# QUALITY CONTROL OF COMBINED ADJUSTMENT OF PHOTOGRAMMETRIC AND GPS DATA

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#### ABSTRACT:

Airborne Global Positioning System (GPS) data are incorporated in a photogrammetric block adjustment algorithm. The quality of the combined system was analyzed using both simulated and real data, and the statistical significance of the parameters of the combined functional model was investigated. Much emphasis was given to the possible reduction of ground control points. The obtained results show that a significant reduction can be achieved while still maintaining requirements for precision.

KEY WORDS: GPS, Combined-Adjustment.

#### INTRODUCTION

The Global Positioning System (GPS) has been in operation for several years, although not all of the planned 24 satellites (21 plus three in-orbit spares) have been placed in orbit. At present, after the 11th Block II Navstar satellite was "set healthy" on August 30, 1991, 16 satellites (including operational Block I prototypes) are broadcasting usable signals accessible around the world.

The potential high positioning accuracy of GPS and in particular the kinematic relative positioning in highly dynamic applications, reportedly being in the range of several centimeters [1,6,10,8], has led to the utilization of GPS in photogrammetric mapping.

GPS use in photogrammetry, has focused on three aspects: GPS-based photo-flight navigation according to a given flight plan; GPS applications in laser profiling for digital terrain models; and GPS-derived positions of camera exposure stations and introduction of these data into block adjustments with the objective of minimizing the need for ground control points.

In this paper we will concentrate on the third aspect, i.e., the combined adjustment of GPS and photogrammetric data.

The introduction of GPS data into block adjustment is basically simple: the GPS coordinates are related to the block coordinate system by a set of transformation terms introduced as additional unknowns to be solved by the combined adjustment.

Various studies conducted with simulated as well as real data [2,4,5,9] have shown that the introduction of GPS control of camera exposure stations in aerial triangulation adjustment reduces substantially the need for ground control points, while the precision requirements of mapping are maintained. They have also revealed a number of questions related to the degree of polynomials used in GPS modelling and the number of parameters utilized in the adjustment.

Constant and linear terms were usually included in the combined adjustment for datum transfer and slope corrections [1,7]. When second order terms were used, however, taking care of quadratic corrections, unfavourable results were observed [5]. Furthermore, the GPS correction parameters may be introduced stripwise or as one set of parameters for several strips, or even for the complete block. In any case, the introduction of an excessive number of parameters should be avoided and their determinability should be assured.

In this study, the GPS data of the camera exposure stations were implemented in a block adjustment

with independent models based on the simultaneous determination of seven planimetric and height parameters. The statistical significance of the GPS correction parameters was investigated and their influence on the adjustment results was analyzed. Different control point configurations were used to confirm the possible reduction of ground controls. The experiments were performed using both simulated and real data.

### THE MATHEMATICAL MODEL

The block adjustment with independent models (BAWIM) program developed at ITC many years ago (utilizing the famous 4-3 method) was modified to determine simultaneously the seven parameters and was further modified to accept the GPS data. The GPS coordinates of the camera stations are related to the block coordinate system by polynomial transformation terms; the transformation terms can be chosen stripwise or may be common for several strips.

The following additional observation equations for each camera station i in strip k were formulated:

V <sub>X</sub> gps ik	=	${\tt x_{ik}^{pc}}$	-	(a <sub>ok</sub> +	a <sub>lk</sub> Sik +	a <sub>2k</sub> S <sup>2</sup> ik)	-	X gps ik
V <sub>Yik</sub> gps	=	Y pc ik	-	(b <sub>ok</sub> +	b <sub>lk</sub> Sik +	b <sub>2k</sub> S <sup>2</sup> ik)	-	Y <sup>gps</sup> ik
V <sub>Zik</sub> gps	=	z <sup>pc</sup> ik	-	(c <sub>ok</sub> +	c <sub>1k</sub> Sik +	c <sub>2k</sub> S <sup>2</sup> ik)	-	Z gps ik

Where:

S<sub>ik</sub>

- (X, Y, Z)<sup>pc T</sup> : The unknown coordinates of the perspective centre i in strip k.
- (X, Y, Z)<sup>gps T</sup>
  ik
  : The observation of camera
  station i given by GPS in strip
  k.
- $(a_{ok}, b_{ok}, c_{ok})^{T}$  : The unknown parameters for the constant term (shift correction) in strip k.
- $(a_{1k}, 1k_1, 1k_1)^T$  : The unknown parameters for the linear term (slope correction) in strip k.
- $(a_{2k}, b_{2k}, c_{2k})^{T}$  : The unknown parameters for the second order term (quadratic correction) in strip k.
- $(v_{\chi}, v_{\gamma}, v_{Z}, v_{Z})_{ik}^{gps T}$ : Vector of the least squares residuals.
  - : Represent the distance of the exposure station i from the first perspective centre of strip k.

