BUNDLE ADJUSTMENT IN NO NEED OF APPROXIMATIONS OF PARAMETERS

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ABSTRACT

Bundle adjustment has been widely used in orientation and camera calibration. But since the observation equations are non-linear, approximations of parameters are necessary at the start of adjustment. This paper discusses a method and a procedure to evaluate the approximations automatically associated with any model or object space coordinate systems. This method can realizes semi-automatic bundle adjustment or photogrammetry without control points. The method is based on relative orientation by the linear coplanarity condition and decomposition of rotation matrices to angular elements. This is validated by experiments of simple orientation of a pair of photographs and a camera calibration without control points using a 3-D target field.

Key Word: Bundle Adjustment, Automatic Adjustment, Camera Calibration, Close Range Photogrammetry, Approximations of Parameters

1. INTRODUCTION

Bundle adjustment has been widely used in camera calibration and triangulation. But since observation equations are non-linear, approximations of all parameters are required at the beginning of computation.

In close-range photogrammetry the approximations of exterior orientation parameters are usually recorded at exposing positions. But it is time consuming and sometimes hard, because a convergent or parallel imaging configuration rather than vertical one is often used. For a digital plotter (digital-image-based plotter) which is now being developed in many organizations (Lohmann, 1989, Ohtani,1989), easy manipulation is substantially required by operators who are not familiar with photogrammetry. Hence an automatic or semi-automatic adjustment procedure is now strongly called for.

This paper shows a method to automatically calculate approximations of exterior orientation parameters and coordinates of object points associated with any model or object space coordinate system. The method is based on relative orientation using the linear coplanarity condition and decomposition of rotation matrices to angular elements (Hattori, 1991).

In practice the purpose of many industrial measurements is focused only on shapes of objects, not absolute coordinates. And camera calibration works also can be executed only by the coplanarity condition in any coordinate system (Fraser,1982). The authors' method solves the problem about the selection of a coordinate system, and realizes photogrammetry without control points. It is very useful in digital plotters, because one can easily define any coordinate system on the screen, observing a model stereo-optically.

2. OUTLINE OF EVALUATION OF INITIAL VALUES OF PARAMETERS

Fig.1 shows an example of an imaging configuration in a camera calibration which will be again referred to in experiments. A three dimensionally allocated targets are imaged convergently at various positions and with various camera rotations. The following is a flow of the procedure to obtain approximations of parameters.

(1) Overlapping photographs are separated to each independent model. Rotation matrices of independent models are evaluated and decomposed to angular elements (see 3.).

(2) The independent models are linked to make a global model(see 4.1).

(3) If necessary, the global model coordinate system is transformed to the object space coordinate system using more than three control points(see 4.2).

(4) Object space coordinates of target points are calculated. Then the rotation matrix of each photograph in the object space coordinates system (or in the global model coordinate system) is decomposed to angular elements(see 4.3).

3. RELATIVE ORIENTATION BY THE LINEAR COPLANARITY CONDITION

3.1 Coplanarity condition

Let us start with a pair of overlapping photographs. The interior orientation is assumed complete. Model coordinates of two corresponding points are expressed, as shown in Fig.2-1, as

$$\begin{bmatrix} Xp_1 \\ Yp_1 \\ Zp_1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ -c \end{bmatrix}$$

$$\begin{bmatrix} Xp_2 \\ Yp_2 \\ Zp_2 \end{bmatrix} = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ -c \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

where $(x_1\ y_1\ -c)^T, (x_2\ y_2\ -c)^T$ are photographic coordinates, $(Xp_1\ Yp_1\ Zp_1)^T$, $(Xp_2\ Yp_2\ Zp_2)^T$ are model coordinates, c is a camera distance and B is a base length (unity with an unknown sign). The coplanarity condition:

$$Yp_1Zp_2-Zp_1Yp_2 = 0 \qquad (2)$$

is rewritten to the form;

$$p_1x_1x_2+p_2x_1y_2 +p_3x_1(-c)+q_1y_1x_2$$
$$+q_2y_1y_2 +q_3y_1(-c)+r_1(-c)x_2+ r_2(-c)y_2$$
$$+r_3(-c)(-c) = 0 \qquad (3)$$

where

$$p_1 = m_{21}n_{31}-m_{31}n_{21},\ p_2 = m_{21}n_{32}-m_{31}n_{22},$$
$$p_3 = m_{21}n_{33}-m_{31}n_{23},\ q_1 = m_{22}n_{31}-m_{32}n_{21},$$
$$q_2 = m_{22}n_{32}-m_{32}n_{22},\ q_3 = m_{22}n_{33}-m_{32}n_{23},$$
$$r_1 = m_{23}n_{31}-m_{33}n_{21},\ r_2 = m_{23}n_{32}-m_{33}n_{22},$$
$$r_3 = m_{23}n_{33}-m_{33}n_{23}. \qquad (4)$$

