s_i(x_{i+1}) = s_{i+1}(x_{i+1})$$
$$s'_i(x_{i+1}) = s'_{i+1}(x_{i+1})$$
$$s''_i(x_{i+1}) = s''_{i+1}(x_{i+1})$$

So, by introducing

$$h_i = x_{i+1} - x_i$$
$$c = (c_2, \ldots, c_{i-1})^T; \qquad c_1 = c_n = 0$$
$$y = (y_1, y_2, \ldots, y_n);$$
$$a = (a_1, a_2, \ldots a_n)^T;$$
$$D = \mathrm{diag}(\delta y_1, \delta y_2, \ldots, \delta y_n)$$

a short manipulation yields

$$b_i = \frac{a_{i+1} - a_i}{h_i} - c_i h_i - d_i h_i^2 \qquad i=1,2,\ldots,n-1 \qquad (4)$$

$$d_i = \frac{c_{i+1} - c_i}{3h_i} \qquad (5)$$

and finally

$$a = y - \lambda^{-1} D^{-2} c \qquad (6)$$

Next, if the Lagrangian parameter λ is given, we obtain by further manipulation the vector **c**, and then the vector **a** and the remaining coefficients b_i and d_i.

The computation of the Lagrangian parameter λ can be done by using an iterative process as, for example, the Newton's method. It consists on repeating iteratively the formula

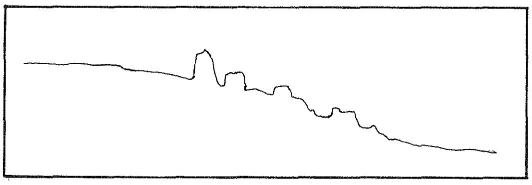
$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \qquad (7)$$

In their article, the authors propose a method for computation of the Lagrangian parameter.

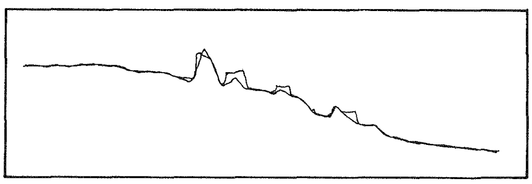
5. THE DEVELOPED ALGORITHM

Our algorithm is completely based on the theory discussed above.

The algorithm consists on applying the approximation technique described above over each profile individually, without considering its neighbors. The method is applied iteratively. After each iteration a new approximation function is generated, and used as input for the next iteration. The process stops after 3 or 4 iterations. Figure 5.1 shows an example of the behavior of a profile after each iteration.

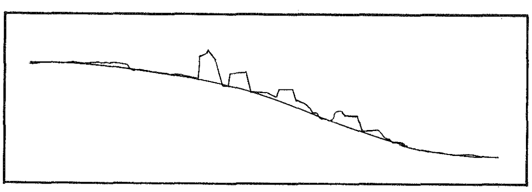


a) First iteration - after pre-filtering



b) second iteration

c) third iteration



d) fourth iteration

Fig 5.1-General representation of the result of applying the spline function algorithm.

The developed algorithm is composed by several operational phases. Figure 5.2 shows the general configuration of the algorithm.

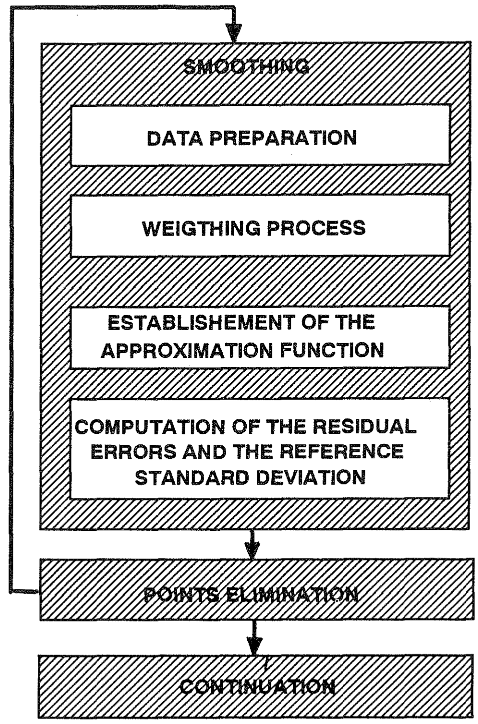


Fig 5.2 - General configuration of the algorithm

Next we describe the relevant points of each phase.

5.1. DATA PREPARATION

The data preparation consists on renumbering the data input, to put them in agreement with the approximation function theory. This is done to consider the eliminated points after each iteration.

The data preparation is done at the beginning of the iteration.