For the stochastic model of the observations, the following assumptions were made: the observations are not correlated, while the weights of the GPS observations and the observations of the ground control points are evaluated with respect to the weight unit of the photogrammetric observations.

The observation equations listed above, together with those derived from the seven-parameter solution, are adjusted simultaneously.

# TESTING THE STATISTICAL SIGNIFICANCE OF GPS PARAMETERS

The test of significance of the GPS parameters was to determine if the parameter values were significantly different from zero and/or if parameter groups were significantly different from each other. In this way insignificant parameters are either eliminated from the adjustments or grouped with other parameters: thus overparametrization of the system is avoided.

The tests of significance were formulated as statistical hypotheses and are tested using the test quantities developed in [3].

A null hypothesis which indicates that the parameter values are not significantly different from zero can be written as

Ho :  $E{Gi} = 0$ 

When testing the significance between groups of parameters (e.g., different GPS parameters introduced per strip), a null hypothesis, which assumes that the parameters in group 1 are not significantly different from the parameters of group 2, can be formulated as

Ho :  $E{Gi^1 - Gi^2} = 0$  i=1,...,m

where

m : number of parameters in the groups Gi : the parameter values

E{.}: indicates mathematical expectation
 superscripts 1 and 2 show the
 parameter groups

To assess the test statistics, the weight coefficient matrix of the GPS parameters should be evaluated, by applying Gaussian reduction in the reduced normal equations. In this way the GPS parameters are orthogonalized with respect to model orientation parameters and their weight coefficient matrix can be isolated.

#### TEST DATA

Using both simulated and real data photogrammetric independent model blocks could be generated at different scales, with different measuring errors and randomly generated model orientation parameters, while the GPS data could be produced with different observation errors and with different kinds of systematic errors modelled with constant, linear and quadratic terms and their combinations.

The real data were taken from the "Flevoland" test field in The Netherlands (the flight took place on June 1987). The block consisted of 16 parallel strips, each with a length of approximately 4 km. A Wild RC 10 aerial camera with a focal length of 213.67 mm was used (photo scale 1:3800).

The photogrammetric measurements were carried out on a Kern DSR1 analytical plotter and the

independent models were analytically formed. The GPS instrumentation consisted of two Sercel receivers, one stationary NR52 receiver at a known reference point (see fig. 1) and one TR5SB receiver onboard the aircraft.

The coordinates of the ground control points were determined using both conventional geodetic methods and GPS. Further information and detailed analysis of the block data can be found in [6,10,11].

A part of the block consisting of four strips with 15 to 18 models per strip (66 models in total) was used for the experiments. The part of the block and the available control points are shown in figure 1 by the dashed lines.



△ Ground control point → 1 KM

Stationary GPS receiver

Figure 1. The test field "Flevoland"

#### EXPERIMENTS AND THEIR ANALYSIS

A number of experiments were carried out with simulated data to test the mathematical and stochastic models and the computer programs, and to verify and enhance the conclusions drawn from the experiments with the real data.

The findings of these experiments were incorporated in the analysis of the experiments with the real data. Here the results of two experiments related to the testing of the significance of the GPS modelling parameters are presented.

In the first experiment, a block of two strips with 10 models per strip was generated. It was controlled with four XYZ points at the block corners and a chain of height control points at both the beginning and end of the block. The generated GPS data contain constant and linear terms. The adjustments were performed using constant (CT), linear (LT) and quadratic (QT) stripwise modelling, i.e., a total of 18 parameters were used.

The three groups of parameters were tested per strip for their significance according to the statistical tests developed in [3]: the results are given in table 1.

test	strip	critical	test quantity			
identification	10.	F .				
		3.∞,.003	СТ	LT	QT	
GPS generated	1		173.0	366.0	2.4	
data with constant and linear terms	2	4./	18.5	121.0	2.5	

# Table 1. Test quantities of the groups of parameters. The GPS data were generated with constant and linear errors

From the above table it is clear that the constant and linear terms in the two strips are rejected, i.e., they are significant, whereas the quadratic terms are not significant. The results correctly indicate the parameters which were expected to be significant.

