OPTIMIZED ALGORITHM FOR THREE DIMENSIONAL OBJECT RECONSTRUCTION

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ABSTRACT

This paper presents the development of the mathematical concept of image reconstruction of three-dimensional object from silhouettes based on functional analysis. This concept leads to the formulation of an algorithm which is also described. The use of this algorithm with an additional optimization procedure eliminates the errors in positioning the camera or the object. The use of silhouettes limits the reconstruction to convex cross sectional objects although two or more convex objects can be reconstructed in the same scene using combinatorial methods. The complete method was used to reconstruct a lamp-shade from four silhouettes of images taken by a camera with good results.

KEY WORDS: 3D-Reconstruction, Algorithm, Optimization

INTRODUCTION

The problem of reconstructing shapes of three-dimensional objects from two-dimensional images is one of the important goals of computer vision. Concerning to the reconstruction from silhouettes, the problem can be viewed as similar to computerized tomography, where the projection mapping used in analysis of interior densities is different from the silhouettes projection and thus, in order to solve the problem, an alternative mathematical analysis is required.

For many different uses the reconstruction of a real body in three dimensional space has become very important. In designing, remote sensing, computer animation, and any other field where an object should be analyzed visually in 3D, the reconstruction is of great interest.

Some authors have written about this subject. (Pai et al., 1990), (Cyganski et al., 1990), (Cernuschi-Frias et al.,1989) and (Bolle et al., 1991) developed algorithms and showed the importance of inference to solve the problem.

In this paper, in order to introduce the algorithm, a mathematical description of the problem is done and is specially helpful for the optimized algorithm for incorrect centralization.

The test using a real object using the complete algorithm are showed in section 5.

1. EXPERIMENTAL DATA

1.1 Data Acquisition

The problem of three-dimensional object reconstruction consists essentially in the surface estimation from images taken from different positions. In this analysis the images are taken over a circumference around the object.

Figure 1 shows a lamp shade to be reconstructed and a video camera connected to a microcomputer. The lamp shade is over a rotatory base. It is not necessary to the body stay at the center of the rotatory axis but the video camera should be fixed so that this axis appears at the same position in all the images.

The rotatory base is positioned in a few different angles over Π rad. No images are taken in opposite directions, as two angles differing from Π rad are symmetrical and together carry no more information for this algorithm.

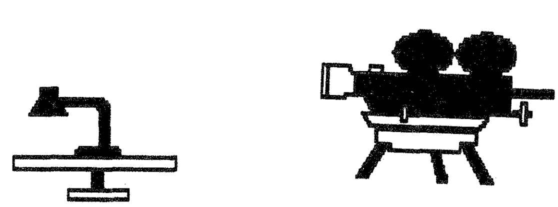

Figure 1

1.2 Cones and Cylinders From Plane Images

Each of the n images filmed by the video and stored in magnetic disk are pre-processed into a binary image, one color level for the object and another for the background.

The binary image is a mapping over the two-dimensional Euclidean space defining a set where the body was identified. This set may be convex or concave and results from projection of the body over a plane positioned straight over one direction. In figure 2 the experimental layout is showed.

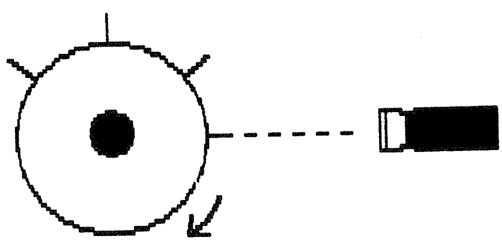

Figure 2

The discussion of the use of cylindrical geometry instead of conical geometry depends specially on the distance from object an camera and the largest dimension of the object itself. The resultant errors from the use of cylindrical geometry can be estimated and used only when applicable, that is, when it is comparable to errors from digitalization.

2. PROBLEM FORMULATION

2.1 Mathematical Formulation

Originally, the three-dimensional object reconstruction problem can be mathematically described as:

Let us define $F: \mathbb{R}^3 \rightarrow \mathbb{R}$ the function that characterizes an object in space. This function can refer to any property of the body itself or a surface property like color. It should be assured that this function can determine the position and the form of any object.

Before we introduce the inverse problem consisting of the three-dimensional reconstruction, we will define the direct problem.