5.2. WEIGHTING PROCESS

The weighting process consists on given values to the constraint δyi of the formula (3)

$$\sum_{i=1}^{n}\left(\frac{g(xi)-yi}{\delta yi}\right)^2 \le S$$

where $S = N - \sqrt{2N}$ and N is the number of points of the profile.

As we will see next, the value of the constraint δyi is given as a function of the variation of the residual error between the measured points and the approximation function.

5.3. ESTABLISHEMENT OF THE APPROXIMATION FUNCTION

The approximation function is established as described by Schoemberg and Reinsch.

5.4. COMPUTATION OF THE RESIDUAL ERRORS AND THE REFERENCE STANDARD DEVIATION

The computation of the residual errors is simply performed by computing the difference between the original points and the approximation function at each point. Next, the reference standard deviation is computed by the following formula:

$$\sigma = \sqrt{\frac{V^{T}PV}{N-1}} \qquad (8)$$

where

σ is the réference standard deviation
V is the vector of the residual errors
P is the weight matrices
N is the number of points of the profile

The residual errors and the reference standard deviation are used as a constraint value for point elimination.

6. WEIGHTING STRATEGY

In an attempt to obtain appropriate and rapid blunder's elimination, the principal point to consider in the process is the weighting strategy. By correctly weighting the δyi value, blunders can be eliminated more rapidly.

The weighting strategy used in our algorithm is applied as follows:

1-In the first iteration, the approximation function is completely modeled with the profile. This can be done by giving a small value to the δyi constraint.

1-Next, the sign and the magnitude of the residual errors are taken into consideration. Hence, to emphasize the existing blunders and given that they will always lie above the approximation function, all points lying above the approximation function are weighted by means of a exponential function of the type

$$\delta y_i = e^{(-k1+vi/\sigma)} \qquad (9)$$

where

k1 is a constant
σ is the reference standard deviation
$0,1 \leq \delta y_i \leq 0.5$

and all points lying below the approximation function are weighted by means of a constant value $k2 \leq 0,1$.

The limit values 0,1 and 0,5 were fixed by experience.

By applying such a weighting strategy, the approximation function is forced to keep of the points lying above it and at the same time to descend toward the points lying below it.

7. BLUNDER'S ELIMINATION

For blunder's elimination two types of errors should be distinguished: (i) measuring errors due to weak correlation; and (ii) points not measured on the ground (obstacles). The first one is eliminated during the pre-filtering process if they are excessively big. When they are not big enough to be automatically eliminated during the pre-filtering process, they will rather be considered as the second type of error. The second type of error can be eliminated by taking into account the residual error generated during the approximation adjustment.

The blunder's elimination method used in our algorithm consists simply on eliminating measurements for which the residual error is greater than a specified value. Threshold values can be fixed by using the computed reference standard deviation value.

8. PRACTICAL RESULTS

For testing the procedure, a computer program was written in FORTRAN 77 for a VAX-8350.

The input data used for testing were a series of 28 simulated profiles handled from a photogrammetric model by means of the analytical plotter KERN DSR-11.

The profiles represent a slightly hilly terrain with sparse buildings and few trees. They are 300 m long (100 points) and are separated 5 m wide. The image was generated using a normal angle camera (c=300 mm) and with a flight high of 1500 m (scale = 1:5000). Figure 8.1 shows an aerial image of this input data.

Figures 8.2 and 8.3 show a perspective view and a representation with contours line of the relief before smoothing. To compare the results, figures 8.4 and 8.5 show a perspective view and a representation with contours line of the smoothed relief manually handled.

The results of the developed algorithm are shown in figures 8.6 and 8.7. The figures show a perspective view and a representation with contours line of the smoothed surface.

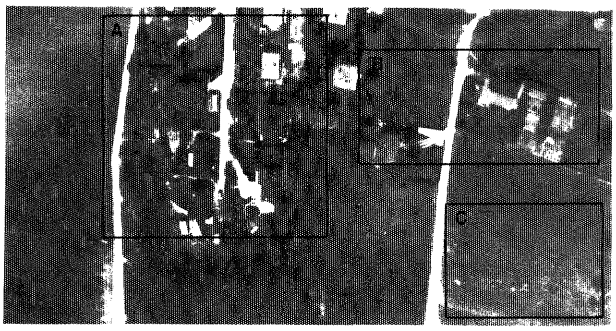


Fig. 8.1 - Aerial view of the testing region

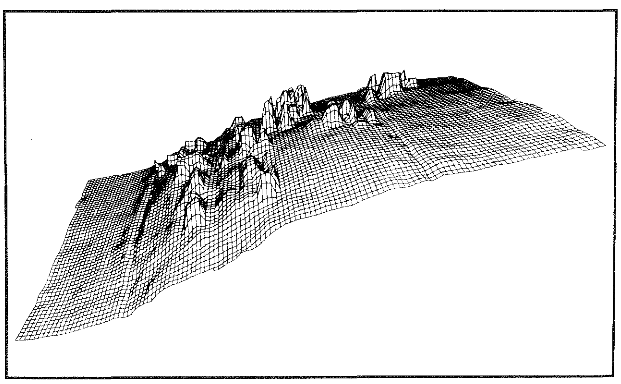


Fig 8.2 - Perspective view of the original surface (with obstacles)

