METHODS AND RESULTS OF COMBINED ADJUSTMENT UTILIZING KINEMATIC GPS POSITIONING AND PHOTOGRAMMETRIC DATA

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Abstract
The paper briefly describes the main problems of progress on GPS- supported aerotriangulation. Based on investigations and by means of general consideration, new algorithms for solving the combined adjustment of kinematic GPS observations and photogrammet ric observations with minimum ground control are discussed. The main part of the paper describes the results of the empirical GPS tests BLUMENTHAL and RHEINKAMP in Germany and analyzes the potential of GPS-supported aerotriangulation in various ways. The accuracy obtained in experiments with minimum ground control indicates that the use of relative kinematic GPS data will become a standard application for aerotriangulation. Some experiments of the near future for GPS-supported aerotriangulation are discus sed.

1 Introduction

In order to achieve higher performance and optimum reliability through mutual control of different type of observations photogrammetric research has also focussed on using geodetic as well as navigational and astronomical data. Today extended mathematical and stochastic models allow to treat geodetic, navigational and astronomical information in the same adjustment process [Ackermann 84], [Hell 79], [Konecny 84], [Kruck 83].

Due to the introduction of NAVSTAR GPS (**NAV**igation **S**ystem with **T**ime **A**nd **R**anging **G**lobal **P**ositioning **S**ystem aerotriangulation recently gained potential for developing into a highly precise method of photogrammetric point determination [Andersen 89], [Brossier 88], [Jacobsen, Li 90], [Dorrer, Schwiertz 90], [Faig 89], [Friess 88], concerning the need for ground control points it will be autonomous to a great extend. From GPS observations registered during a photo flight highly accurate GPS antenna coordinates can be computed. In a subsequent combined bundle block adjustment these serve as additional observations of the exterior orientation parameters of the aerial photos [Jacobsen 91]

In this article the mathematical equations for combined bundle block adjustment are derived in separate steps for photogrammetic observations, geodetic observations and navigational data. Finally, the results of the empirical investigations of GPS- supported photogrammetric point determination are presented in datail. The article ends up with a summary and prospects for further developments and applications.

2 Extended mathematical model for GPS-supported bundle block adjustment

The mathematical model constitutes the mathematical representation of an actual physical process. Concerning the incorporation of GPS-data into the process of bundle block adjustment the question for the mathematical model can be specified as follows: based on measured image coordinates and highly precise excentric GPSantenna coordinates, how to obtain adjusted object coordinates in the corresponding system and how to achive an increase in an adjustment's efficiency.

2.1 Photogrammetric observations and modified collinearity equations

Provided all approximate values of the unknowns are at hand the linearized observation equation in matrix representation reads as follows [Konecny 84]:

$$\mathbf{V} = \mathbf{A} \cdot \mathbf{X}_1 + \mathbf{B} \cdot \mathbf{X}_2 + \mathbf{C} \cdot \mathbf{X}_2 \quad (1)$$

Prior to an aerotriangulation the focal length of the used camera and its principal point have usually been determined by means of a camera calibration; thus they are set up as weighted observations in the process of bundle block adjustment. Apart from the collinearity equations the registered GPS-positions can be entered into the adjustment process as they were "pseudo aerial control points". Equation (2) describes the transformation of the GPS antenna coordinates into image space [Li 92]:

$$\mathbf{X}_{GPS} - \mathbf{X}_0 \;=\; \mathbf{R} \cdot \mathbf{E} \qquad (2)$$

In the collinearity equations (1) used for conventional aerotriangulation the coordinates of the camera projection centers have to be replaced by the ones of the GPS antenna [Dorrer 88], [Dorrer, Schwiertz 88].

2.2 Excentric GPS observations of the camera projection center

Recent developments in Satellite Geodesy resulted in observation methods allowing the determination of the camera station with sufficient accuracy during the moment of exposure [Seeber 91]. Utilizing these observations in aerotriangulation leads to a significant reduction of the amount of necessary ground control points.