For describing the direct problem some definition are missing:

Considering that $F \in \Omega \subset C[\mathbb{R}^3]$, the piece wise continuous function set, let $P : \Omega \rightarrow \Gamma$ where:

$$\Gamma = \{ h \in C[\mathbb{R}^2] \mid h : \mathbb{R}^2 \rightarrow \mathbb{R} \},$$

the projection mapping such that,

$$P(F) = h \quad \text{and} \quad P(\underset{\sim}{0}) = \underset{\sim}{o}$$

where $\underset{\sim}{0}$ is the null vector in Ω and $\underset{\sim}{o}$ is the null vector in Γ

Another property of a projection mapping is that if $P(F_i) = h_i$ for all i where

$F_i : \theta_i \rightarrow \mathbb{R}$ and $h_i : \alpha_i \rightarrow \mathbb{R}$;

$\theta_i \subset \mathbb{R}^3$ and $\theta_i \cap \theta_j = \phi$ for $i \neq j$;

$\alpha_i \subset \mathbb{R}^2$ and $\alpha_i \cap \alpha_j = \phi$ for $i \neq j$;

then, for $\mathcal{F} : \mathbb{R}^3 \rightarrow \mathbb{R}$ such that $\mathcal{F}(x) = F_i(x)$ for all $x \in \theta_i$ and $\mathcal{F}(x) = 0$ elsewhere and for $\mathcal{H} : \mathbb{R}^2 \rightarrow \mathbb{R}$ such that $\mathcal{H}(y) = h_i(y)$ for all $y \in \alpha_i$ and $\mathcal{H}(y) = 0$ elsewhere, the application of the projection mapping results in:

$$P(\mathcal{F}) = \mathcal{H} \tag{1}$$

The silhouette is a kind of projection mapping that for $F : \theta \rightarrow \mathbb{R}$ where $\theta = \mathrm{supp}(F)$, that is, support of function F,

$$P(F) = h \quad \text{such that} \quad h(y) = K \text{ for all } y$$

It can be proved from this definition of silhouette, that the inverse mapping such that,

$$P\, P_R^{-1}(\, h\,) = h$$

can be found and it is the right inverse transforming of P. Nevertheless the projection P_R^{-1} is not unique because $P_R^{-1} \neq F$ as P is not an injective transform. Thus, the left inverse transform P_L^{-1} such that

$$P_L^{-1} P\,(F) = F$$

does not exist.

This preliminary analysis leads us to the conclusion that the initial purpose of finding F from a finite number of silhouettes is not feasible.

However, finding the body function may not be the better way of reconstruct the body form. If this is our interest, finding the support of F, will be enough. Colors, texture and densities become important in other kinds of reconstruction. So, after changing the goal, the problem is:

Given $h_i : \mathbb{R}^2 \rightarrow \mathbb{R}$ and P_i, $i=1,\ldots,J$, find $\mathrm{supp}(F) \subset \Omega \subset \mathbb{R}^3$ according to the following correlation:

$$P_i(F) = h_i \tag{2}$$

In order to solve this problem is necessary to define inverse mapping P_i^{-1} so that:

$$P_i^{-1}(P_i(F))=P_i^{-1}(h_i)=F \quad \text{for } i=1 \text{ to } n$$

As the left inverse transform of P_i does not exist, there is infinite set of functions F which satisfy (1). This set $\mathscr{F}$ is,

$$\mathscr{F} = \{ F \mid P_i(F) = h_i \text{ for } i=1 \text{ to } n \} \quad (3)$$

The modification of purpose is not sufficient to solve our problem although, using this restricted reconstruction a special class of subset of $\mathbb{R}^3$ can be exactly reconstructed for silhouettes take from some special directions.

In practical the solution set can be defined in another way that leads to the algorithm:

$$\mathscr{F} = \bigcap \mathscr{F}_i$$

where

$$\mathscr{F}_i = \{ F \in \Omega \mid P_i^{-1}(F) = h_i \}$$

Once the problem is intrinsically ill posed concerning to uniqueness, we have to choose one element of $\mathscr{F}$. As the choice is arbitrary, any function could be used, however, the method chooses the function which has the largest support. The new problem changes to:

Let M(F) be the measure of the support of F, that is M (supp(F)) = volume (supp(F)). Find s_o= supp(F_o) that satisfies:

$$M(s_o) = \max_{F \in \mathscr{F}} M(\text{supp}(F))$$

Before start solving the problem, a further analysis in the new formulation should be helpful. In fact, the solution $s_o \subset \Omega$ is the whole region where it is possible to find a body.