It is easy to see that a vector

$$\underline{a}= (p_1\ p_2\ p_3\ q_1\ q_2\ q_3\ r_1\ r_2\ r_3)^T$$

has a relation;

$$\underline{a}^T\underline{a} =2.$$

Expressing eq.(3) in the form of an observation equation

$$X\underline{a} = \underline{v}\ , \qquad (5)$$

where X is a design matrix and $\underline{v}$ is a residual vector, one can solve $\underline{a}$ by minimizing v^Tv. An objective function for this purpose becomes with a Lagrangean multiplier u

$$U = \underline{a}^TX^TX\underline{a} - u(\underline{a}^T\underline{a} -2). \qquad (6)$$

By diferentiating eq.(5) with X, one gets

$$(X^TX-uI)\underline{a} = 0. \qquad (7)$$

Namely $\underline{a}$ is an eigen-vector and u is a variance of residuals; $|v|^2/2$. If an imaging configuration is good, only one u that is near zero is obtained. Or otherwise multiple candidates of u may be obtained, out of which the correct one is determined by the following procedure.

3.2 Determination of the rotation matrices and angles

Then the rotation matrices (m_{ij}) and (n_{ij}) are evaluated from the vector $\underline{a}$. Even though Fig. 2-1 is assumed to be correct, Figs.2-2,2-3,2-4 as well as 2-1 are included in solutions. Figs.2-1 and 2-2 are equivalent, whereas Figs.2-3 and 2-4 are false, because they are turned over into a negative position.

The rotation matrices must be defined as;

$$(m_{ij})=$$

$$\begin{bmatrix} \cos\phi_1 & 0 & -\sin\phi_1 \\ 0 & 1 & 0 \\ \sin\phi_1 & 0 & \cos\phi_1 \end{bmatrix}\begin{bmatrix} \cos k_1 & \sin k_1 & 0 \\ -\sin k_1 & \cos k_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi_1\cos k_1 & \cos\phi_1\sin k_1 & -\sin\phi_1 \\ -\sin k_1 & \cos k_1 & 0 \\ \sin\phi_1\cos k_1 & \sin\phi_1\sin k_1 & \cos\phi_1 \end{bmatrix} \qquad (8-1)$$

$$(n_{ij})=$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos w_2 & \sin w_2 \\ 0 & -\sin w_2 & \cos w_2 \end{bmatrix}\begin{bmatrix} \cos\phi_2 & 0 & -\sin\phi_2 \\ 0 & 1 & 0 \\ \sin\phi_2 & 0 & \cos\phi_2 \end{bmatrix}$$

$$\begin{bmatrix} \cos k_2 & \sin k_2 & 0 \\ -\sin k_2 & \cos k_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi_2\cos k_2 & \cos\phi_2\sin k_2 & -\sin\phi_2 \\ -\cos w_2\sin k_2 + \sin w_2\sin\phi_2\cos k_2 & \cos w_2\cos k_2 + \sin w_2\sin\phi_2\sin k_2 & \sin w_2\cos\phi_2 \\ \sin w_2\sin k_2 + \cos w_2\sin\phi_2\cos k_2 & -\sin w_2\cos k_2 + \cos w_2\sin\phi_2\sin k_2 & \cos w_2\cos\phi_2 \end{bmatrix} \qquad (8-2)$$

It should be noted that the rotation order in the definition is unique. For other orders it can be shown that there are some angles at which the rotation matrix becomes singular and fails to be decomposed to angular elements.