In the second experiment, the same block and same type of adjustments were used, but the generated GPS data contain only quadratic systematic errors. The results of the test are given in table 2.

test	strip	critical	test quantity			
Identification	10.	F 3.∞,.003	СТ	LT	QT	
GPS generated	1	4.7	1.0	678.0	2.5	
quadratic term	2	4./	8.6	116.0	2.9	

Table 2. Test quantities for the groups of parameters. The data were generated with only quadratic errors

As we see, despite the fact that the generated GPS data contain only quadratic errors, the linear terms in the two strips are strongly rejected, while the constant and quadratic terms are not seen to be significant (the constant term is rejected in the second strip).

The results indicates that there is a curious interaction between linear and quadratic terms. It appears that the linear term approximates quite well the existing quadratic error.

To examine the influence of the GPS modelling on the accuracy of the combined adjustment, the previously used data sets were adjusted without GPS, with GPS using constant and linear terms for the GPS modelling (six parameters) and finally with constant, linear and quadratic terms (nine parameters). The results are summarized in table 3. Tests 2 and 3, refer to the set of data in which the GPS data were generated with constant and linear errors. Tests 4 and 5 refer to the generated GPS data with only quadratic errors.

Case	Test	No. of	absolute accuracy (meter)				
	110.	points	μ x	и У	μ z	μ xy	
without	1	80	.170	.130	.230	.150	
with 6	2	80	.102	.091	.095	.097	
par. cor. with 9	3	80	.175	.125	.603	.152	
par. cor. with 6	4	80	.098	.087	.094	.093	
par. cor. with 9 par. cor.	5	80	.179	.126	.608	.155	
part cort						L	

# Table 3. The accuracy results with different GPS parameters

Comparing the absolute accuracy of test 1 with tests 2 and 4, we see that in the latter better absolute accuracies in planimetry and height are obtained.

The comparison of test 1 with tests 3 and 5, in which constant, linear and quadratic terms were used for the GPS modelling, shows that the planimetric absolute accuracy remains on the same level of 15 cm, while the height accuracy deteriorates from 23 cm to 60 cm. This indicates that when non significant parameters (in our case the quadratic terms) are included in the mathematical model of the adjustment, the results deteriorate. This showed up consistently in all experiments with generated as well as real data.

In the experiments with the real data, the influencing factors which were investigated were the control configuration, where five configurations were considered shown schematically in figure 2, and also the GPS modelling parameters. The blocks were adjusted with the use of constant and linear terms, six correction parameters, and with the use of constant linear and quadratic terms, i.e., nine parameters per strip.



Figure 2. Various types of control distributions

The statistical significance of the GPS parameters was also tested. For these experiments, the combined adjustments were executed with the GPS observations being modelled with constant linear and quadratic terms. Three configurations were chosen (C1, C3 and C5), to check if the significance of the correction parameters depended upon the control configuration.

The tests were carried out per parameter group, and the results are given in table 4.

configuration	strip no.	critical value	test quantity				
		r 3,∞,.003	CT	LT	QT		
C1	1 2 3 4		292.0 43.0 10.8 29.0	184.7 116.0 63.5 49.4	0.5 0.5 0.9 0.3		
СЗ	1 2 3 4	4.7	549.0 527.0 664.0 6.0	6.0 121.0 15.6 8.0	0.9 0.5 0.1 0.1		
C5	1 2 3 4		300.0 726.4 791.0 6.7	31.6 21.9 24.2 21.9	0.1 0.3 0.5 0.3		

Table 4. Test quantities for the groups of parameters.

From the above table we see that the test quantities for both linear and quadratic terms are rejected for all strips in all configurations. This indicates that these two terms are significant. On the other hand, the test quantities for the quadratic terms are smaller than the critical value, which indicates that the quadratic terms are not significant.

The conclusion does not necessarily imply that the quadratic deformation is not present in the GPS data. As we have seen in the experiments on the statistical significance with generated data, the quadratic deformation can be very well approximated by the linear terms used in the adjustment. It is also observed that the results are consistent and independent of the configuration used.

Based on these outcomes, the proper modelling for the combined adjustment seems to be the one which takes into account the constant and linear terms. The introduction of the quadratic term may negatively effect the results.

The GPS modelling was done per strip, and it was of interest to assess if the parameters introduced per strip were significantly different. The individual parameters of strip 1 and strip 2 in configuration C3 were tested. The critical value for this test was  $F_{1,\infty} = 10.8$ . Since it had been already proved that the quadratic term was not significant, the test among parameters was performed for only the constant and linear terms. The results are given in table 5.