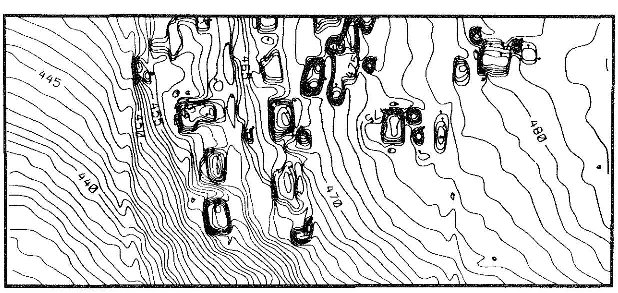


Fig. 8.3 - Representation with contours line of the original surface before smoothing

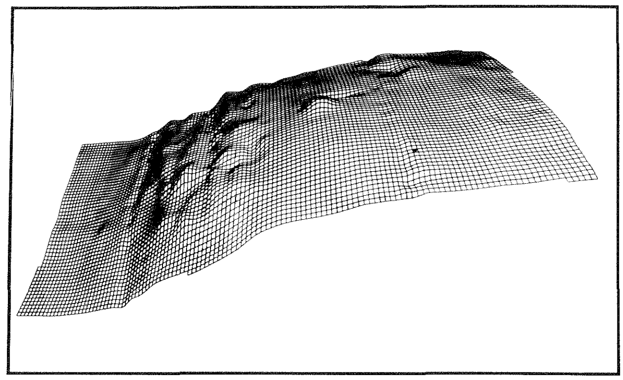


Fig 8.4 - Perspective view of the surface smoothed manually

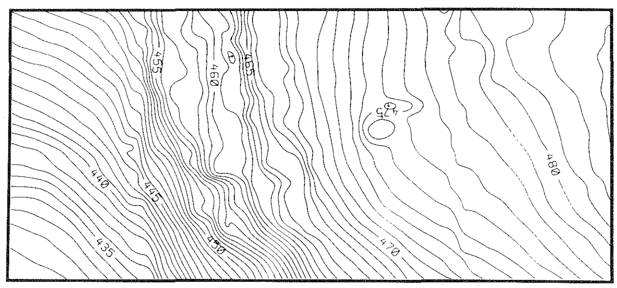


Fig. 8.5 - Representation with contours line of the surface smoothed manually

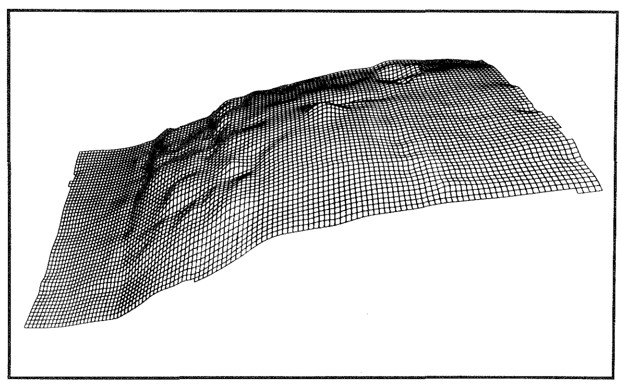


Fig 8.6 - Perspective view of the smoothed surface by applying the developed algorithm

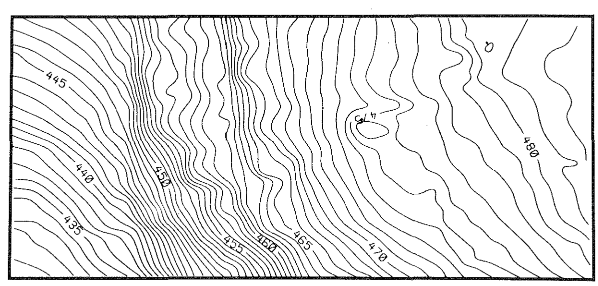


Fig. 8.7 - Representation with contours line of the smoothed surface by applying the developed algorithm

To compare the results of the developed algorithm with the finite element algorithm, the figures 8.8 and 8.9 show the region test represented by a set of profiles smoothed by applying the developed algorithm and the finite element algorithm respectively. Also, the main results of each method is presented in table I.