3.3 Evaluation of ϕ_1

Since $m_{23}=0$, from eqs.(4)

$$m_{33}n_{21} =-r_1,$$
$$m_{33}n_{22} =-r_2,$$
$$m_{33}n_{23} =-r_3. \qquad (9)$$

And then

$$m_{33}^2(n_{21}^2+n_{22}^2n_{23}^2) = r_1^2+r_2^2+r_3^2.$$

Since the photographs are assumed diapositive, $m_{33} > 0$. From the orthogonality of (n_{ij}),

$$m_{33} = \sqrt{r_1^2 +r_2^2+ r_3^2.} \qquad (10)$$

From eq.(10) two candidates of ϕ_1 are obtained. Which is correct is suspended here. Then from eqs.(9)

$$n_{21}=-r_1/m_{33},$$
$$n_{22}=-r_2/m_{33},$$
$$n_{23}=-r_3/m_{33}. \qquad (11)$$

Multiplying the first, second and third of eqs.(4) with n_{21}, n_{22} and n_{23} respectively and summing them up, one obtains

$$m_{31} = -(p_1n_{21}+p_2n_{22}+p_3n_{23}). \qquad (12-1)$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = S \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} , \quad (18)$$

is commonly used for 3-D space transformation, where S is a scale, $A = (A_{ij})$ is an orthogonal matrix and $B=(B_i)$ is a translation vector. B and S are evaluated from gravity centers and a scale ratio of two coordinate systems. Thus eq.(18) is reduced to the form;

$$\underline{X}_i = A\,\underline{X}_{Mi}, \quad (i=1,2,,n) \quad (19)$$

where suffix i means control point No.. $\underline{X}_i$ and $\underline{X}_{Mi}$ are coordinate vectors associated with the object space coordinate system and the global model coordinate system respectively. Their origins are assumed already shifted to respective gravity centers and $\underline{X}_i$ are assumed to be scaled by S. The matrix A is determined so as to minimize

$$E = \sum_{i=1}^{n} (A\,\underline{X}_{Mi} - \underline{X}_i)^T \quad (A\,\underline{X}_{Mi} - \underline{X}_i) \quad (20)$$

This problem was already solved by some researchers (Arun,1987, Horn,1988). The authors adopted the Arun's method: By expanding eq.(20) one obtains

$$E = \sum_{i=1}^{n} (\underline{X}_i{}^T \underline{X}_i + \underline{X}_{Mi}{}^T \underline{X}_{Mi} - 2\underline{X}_{Mi}{}^T A^T \underline{X}_i). \quad (21)$$

E is minimized when

$$\mathrm{Trace}\left(\sum_{i=1}^{n} (\underline{X}_{Mi}{}^T A^T \underline{X}_i) \right) = \mathrm{Trace}\left(A^T \sum_{i=1}^{n} (\underline{X}_{Mi}{}^T \underline{X}_i) \right)$$

is maximized. With appropriate orthogonal matrices U, V which singular-value-decompose $\Sigma\,(\underline{X}_{Mi}{}^T \underline{X}_i)$ to

$$\sum_{i=1}^{n} (\underline{X}_{Mi}{}^T \underline{X}_i) = V \Lambda U^T, \quad (22)$$

where Λ is a diagonal matrix, the solution of the matrix A is given as

$$A = VU^T. \quad (23)$$

4.3 Evaluation of angular elements

After all rotation matrices (M_{ij}) associated with the object space coordinate system (or global model coordinate system) are obtained, they are decomposed to angular elements. Let the matrices related to angular elsements K, Φ and Ω be expressed simply as $[K]$, $[\Phi]$ and $[\Omega]$. Here angles are expressed by capital letters. If the rotation order of angles is fixed, the matrix (M_{ij}) can be singular and unable to be decomposed to unique angular elements. In order to assure unique decomposition, one has to change the order of rotations depending on the values of elements of the rotation matrix; i.e.,

a) If $M_{13} = \pm 1$, $(M_{ij}) = [\Omega][\Phi][K]$

b) If $M_{31} = \pm 1$, $(M_{ij}) = [K][\Phi][\Omega]$

c) If $M_{13} = \pm 1$ and $M_{31} = \pm 1$,

$$(M_{ij}) = [K][\Omega][\Phi]$$

Since the treatments for any cases are similar, here only case a) is discussed. From equation $\sin\Phi = -M_{13}$, one gets two candidates for Φ for $-\pi<\Phi\leq\pi$. Since $\cos\Phi \neq 0$,

$$\sin\Omega = M_{23}/\cos\Phi,$$
$$\cos\Omega = M_{33}/\cos\Phi,$$
$$\cos K = M_{11}/\cos\Phi,$$
$$\sin K = M_{12}/\cos\Phi. \quad (24)$$

For each candidate for Φ, Ω and K are determined uniquely. They are tested on whether to satisfy the following equations.

$$\begin{aligned} -\cos\Omega\sin K + \sin\Omega\sin\Phi\cos K &= M_{21} \\ \cos\Omega\cos K + \sin\Omega\sin\Phi\sin K &= M_{22} \\ \sin\Omega\sin K + \cos\Omega\sin\Phi\cos K &= M_{31} \quad (25) \\ -\sin\Omega\cos K + \cos\Omega\sin\Phi\sin K &= M_{32} \end{aligned}$$

Sets of candidates which do not satisfy all the equations are discarded.