For incorporating kinematic positioning of the GPS antenna into a terrestrial geodetic system the formulation of GPS-supported aerotriangulation has to be extended. Components which describe the spatial excentricity between the GPS antenna and the camera projection center plus the corresponding coordinate transformation have to be added as well as parameters treating systematic effects within the GPS-data. In matrix representation this is expressed by [Li 90]:

$$\mathbf{X}_{gps} = \mathbf{X}_d + (1 + dM) \cdot \mathbf{R}_0 \cdot (\mathbf{X}_0 + \mathbf{R} \cdot \mathbf{e}) + t \cdot \mathbf{D}_1 + \mathbf{D}_0 \quad (3)$$

The components of the spatial excentricity between the camera projection center and the GPS antenna are defined in the image coordinate system. To be able to meet the requirements of practical applications it showed be introduced as observed unknown. The high precision determination of the excentricity is possible only via indirect and time consuming measurements [Schwiertz, Dorrer 91]. The resulting components can be entered into the adjustment through equation:

$$\mathbf{V}_e \;=\; d\mathbf{e} - (\mathbf{e}_{obs} - \mathbf{e}^0) \qquad (4)$$

Apart from the fixed relation between camera projection center and GPS antenna the coordinates of the antenna may serve as "aerial control point" coordinates. They are direct observations of the unknowns and the cooresponding observation euqation reads:

$$\mathbf{V}_{X_{GPS}}(aeri.) \;=\; d\mathbf{X}_{GPS} - (\mathbf{X}_{GPS_{obs}} - \mathbf{X}^0) \qquad (5)$$

2.3 GPS antenna coordinate differences

A widely used practice for eliminating errors common in the measurements is to form differences of these measurements. Based on this principle and on equation (3) a formulation for combined bundle block adjustment will be developed, the utilized GPS antenna coordinates have been interpolated onto each instant of exposure.

The application of differencing makes use of the fact that certain effects on neighbouring GPS positions within strips or blocks are of a systematic nature. After experimental investigations [Jacobsen 91], [Li 92] the effects of spacial excentricity, time shifts, interpolation and of geoid undulation [Haug 80] have appeared to be systematic. The same applies to transformations onto excentric GPS antennas. By means of appropriate coordinate differencing these systematic effects can be eliminated.

Differencing two GPS antenna positions results in an observation equation for coordinate differences:

$$\Delta\mathbf{X}^{i,j}_{GPS} = \mathbf{X}^j_0 - \mathbf{X}^i_0 + (\mathbf{R}^j - \mathbf{R}^i) \cdot \mathbf{e} + (t^j - t^i) \cdot \mathbf{D}_1 \quad (6)$$

In comparison to equation (3) term D0 and thus part of the systematic errors of the GPS data is canceled. The expression $(t_j - t_i) \cdot D1$ is depending on the time shift. For neighbouring GPS positions within the same strip it has the same order; thus they are nearly eliminated by differencing.

2.4 GPS-based geodetic observations

Besides utilizing GPS antenna coordinates in the image system also incorporating geodetic GPS information into combined bundle block adjustment raises the economic efficiency of photogrammetric point determination. Furthermore this measure allows for an improvementin accuracy and reliability of the results.

Summarizing all possibilities of applying geodetic information derived from GPS observations leads to observation equations that are based on simular formulations [Kruck 83] set up for: Object coordinates derived from GPS observations:

$$\mathbf{V}_{X_{GPS}}(terr.) \;=\; d\mathbf{X}_{GPS} - (\mathbf{X}_{GPS_{obs}} - \mathbf{X}^0) \qquad (7)$$

Coordinate differences:

$$\mathbf{V}_{\Delta X} \;=\; d\mathbf{X}_j - d\mathbf{X}_i - (\Delta\mathbf{X}_{obs} - \mathbf{X}^0_j + \mathbf{X}^0_i) \qquad (8)$$