GPS modelling term	parameter diff in	test quantity
constant	x y z	29.0 89.3 12.3
linear	x y z	25.2 26.7 16.5

Table 5. Test quantities between individual parameters

We can observe that the differences of the parameters are significant. Similar results were obtained by testing the significance of the parameter differences among the other strips. This suggests that modelling of the GPS parameters as strip invariant is appropriate for the available set of data used in the experiments.

Table 6 contains the accuracy results of the five configurations shown in figure 2, being adjusted as follows:

- without GPS data
- with GPS modelled with linear and constant terms with GPS modelled with linear, constant and

quadratic termș

Case	No. of check config.		rel. accuracy (meter)			abs. accuracy (meter)			
	points		σ x	σ y	σ z	μ x	и У	μ z	μ xy
without GPS data	23 19 13 13 24	C1 C2 C3 C4 C5	.017 .017 .020 .020 .018	.019 .019 .020 .020 .020 .019	.032 .032 .033 .033 .033 .032	.090 .108 .097 .039 .052	.070 .043 .042 .042 .042 .037	.533 .418 .109 .108 .255	.081 .092 .074 .041 .045
with GPS data (constant & linear term)	23 19 13 13 24	C1 C2 C3 C4 C5	.019 .020 .021 .021 .021 .020	.020 .020 .020 .020 .020 .020	.033 .034 .034 .035 .033	.092 .120 .097 .035 .068	.054 .047 .051 .030 .036	.429 .130 .108 .098 .139	.074 .091 .078 .033 .054
with GPS data (con- stant lin- aer & qua. terms)	23 19 13 13 24	C1 C2 C3 C4 C5	.018 .019 .020 .021 .019	.019 .020 .020 .020 .020 .019	.032 .033 .033 .034 .034	.091 .121 .118 .030 .055	.072 .047 .042 .035 .032	.598 .434 .106 .103 .259	.082 .091 .088 .037 .045

### Table 6. Variation in control point configuration and in GPS modelling.

Comparing the results of the adjustment "without GPS data" with "GPS constant and linear terms", it can be observed that the planimetric precision is approximately the same, while the height precision shows considerable improvement. The highest improvement, by a factor of 3.2, is observed for configuration C2.

Comparison of the results shows that with the introduction of the quadratic term the height accuracies deteriorate quite appreciably for configurations C1,C2 and C5, while the accuracies remain approximately the same for configurations C3 and C4 because these configurations are very well controlled in height.

Comparing the results of the different control configurations in the combined adjustment with constant and linear terms for the GPS modelling, we see that

- One additional control point in the middle of the block configuration C5 compared with C1 improves the height precision by a factor of 4.
- Configuration C2 gives height precision similar to C5.
- The height accuracy of configuration C2 is only 2 cm more than the accuracy of C3, but C2 has only two chains of height control points, while C3 has 3 chains.

## CONCLUSIONS

The results demonstrate that GPS-controlled photogrammetry has the potential to reduce substantially the need for geodetic control points while the accuracy requirements are maintained. The improvement in height accuracy is significant: improvement by as much as a factor of 3.2 has been observed in the experiments with the real data. Improvement in planimetric accuracies was not noticed in these experiments, probably because the large-scale block used has a high planimetric accuracy (within 10 cm) when it was conventionally adjusted. Improvement in planimetry was observed in the experiments with generated data and in particular in medium size blocks (10 strips, 40 models/strip). The use of only four ground control points, without additional height points, plus GPS data does not yield satisfactory results, while the addition of one XYZ point in the block centre gives much better results. It should be noted, however, that these control configurations may cause numerical instabilities in practical applications where the blocks are not symmetric or as well prepared as simulated or test field blocks.

From the different control configurations used in the combined adjustments, the most effective with respect to block precision and yet with substantially reduced ground control points is four XYZ points at the block corners and a chain of height controls at both the beginning and end of the block. In aerial triangulation using GPS data, there is no need for ground control points in the inner area of the block.

It is clear from the presented results that, GPS modelling with constant, linear and quadratic terms worsens the accuracy and in particular the height accuracy. It seems that the block deformation in heights adapts itself to the GPS modelling rather than being controlled. On the other hand, GPS modelling with constant and linear terms gives a strong support for controlling the height block deformation.

The statistical test applied for detecting if parameter values are significantly different from zero is shown to be very effective, and provides the means of avoiding the detrimental effect of insignificant parameters. The test to determine if parameter groups are significantly different from each other should also be incorporated in combined adjustments software, thus avoiding the introduction of an excessive number of parameters in the system.

It is evident that the high accuracy and functionality of the kinematic GPS will require the photogrammetric community to reconsider the planning of photogrammetric projects.

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