5. EXPERIMENTS

The procedure was applied to two experiments for validity check; A simple relative orientation of a pair of stereo photographs and a camera calibration without control points.

5.1 Relative orientation of a pair of stereo photographs

A target field of 5m x 5m x 0.5m (depth) was imaged by a 35mm metric camera, PENTAX PAMS 645, f= 44.979mm. Two photographs were taken vertically in stereo with a base length of 1.5m, overlapping each other 50%. Common pass-points are 12 in number (minimum requirement is 8). This configuration is not good for the procedure of automatic adjustment but very commonin industrial photogrammetry.

In nine eigen-values obtained from eq.(6), three of them were 0.0598,0.146 and 1.02, while others are greater than 100,000. As a result of applying the procedure mensioned in 3., a set of rotation angles with respect to the model coordinate system were obtained only for the third minimal eigen-value. The other eigen-values did not produce misleading false solutions. Residual y-parallaxes obtained in the ensuing precise orientation were 7 um in RMS. Table 2 shows the approximations and precise values of angles.

Likely one gets

$$m_{32} = -(q_1 n_{21} + q_2 n_{22} + q_3 n_{23}) \quad (12\text{-}2)$$

$$m_{33} = -(r_1 n_{21} + r_2 n_{22} + r_3 n_{23}) \quad (12\text{-}3)$$

where eq.(12-3) is identical to eq.(10).

3.4 Evaluation of k_1

Writing the first six expressions of eqs.(4) in the form of

$$m_{21} n_{31} = p_1 + m_{31} n_{21},$$

$$m_{21} n_{32} = p_2 + m_{31} n_{22},$$

$$m_{21} n_{33} = p_3 + m_{31} n_{23},$$

$$m_{22} n_{31} = q_1 + m_{32} n_{21},$$

$$m_{22} n_{32} = q_2 + m_{32} n_{22},$$

$$m_{22} n_{33} = q_3 + n_{32} n_{23},$$

multiplying the first with the forth, the second with the fifth and the third with the sixth of each side of the above expressions and summing up them, one can calculate the right side of it. And the left side becomes

$$m_{21} m_{22} (n_{31}^2 + n_{32}^2 + n_{33}^2) = m_{21} m_{22}$$

$$= -\sin k_1 \cos k_1 = -1/2 \sin 2k_1.$$

This procedure produces four candidates for k_1.

Then n_{31}, n_{32} and n_{33} are evaluated for each candidate for k_1. They are evaluated from following different equations for better precision.

a) for $-3/4\pi \le k_1 < -\pi/4$ or $\pi/4 \le k_1 < 3/4\pi$

$$n_{31} = (p_1 + m_{31} n_{21})/(-\sin k_1),$$

$$n_{32} = (p_2 + m_{31} n_{22})/(-\sin k_1),$$

$$n_{33} = (p_3 + m_{31} n_{23})/\cos k_1 \quad (13\text{-}1)$$

b) for $-\pi/4 \le k_1 < \pi/4$ or $3/4\pi \le k_1 \le 5/4\pi$

$$n_{31} = (q_1 + m_{32} n_{21})/\cos k_1,$$

$$n_{32} = (q_2 + m_{32} n_{22})/\cos k_1,$$

$$n_{33} = (q_3 + m_{32} n_{23})/\cos k_1 \quad (13\text{-}2)$$

3.5 Evaluation of ϕ_2, w_2

From eqs.(8-2)

$$\sin w_2 \cos \phi_2 = n_{23},$$

$$\cos w_2 \cos \phi_2 = n_{33}. \quad (14)$$

Since $n_{33} > 0$, which means $\cos \phi_2 \ne 0$,

$$\cos \phi_2 = \sqrt{n_{23}^2 + n_{33}^2}. \quad (15)$$

There are four candidates for ϕ_2. And for each candidate for ϕ_2, angle w_2 is evaluated by

$$\sin w_2 = n_{23}/\cos \phi_2,$$

$$\cos w_2 = n_{33}/\cos \phi_2. \quad (16)$$

3.6 Evaluation of k_2

From eqs.(8-2);