Height differences:

$$\begin{aligned} \mathbf{V}_{\Delta h_i j} &= \Delta\mathbf{h}_1 + \Delta\mathbf{h}_2 - (\Delta h_{obs} - \Delta h^0) \\ \Delta\mathbf{h}_1 &= -\frac{X_i}{R} dX_i - \frac{Y_i}{R} dY_i - \frac{Z_i}{R} dZ_i \\ \Delta\mathbf{h}_2 &= \frac{X_j}{R} dX_j + \frac{Y_j}{R} dY_j + \frac{Z_j}{R} dZ_j \end{aligned} \qquad (9)$$

Distances:

$$\begin{aligned} \mathbf{V}_s &= \mathbf{S}_1 + \mathbf{S}_2 - (S - S^0) \\ \mathbf{S}_1 &= \frac{\Delta X_{ij}}{S} dX_j + \frac{\Delta Y_{ij}}{S} dY_j + \frac{\Delta Z_{ij}}{S} dZ_j \\ \mathbf{S}_2 &= -\frac{\Delta X_{ij}}{S} dX_i - \frac{\Delta Y_{ij}}{S} dY_i - \frac{\Delta Z_{ij}}{S} dZ_i \end{aligned} \qquad (10)$$

2.5 Variance Component Estimation

The stochastic model containing a formulation for standard deviations (a priori) and the arising weight coefficients (a posteriori) have great influence on the results of combined bundle block adjustment for large blocks [Kruck 85]. For verifying the stochastic model of each group of observations within GPS- supported bundle block adjustment the method of variance component estimation has been applied. This method plays an important role for fine tuning the stochastic model in that it helps to find out the optimal relation between the unknown and heterogenous accuracies. The empirical standard deviation for each group of observations, e.q. for all GPS data, can be calculated:

$$S_{0_g} = \frac{V_g^T Q_g^{-1} V_g}{S_p[Q_g^{-1} Q_{vv_g}]} \quad (11)$$

The actual estimation process will be repeated until all group variances are converging. The computed empirical standard deviations of all group of observations are compared to their theoretical standard deviation. In case of a deviation between two corresponding variances the theoretical one has to be altered during another adjustment procedure. This way the estimate Sog a altered a postriori in an iterative procedure until the variance components – given a sufficient redundance – show a proper relation, that is to say

$$\frac{theoretical - standard - deviation}{empirical - standard - deviation} --- > 1 \quad (12)$$

3 Empirical investications on testblocks for GPS-supported block adjustment

In cooperation with the "Institut für Erdmessung"(IFE), the photogrammetric application of GPS has been investigated on two projects within Germany, "BLUMENTHAL" and "RHEINKAMP". The obtained data from the base for analyzing the systematic and stochastic propertics of GPS-supported bundle block adjustment. Starting from the results of these investigations the mathematical models set up in section 2 are to be tested. The aim is to find an operational solution for GPS-supported aerotriangulation using no ground control points or only the minimum amount. In this section the two testblocks are briefly described under a photogrammetric aspect; the results will be presented.

3.1 GPS-supported bundle block adjustment "RHEINKAMP"

The testarea "RHEINKAMP" is situated in the mining area of the "Ruhrkohle" Corporation. The whole area is controlled for soil movements every three years by means of precise point determination within conventional bundle block adjustment. During the second last campaign in 1988 the object points were precisely determined once again. The aerial photogrphs taken in the strips with an endlap of 80% and a sidelap of 30% have an average scale of 1:4000. Five cross-strips were flown in order to stabilize the block geometry apart from the utilized ground control points.

The technical data of the photo flight are listed below:

GPS receiver	: 4 TI4100
survey aircraft	: Cessna 404 of HL
ground speed	: 210 km/h
flight duration	: 90 min
aerial camera	: RMK 30/23
focal length	: 305.007mm (k=223mm)
block size	: 60 km
flight altitude	: 1200m
number of photos	: 454
exposure interval	: 3 sec
antenna-camera-offset	: x=-0.53m y=0.52m z=1.16m
number of object points	: 4856
number of control points	: 21 full, 133 vertical
accuracy of control points	: Sx=Sy=±2cm, Sz=±3cm
number of image points	: 2950

With respect to satellite positioning all the data of the "RHEINKAMP" testblock have been analyzed using two different GPS solutions. For survey aircraft positioning the navigational solution relying on smoothed code ranges was applied first (absolute camera positions). The second solution was computed with range corrections derived from simultaneous GPS observations on a coordinated reference point within the testarea (relative camera positions). As stated in [Li 92], the second method came up with inconsistent GPS positions partly showing large shifts due to technical problems and frequently changing satellite constellations. On the other hand absolute positioning with the navigational solution provides good internal accuracies of the excentric camera positions to each other but only poor external accuracies.