Note that the solution of maximum volume is always convex in planes parallel to the image rays.

Regarding to the conclusion above and assuming the data of projections, h_i i=1,J is consistent, s_o is the intersection of the cones contained in Ω, showed in figure 2. If the distance of the camera from Ω is large enough to consider that the rays coming into camera are parallel, s_o can be find as intersection of cylinders.

3. ALGORITHM

3.1 General Overview

From the problem discussed over, reconstruction of forms from shadows consists in finding the intersection of cones or cylinders whose bases are the silhouettes. This interception occurs in space at the center of the circle corresponding to the experimental layout. In this paper the approximation of cylinders is used.

The algorithm is prepared to identify more than one object, what mean that more than one cylinder for each view is intercepted.

As cylinder approximation is used, the intersections of the solids can be numerically done in intervals in the direction of the axis. This method makes easier finding the intersections because instead of solids, we use slices of planes which intercept forming polygons.

In each plane, we can find more than one polygon even if there is only one object. This happens when there are non convex silhouettes, figure 3.

The steps of the algorithm are:

i) Pre-processing of projection data, that is, finding the binary limit (BL) that differ, in each projection, the existence or not of an object.

ii) Reading data of a height interval of each view, this data corresponds to a horizontal line in digitized image.

iii) Determining compact intervals (CI), line segments that are over the binary limit, that is, $I = [z_1,z_2]$ is an compact interval if $\forall z \in I$, $f_i(z) > BL$. Compact intervals of one view do not depends on others views.

iv) Finding the intersections of the compact interval of different views.

The fourth step is the reconstruction itself and include three specific steps: COMBINATION, POLYGON and POLYHEDRAL. COMBINATION is the routine that makes all the possible combination of compact intervals in order to find a non empty polygon, POLYGON is the routine which do the verify if it is empty or not and POLYHEDRAL construct the lateral polygons covering the volume.

3.2 Optimized Algorithm For Incorrect Positioning

This step of the reconstruction algorithm is required when there are systematic errors of positioning the central axis.

Considering the centralization is not known for all views, the correct relative positioning is found using the mathematical programing problem of maximization of volume as in the original algorithm but with different constraints:

$$P_i^{-1} \; (\; h_i(x-a) \;) = F \quad \text{for any } a \in \mathbb{R}$$

The maximization of volume has two physical consequence on the resultant shape, first it eliminates the empty intersection of any two strips of two silhouettes, that is,

$$P_i P_i^{-1}(h_i) = h_i . \qquad (3)$$

If the data contain errors of positioning, the choice of one $F \in \mathcal{F}$ does not imply in $P_i(F) = h_i$.

Second, in some special cases it is not possible to find the correct centralization of one view using (6) so the maximization of volume find the centralization of maximum volume only but not with the correct centralization. This choice makes the probability of the object is contained in the volume found.

Note that this is the only case the object may not be contained in the solution volume.

3.3 Combination

For each investigated plane, there is a set of compact intervals of all views. The set of compact interval of each view corresponds to a set contained in the cross section of Ω, these longitudinal stripes in the cylinder is called effective stripes (ES).

$S=\{\underset{\sim}{x} \in \mathbb{R}^2 \,|\, G_i(\underset{\sim}{x})=z, z \in I, I$ is CI of ith view$\}$

The procedure of enumerate all the combinations to be tested, adopts another numeric basis to write the number of combinations tested and to identify the next combination.

This new numeric basis is constructed like that with J : the first figure in the representation varies from 0 to the total number of compact intervals in the first view, the second varies from 0 to the total number of CI of the second view and so on.

For instance, let N_1, N_2, ..., N_J total numbers of compact intervals respectively for each view, the representations become:

$$\begin{array}{ll} 0\;0 \ldots 0 & - \; 0 \\ 1\;0 \ldots 0 & - \; 1 \\ \vdots & \\ a_1 a_2 \ldots a_J & - \; \sum_{i=1}^{J} a_i N_i^{i-1} \end{array}$$

Besides numbering the combinations tested, this representation also an easy way of classify an combination. The combination number a_1, a_2, ..., a_J corresponds to the a_1th compact interval of first view and so on.