$$(-\cos w_2)\sin k_2 + (\sin w_2 \sin \phi_2)\cos k_2 = n_{21}$$

$$(\cos w_2)\cos k_2 + (\sin w_2 \sin \phi_2)\sin k_2 = n_{22}$$

$$(\sin w_2)\sin k_2 + (\cos w_2 \sin \phi_2)\cos k_2 = n_{31} \quad (17)$$

$$(-\sin w_2)\cos k_2 + (\cos w_2 \sin \phi_2)\sin k_2 = n_{32},$$

one solves the first two equations to get $\sin k_2$ and $\cos k_2$. They are always solvable, even if $\sin \phi_2$ is zero. And this k_2 is tested by substituting it into the third and forth equations. Any sets of candidates for ϕ_2 and w_2 that do not satisfy both are abandoned.

3.7 Strict relative orientation and determination of the sign of a base length

Since the precision of approximations evaluated above is usually not sufficient, one should execute relative orientation again using those approximations. An independent model is thus obtained, which is either Fig.1-1 or 1-2.

Next the sign of a base length is determined the way that if Zp coordinates of objects in the independent model coordinate system are lesser than 0, it is set plus, and if Zp coordinates are greater than 0, it is set minus.

4. EVALUATION OF ORIENTATION PARAMETERS IN THE OBJECT SPACE COORDINATE SYSTEM

4.1 Model connection in the global model coordinate system

Independent models thus produced are linked to make a global model by usual successive orientation. Scales of successive models are adjusted by scaling base lengths. As a result exposing positions and rotation matrices associated with the global coordinate system $X_M Y_M Z_M$ are determined.

4.2 Transformation from the global model coordinate system to the object space coordinate system

When an object space coordinate system XYZ is given, global model coordinates $X_M Y_M Z_M$ are further transformed to the object coordinates. Here let us consider the case the object space coordinate system is implicitly given in the form of a few of 3-D control points. In most industrial measurements this is common. And in this case one can calculates orientation parameters automatically in the following way.

Similar transformation

points

A target field shown in Fig.1 was imaged by a metric camera, GEODETIC SERVICE CRC1 f ≃ 240.0mm (changeable), film size =23 cm. The camera is designed to determine precise coordinates of object points by simultaneously adjusting with all other parameters; interior orientation parameters of the camera and exterior orientation parameters of photographs (Fraser, 1982).

The field was 4m(height) x 5m(width) x 2m(depth) in size. 63 target points were allocated three dimensionally. Most of points were imaged in most photographs. Ten photographs were taken, rotating kappa by 90 degrees to each other. The order of linking photographs adopted in the experiment is shown in Fig.3, where the photographs 3 and 8 make a datum model, and others are linked to this model. The base length of the datum model was set to unity (1m).

By applying the procedure mentioned in 3., relative orientation parameters associated with each independent model coordinate system were uniquely determined. All approximations of interior orientation parameters but a camera distance were set to zeros. The camera distance was set initially to 249.5mm, which were read out from a micro-meter-based indicator of the camera. No false solutions did not appear. In additional experiments the authors confirmed that any other combination of photographs than in Fig.3 could make models, as long as their convergent angles were not near 90 degrees.

In the case of no control points adjustment can be done by the method of free-network or by the method of minimal constraints. The authors adopted the latter. Seven degree of freedom was fixed by giving the infinite precision to Z_M of point a and X_M, Y_M, Z_M of point b,c. in Fig.1. As a result of the procedure in 4. Table 2 was obtained, which includes the approximations and the adjusted values of the interior orientation parameters of the camera (except for ones related to lens distortions) and exterior orientation parameters for photo 1 and 10 as well as a RMS difference between approximations and adjusted values of target point coordinates. The rotation matreces determined by the procedure in 4.3 are both in the form $[\Omega][\Phi][K]$.

Table 1 and 2 prove that the algorithm produces approximations of parameters precise enough for ensuing bundle adjustments.

6. CONCLUSION

This paper discusses the algorithm for automatic calculation of approximations of parameters in bundle adjustment. Relative orientation parameters of each pair of photographs are evaluated from the linear coplanarity condition. All models are linked to form a global model. Then their rotation matrices are uniquely decomposed to angular elements. If the object space coordinate system is given, the transformation parameters are also automatically evaluated.