Due to the large amount of data the testblock is subdivided into sub-block for evaluation with the modified software package BINGO (Buendelausgleichung fuer INGenieurtechnische Objekte). The empirical results of the photo flight "RHEINKAMP" are presented in tables 1 to tables 4 for both absolute and relative positioning.

Tab.1 Results of bundle block adjustment utilizing all ground control points

Positioning without/with range corre.	Version (method of bundle block adjustment)	Stand. devia. (m)		
without/with	(AT)/(GPS/AT)	$\pm Sx$	$\pm Sy$	$\pm Sz$
without	AT	0.029	0.030	0.067
without	GPS/AT 1.	0.030	0.030	0.067
without	GPS/AT 2.	0.035	0.035	0.067
without	GPS/AT 3.	0.030	0.030	0.068
with	GPS/AT 1.	0.031	0.031	0.069
with	GPS/AT 2.	0.033	0.033	0.067
with	GPS/AT 3.	0.030	0.031	0.069

Tab.2 Results of bundle block adjustment based on four ground control points

Positioning without/with range corre.	Version (method of bundle block adjustment)	Stand. deviat. (m)		
without/with	(AT)/(GPS/AT)	$\pm Sx$	$\pm Sy$	$\pm Sz$
without	AT	0.043	0.043	0.176
without	GPS/AT 1.	0.038	0.039	0.105
without	GPS/AT 2.	0.042	0.043	0.106
without	GPS/AT 3.	0.039	0.040	0.107
with	GPS/AT 1.	0.039	0.039	0.101
with	GPS/AT 2.	0.039	0.038	0.103
with	GPS/AT 3.	0.037	0.040	0.125

Tab.3 Results of bundle block adjustment based on one ground control point

Positioning without/with range corre.	Version (method of bundle block adjustment)	Stand. deviat. (m)		
without/with	(AT)/(GPS/AT)	$\pm Sx$	$\pm Sy$	$\pm Sz$
without	AT			
without	GPS/AT 1.	0.075	0.078	0.152
without	GPS/AT 2.	0.089	0.094	0.153
without	GPS/AT 3.	0.144	0.147	0.196
with	GPS/AT 1.	0.080	0.084	0.120
with	GPS/AT 2.	0.088	0.093	0.143
with	GPS/AT 3.	0.147	0.115	0.188

Tab.4 Results of bundle block adjustment computed without ground control points

Positioning without/with range corre.	Version (method of bundle block adjustment)	Stand. deviat. (m)		
without/with	(AT)/(GPS/AT)	$\pm Sx$	$\pm Sy$	$\pm Sz$
without	AT			
without	GPS/AT 1.	0.113	0.120	0.151
without	GPS/AT 2.	0.133	0.145	0.199
without	GPS/AT 3.	0.159	0.173	0.217
with	GPS/AT 1.	0.116	0.126	0.156
with	GPS/AT 2.	0.132	0.145	0.174
with	GPS/AT 3.	0.231	0.233	0.247

* AT : conventional bundle block adjustment
GPS/AT 1. : antenna coordinates from pre-corrected GPS data
GPS/AT 2. : antenna coordinates without pre-corrected of GPS data
GPS/AT 3. : GPS antenna coordinate differences
For combined bundle block adjustment of the "RHEINKAMP" testblock the following conclusions can be drawn:

a) With respect to conventional adjustments utilizing all ground control points additional support through GPS positioning does not lead to an increase in accuracy. The whole block is already sufficiently supported by the set of ground control points and the internal stability of the block geometry (see tab. 1). The gain in accuracy utilizing GPS data and only four ground control points becomes apparent in tab. 2 for the height.