A subroutine which can be used for this is called once before each test:

```
COMBINATIONS:

     for i = 1 to J
          if a_i = N_i then
               a_i = 0
          else
               a_i = a_i + 1
               return
          end if
     next i
return
```

3.4 Polygons

The Polygon algorithm follows a systematic path in order to identify all the vertices of each polygon in order.

For defining a polygonal intersection of n straight lines some point should be considered:

(i) not all intersection points of two of the lines are vertices of the polygon;

(ii) not all lines that pass through one vertices of the polygon form a side of it;

(iii) for all line that form a side of the polygon pass through two and only two vertices of the polygon.

This aspects although obvious are essential for a precise definition of the polygon.

The process starts with the parameters of all lines that limit the slices in plane.

One line is chosen for being the the first line that will be the parameter for finishing the search. Another line is chosen and it is verified if the point of intersection belong to the polygon, if it does, other lines are tested until the second point of the second line is find. This process continue until the second intersection point of the first line is defined.

It is important to neglect the lines that have two different intersection at the same point, that is the tangent lines.

3.5 Polyhedral

Polyhedral is the part of the algorithm that uses the polygons the limit the contour of the object to construct the lateral polygons of the reconstructed shape.

The feature of the polygonal routine that the lines are parallel to a side of a regular polygon let us compare the parallel lines of polygons in adjacent planes in order to link then in a quadrangle or in a triangle

4. TEST

4.1 Reconstructing a Lamp Shade

The algorithm was tested reconstructing real lamp shade from four silhouettes taken from angle of 0, $\pi/4$, $\pi/2$ and $3\pi/4$ rad.

The pre-processing digitalized the silhouettes in 127 x 127 pixel each and the reconstruction used all data.

The reconstruction spend 10 minutes for the definition of the polygons and 3 minutes for the polyhedral part in a PC/XT with mathematical co-processor.

The silhouettes are shown in figure 3 and the reconstruction are shown in figure 4 and 5.

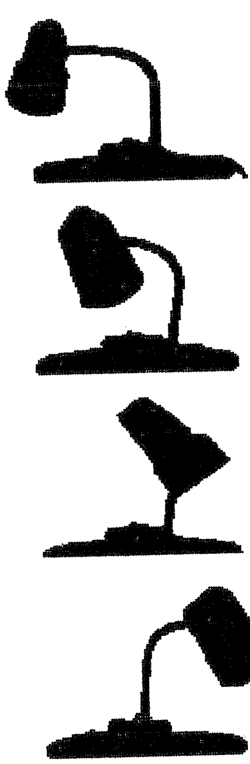

Figure 3

5. CONCLUSIONS

The algorithm introduced is easy to implement and the test showed that with approximations and the optimization, we can have good performance and good results.

As mentioned before, the algorithm cannot reconstruct concavities parallel to the image rays because the information is not contained in the silhouette image. The method can only reconstruct separate convex regions in the planes perpendicular to the rotational axis.

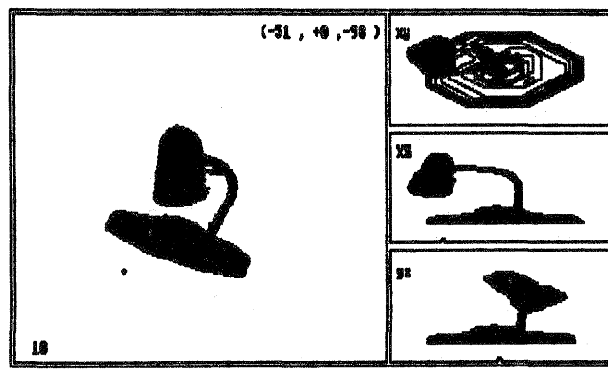

Figure 4. Reconstruction of the lamp shade after the definition of polygons.

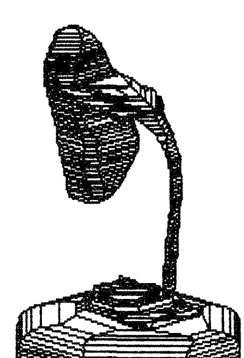

Figure 5. Complete reconstruction of the lamp shade.

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