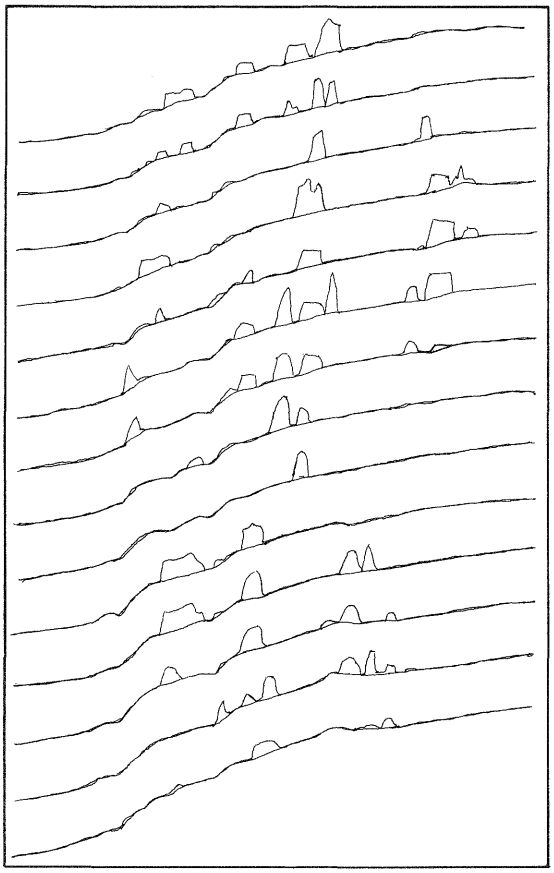


Fig 8.8-Set of profiles smoothed by applying the developed algorithm

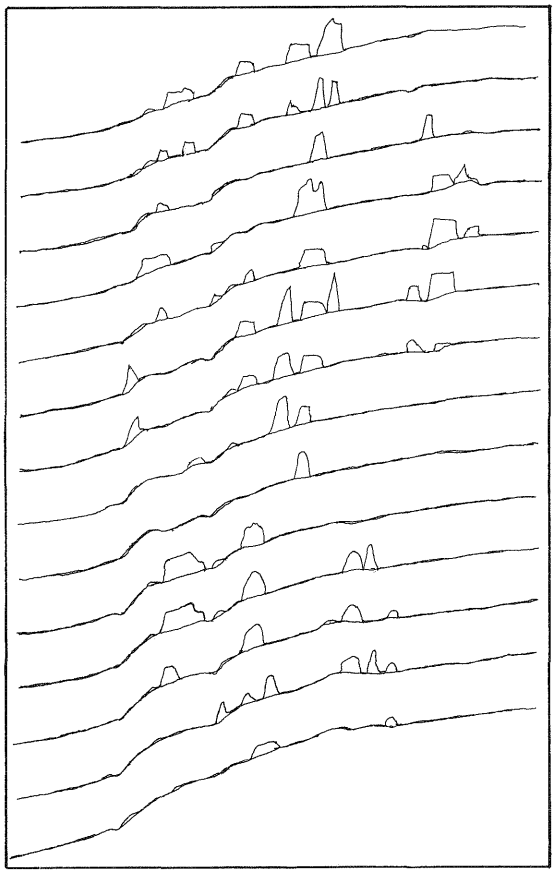


Fig. 8.9-Set of profiles smoothed by applying the finite element algorithm

				CHARACTERISTIC VALUES			
				STANDARD DEVIATION OF THE DIFFERENCE BETWEEN THE ORIGINAL SURFACE AND THE SMOOTHED SURFACE			
SMOOTHED METHODS	Total number of points	Quantity of eliminated points	% of eliminate d points	A 4.254 m2 161 points	B 9.347 m2 358 points	C 6.375 m2 243 points	TOTAL 42.524 m2 1553 points
Spline functions algorithm	4692	2700	57,4%	63 cm	77 cm	77 cm	70 cm
Finite element algorithm	4692	2205	47,0%	64 cm	70 cm	60 cm	65 cm

Table I - Characteristic values obtained by applying the developed algorithm

9. CONCLUDING REMARKS

As demonstrated, the spline function approximation, can also be used for noise and object elimination from automatic correlation data. It is, however, less sensible to the variation of the terrain than the finite element algorithm, what means that it smoothes more. Also, because the spline function approximation does not consider the neighbors profiles, it is less stable than the finite element algorithm, what means that the smooth control is easily controlled on the finite element algorithm.

On the other hand, the spline function approximation method is less time consuming than the finite element algorithm (at least 10 times). So, even if it is less efficient then the finite element algorithm, it can be used for rough analysis or in the case where the terrain has few obstacles.

10. ACKNOWLEDGMENTS

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