b) For the block configuration without cross strips, the adjustment diverges when only one or no ground control points is utilized within the "GPS coordinate differential method"; but the adjustment of the block configuration containing cross strips can be performed. Obviously the block's geometry has a significant influence on the results. The combined bundle block adjustment free from ground control points but relying on GPS data makes it to an accuracy of 0.113m to 0.233m in position and of 0.151m to 0.247m in the highest component.

c) When applying range corrections from relative positioning the adjustments lead to generally higher accuracies than computations performed with absolute positions. This is clearly demonstrated by entering only one ground control point or even none; in these cases relative positioning provides more accurate control information for stabilizing the whole block.

d) After processing of the various adjustment methods the one utilizing "GPS antenna coordinate differences" without cross strips did not come up with any results. These data can only provide relative control information which is insufficient for supporting bundle block adjustments at weak geometric propertions of blocks. For practical applications of "GPS antenna coordinate differences" four terrestial control points showed be determined; otherwise a combination plain antenna coordinates has to be taken into consideration.

3.2 GPS-supported bundle block adjustment of the "BLUMENTHAL" testblock

The "BLUMENTHAL" testarea in the northern part of the German "Ruhrgebiet" covers 3km × 5km of farmland. The block consists of five strips at an image scale of 1:6300 with an endlap of 80% and a sidelap of 60%. Further technical data of the GPS- supported photo flight "BLUMENTHAL" are listed:

GPS receiver	: 2 TI4100
survey aircraft	: Cessna 404 of HL
ground speed	: 240 km/h
flight duration	: 40 min.
aerial camera	: Zeiss RMK 15/23
focal length	: 152.934 mm (k=237mm)
image scale	: 1:6300
flight altitude	: 900 m
overlap	: P=80%, Q=60%
number of photos	: 69
object points(num.)	: 1040
photos/obj.	: 5.5
points/photo	: 85.5
number of control	: 263

These relative kinematic GPS data plus suitable evaluation models and algorithms [Seeber, Wuebenna 89] serve as high precision "aerial control point network" allowing to drow conclusions about kinematic GPS applications for high precision positioning as well as to perform investigations for GPS-supported bundle block adjustment. The investigations directed towards a correction of erroneous GPS data and at the use of original interpolated GPS data have been carried out with the models and combination of methods described in section 2. Additionally different endlaps, sidelaps, aerial and terrestial control point distributions have been entered for computations with the modified software package BINGO. The results arranged in the number of control points are presented in tables 5 to 7.

Tab.5 Results of bundle block adjustment utilizing four ground control points

(P Stan.	Q) devi.	80/60	(P Stan.	Q) devi.	60/60 (m)	(P Stan.	Q) devi.	60/20
$\pm Sx$	$\pm Sy$	$\pm Sz$	$\pm Sx$	$\pm Sy$	$\pm Sz$	$\pm Sx$	$\pm Sy$	$\pm Sz$
a)0.110	0.061	0.175	0.122	0.068	0.229	0.265	0.101	0.350
b)0.046	0.045	0.080	0.061	0.056	0.131	0.067	0.058	0.102
c)0.055	0.058	0.124	0.072	0.055	0.135	0.103	0.155	0.172
d)0.062	0.062	0.118	0.105	0.045	0.094	0.152	0.082	0.125
e)0.064	0.059	0.133	0.063	0.060	0.121	0.083	0.078	0.114
f)0.094	0.084	0.098	0.123	0.062	0.115	0.152	0.153	0.154
g)0.130	0.088	0.117	0.146	0.090	0.114	0.149	0.082	0.108
h)0.146	0.061	0.101						
i)0.137	0.061	0.105						
j)0.125	0.060	0.102						