The procedure realizes photogrammetry without control points or easy orientation and camera calibration. It is very useful for digital-image-based plotters (digital plotters), which features easy manipulation for everybody who are not familiar with photogrammetry. Actually this has been already implemented into a digital plotter, TOPCON PI-1000 and now in test use.

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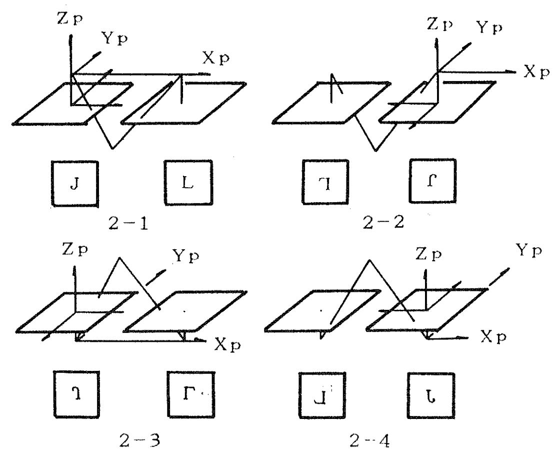

Fig.1 Imaging configuration in camera calibration

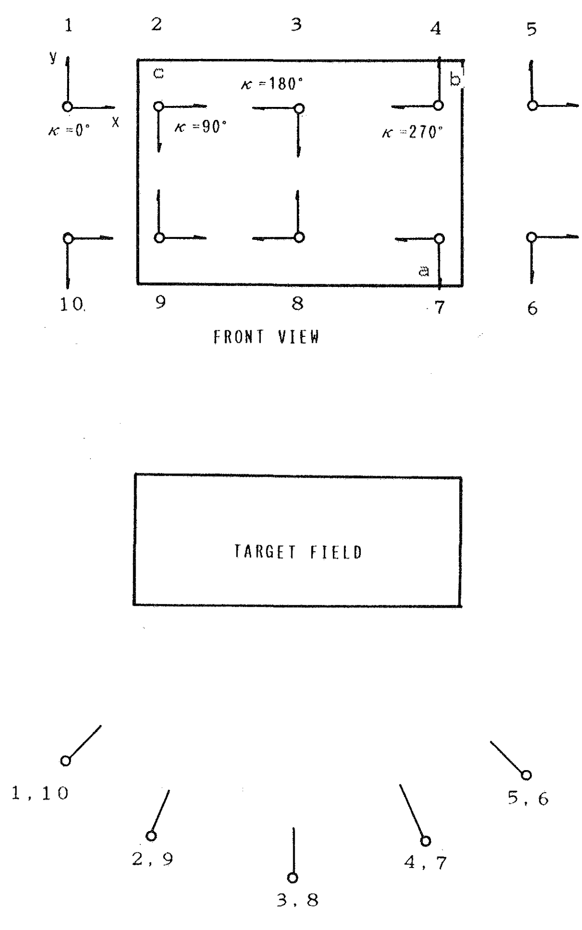

Fig.2 Four solutions retrieved from the coplanarity condition

Table 1 Approximations and the most probable values of relative orientation parameters

angles(°)	Approx.	M P V
ϕ_1	349	359
k_1	345	360
w_2	-2.43	360
ϕ_2	0.0	-1.07
k_2	-1.49	360

```
3--8--6
|  |--7
|  |--9
|  |--10
|
|--5
|--4
|--2
|--1
```

Fig.3 Photograph connection
The number stands for photograph No.
Photographs 3 and 8 make a datum model

Table 2 Approximations and the most probable values of parameters in the camera calibration

Int. orientation parameters		
	approx.	mpv.
camera dist.(mm)	249.5	249.575
principal x_0(mm)	0.0	-0.076
point y_0	0.0	-0.341
Ext. orientation parameters		
photo 1	approx.	mpv.
Ω(°)	-15.258	-14.952
Φ(°)	-0.717	-0.182
K(°)	-88.176	-88.379
X_0(m)	0.971	0.959
Y_0(m)	-0.025	-0.255
Z_0(m)	-0.236	-0.255
photo 10	approx.	mpv.
Ω(°)	-47.681	-47.327
Φ(°)	-16.021	-15.588
K(°)	-97.909	-97.966
X_0(m)	1.969	1.960
Y_0(m)	0.495	0.489
Z_0(m)	-1.139	-1.121
RMS difference of the approximations and the most probable values of target point coords. in the global model coordinate system 0.918(mm)		