Tab.6 Results of bundle block adjustment utilizing one groun control point

(P Stan.	Q) devi.	80/60	(P Stan.	Q) devi.	60/60 (m)	(P Stan.	Q) devi.	60/20
$\pm Sx$	$\pm Sy$	$\pm Sz$	$\pm Sx$	$\pm Sy$	$\pm Sz$	$\pm Sx$	$\pm Sy$	$\pm Sz$
a)								
b)0.061	0.067	0.166	0.123	0.087	0.223	0.138	0.082	0.183
c)0.249	0.293	0.331	0.256	0.307	0.356	0.279	0.346	0.404
d)0.085	0.141	0.240	0.190	0.228	0.270	0.217	0.271	0.296
e)0.133	0.077	0.251	0.141	0.099	0.251	0.267	0.332	0.252
f)0.101	0.126	0.106	0.210	0.062	0.135	0.197	0.113	0.255
g)0.191	0.149	0.166	0.208	0.100	0.159	0.116	0.282	0.297

Tab.7 Results of bundle block adjustment computed without ground control points

(P Stan.	Q) devi.	80/60	(P Stan.	Q) devi.	60/60 (m)	(P Stan.	Q) devi.	60/20
$\pm Sx$	$\pm Sy$	$\pm Sz$	$\pm Sx$	$\pm Sy$	$\pm Sz$	$\pm Sx$	$\pm Sy$	$\pm Sz$
a)								
b)0.110	0.076	0.211	0.117	0.123	0.246	0.136	0.111	0.279
c)0.160	0.215	0.438	0.286	0.346	0.450	0.303	0.356	0.461
d)								
e)0.271	0.097	0.160	0.174	0.350	0.263	0.197	0.324	0.228
f)0.133	0.210	0.152	0.187	0.361	0.210	0.225	0.382	0.234
g)								

* a) : conventional bundle block adjustment
b) : antenna coordinates from pre-corrected GPS data
c) : antenna coordinates without pre-corrected of GPS data (all GPS data)
d) : GPS antenna coordinate differences
e) : combination method (antenna coordinates and coordinate differences)
f) : antenna coordinates with driftparameter
g) : antenna coordinates with dirftparameter (4 GPS data)
h) : antenna coordinates with dirftparameter (10 GPS data)
i) : antenna coordinates with dirftparameter (15 GPS data)
j) : antenna coordinates with dirftparameter (20 GPS data)

The different aspects of the results are summarized in the following paragraphs.

- Higher accuracy through introduction of GPS antennacoordinates being excentric observation of the projection center coordinates

The increase in accuracy due to adding GPS data into combined bundle block adjustment turned out differently in position and height. Compared to conventional adjustment relying on four ground control points the accuracy of the height component found after the GPS-supported adjustment increases significantly (e.g. at P = 60%, Q = 20% three times); this is merely the case for the position accuracy. Comparing the results of GPS-supported bundle block adjustment and conventional bundle block adjustment confirms the present assumption that a bundle solution based on four control points and GPS data has a high stability [Stueckmann-Petring 91]; without supporting GPS information local deformations of the block's geometry may occur. Especially with large control point intervals the application of GPS data is of advantage.

- Effect of the amount of GPS data

Computing GPS-supported bundle block adjustments with various distribution and densities of GPS data lead to the conclusion that the combined approach has been effective in any case. Taking only four "aerial control points" at the corners of the block resulted in better results than conventional adjustment. With increasing amount of incorporated GPS information. e.g. 10, 15, 20 "aerial control points" evently distributed within all tie strips, the results improved.

- Effect of different overlaps on the results of GPS-supported bundle block adjustment

For investigating how the accuracy is depending on the overlap it has been altered in three steps for time involoing the whole block; thus the database for the appropriate strips with different endlap and sidelap is always the same. In conventional adjustment the gain in accuracy due to an increasing overlap is essentially higher in the height component than in position. This is not the case with GPS-supported bundle block adjustment. After increasing the overlap the standard deviation of the height component tends to be more homogenous for the whole block. The example in tab. 5 demonstrates good results even at a minimum overlap at P = 60% and Q = 20%.

- Control point configuration

Applying GPS-supported bundle block adjustment utilizing only one or even no ground control points presently cannot be recommended. To guarantee sufficient reliability and sufficient accuracy four terrestial control points in the corners of a block plus high precision GPS data should be at hand. Empirical investigations contribute to further fine tuning of evaluation methods so that homogenous accuracies and high reliability sufficient for many applications can be reache.

- Assessment of the methods applied for GPS-supported bundle block adjustment

The effect of GPS antenna coordinate differences (see section 2.3)on the geometry of the bundles of rays is derived by means of the relation antenna-camera. When applying this method the absolute information of the original GPS data cannot be respected; however, relations between neighbouring GPS antenna coordinates can be respected. The practical example clearly proves that this method is operable with the use of only four ground control points. As illustrated in tables 6 and 7, a combination of the "GPS antenna coordinate method" (see section 2.2) with the "antenna coordinate difference method" (see section 2.3) even offers the chance of limiting the amount of ground control point to zero. These tables together with table 5 also confain the results achieved with and without precorrection of the systematic GPS effects. The external accuracy of the adjustments is being improved varyingly by a priori corrections of systematic errors.

Combined bundle block adjustment computed with this method and an overlap of P = 80%, Q = 60% came up with the best results: Sx=±0.046m, Sy=±0.045m and Sz=±0.080m. As can be seen form the listed results the effects of adjustments containing linear drift parameters are also varying. The assumption is being confirmed that uncompensated systematic errors and probably unconsidered correlations effected the empirical results. At an overlap of P = 60%, Q = 20% on average correlation between the coordinate of the projection center and the linear drift parameters turns out to Pxo,dox=-0.38, Pyo,doy=-0.51, Pzo,doz=-0.58.

4 Conclusions and outlook

This paper deals with extension of models for combined bundle block adjustment including GPS data. For various formulations have been developed allowing to compensate systematic effects and to defect gross errors of models as well as of data. Whereas many authors are starting from pre corrected eccentric GPS observation, a simple algorithm (GPS antenna coordinate differences) offers the chance to perform bundle block adjustment with minimum ground control utilizing GPS data not pre corrected for systematic errors. In summary it may be said that introduce entering highly precise kinematic GPS positions plus four ground control points into combined bundle block adjustment results in sufficient accuracies. Bundle block adjustment based on one or even no ground control points can at moment only be recommended in case the absolute position is of no interest. Besides four ground control points stable geometric properties of a block are important for achieving good reliability. Due to varying systematic errors bundle block adjustments computed without ground control points will most probably be limited to low accuracy requirementts.

Writing this paper did not aim at the presentation of a complete concept for GPS-supported bundle block adjustment. However, for practical applications of combined bundle block adjustmen tutilizing kinematic GPS positioning the following key aspects should be taken into consideration:

- High accuracies and thus economic processing can only be achieved by GPS-supported bundle block adjustment when systematic errors are compensated for. In this respect it is quite useful to implement a bundle block adjustment program with appropriate algorithms (e.g. with error correction models or differencingsubroutines).
- For fixing the excentricity between GPS antenna and aerial camera on board of survey aircraft the ideal antenna position is vertically above the camera projection center; otherwise the excentricity has to be precisely determined by indirect methods. The components are entered into the adjustment process as weighted observations [Schwiertz, Dorrer 91].
- Continuous GPS-observations have to be synchronited with the instances of exposure. For practical applications a new generation of GPS receivers adapted to photogrammetric purposes in combination with new camera systems Zeiss RMK TOP, LMK 2000 or Leica RC 20 come in handly; on one hand these camera systems allow to register the mid point of exposure, on the other hand new GPS receivers like the ASHTHCH XII are able to receive and process signals of aerial cameras.
- A complete concept for GPS-supported bundle block adjustment should contain tools for respecting correlations between coordinates of the projection center and GPS data and linear driftparameters respectively.

With respect to the observation of excentric camera projection centers the accuracy requirements of aerotriangulation can certainly be met by kinematic GPS positioning in the relative mode. The image rotations at the moments of exposure may be determined with sufficient accuracy by further modifications. Regarding further limitation of ground control and gains in accuracy these techniques open a large field of scientific activities practical applications.

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References

[Ackermann 84] Ackermann, F. (1984): Utilization of Navigation Data for Aerial Triangulation. IAP, vol. XXV, Part A3.

[Andersen 89] Andersen, O.(1989): Experience with kinematic GPS during Aerial Photography in Norway, 42nd Photogrammtric Week, Stuttgart.

[Brossier 88] Brossier, R.; Million, C.; reynes, A. (1988): Photogrammetric Applications of Sercel GPS TR5S-B Receiver at Institute Geographique National, ISPRS Kyoto 1988

[Dorrer 88] Dorrer, E. (1988): GPS-gestuetzte Aerotriangulation - Grundlegendes und Experimente. VRK/MPG, Vol. 86/11.

[Dorrer, Schwiertz 88] Dorrer, E.; Schwiertz, Ch. (1988): Experimental GPS-supported Aerial Triangulation, ISPRS Kyoto 1988

[Dorrer, Schwiertz 90] Dorrer, E.; Schwiertz, Ch. (1990): Kinematic GPS and AerialTriangulation - A Coparative Study. ISPRS Com. III, Wuhan.

[Faig 89] Faig, W.; Shih, T. (1989): Should One Consider Combining Kinematic GPS with Aerial Photogrammetry? PERS, 12/1989.

[Friess 88] Friess, F. (1988): Empirical Accuracy of Positions Computed from Airborne GPS Data, ISPRS Kyoto 1988.

[Haug 80] Haug, G. (1980): Bestimmung und Korrektur systematischer Bild- und Modelldeformationen in der Aerotriangulation am Beispiel des testfeldes "Oberschwaben". NaKaVerm, Heft Nr. D-13, Frankfurt a.M.

[Hell 79] Hell, G. (1979): Terrestische Bildtriangulation mit Beruecksichtigung zusaetzlicher Beobachtungen. DGK, Reihe C, Heft 252.

[Jacobsen, Li 90] Jacobsen, K.; Li, K. (1990): Bundle Block Adjustment using Kinematic GPS Positions. ISPRP Com. I Manaus.

[Jacobsen 91] Jacobsen, K. (1991): Trends in GPS Photogrammtry. ASPRS 1991.

[Konecny 84] Konecny, G.; Lehmann, G. (1984): Photogrammetrie. Walter de Gruyter, Berlin.

[Kruck 83] Kruck, E. (1983): Loesung grosser Gleichungssysteme fuer photogrammetrische Blockausgleichungen mit erweitertem funktionalen Modell, WA UH Nr.128.

[Kruck 85] Kruck, E. (1985): Kombinierte Ausgleichung mit Varianzkomponentenschaechung und Simultankalibrierung. DGK Reihe B, Nr. 275.

[Li 90] Li, K. (1990): Neue Ergebnisse zur kombinierten Buendelblockausgleichung mit kinematischer GPS-Positionierung. ISPRS Com. III, Wuhan.

[Li 92] Li, K. (1992): Empirische Untersuchungen zur GPS-gesuetzte kombinierten Blockausgleichung, WA UH Nr. 177.

[Schwiertz, Dorrer 91] Schwiertz, Ch.; Dorrer, E.(1991): Bestimmung der Exzentrizitaet Kamera-Antenne in der GPS-gestuetzten Aerophotogrammetrie. ZPF 3/1991.

[Seeber, Wuebenna 89] Seeber, G.; Wuebbena, G. (1989): Kinematic Positioning with Carrier Phase and "On the Way" Ambiguity Solution. Proceedings of the Fifth International Geodetic Symposium on Satellite Positioning, Las Cruces, New York.

[Seeber 91] Seeber, G. (1991): Konzepte und Einsatzmoeglichkeiten fuer die kinematische Nutzung von GPS. 25. DVW-Seminars, Hannover.

[Stueckmann-Petring 91] Stueckmann-Petring, J. (1991): Blockausgleichung mit kinematischen GPS-Daten, Diplom-Arbeit an der Universitaet Hannover.