# RASTER ALGORITHMS FOR SURFACE MODELLING

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ABSTRACT:

Two raster algorithms for solving problems in surface modelling are presented. A raster-based triangulation allows for a simple consideration of constraint edges and thus improves computational complexion of the constrained triangulation to a great extent. A medial axis method derives geomorphological elements, e.g. peaks, pits, saddle points, ridge and drainage lines, from a given set of contours and actually assists in generating a high-quality digital terrain model from contours. Examples demonstrate the efficiency of a joint use of the two algorithms in terrain modelling.

**Keywords:** raster algorithm, surface modelling, Delaunay triangulation, Voronoi diagram, distance transformation, medial axis, geomorphological element, digital terrain model.

## 1. INTRODUCTION

Digital terrain models (DTMs) find more and more applications in a variety of branches of science and management of today. This leads to the fact that demand for high-quality DTMs (HQ-DTMs) is increasing. A HQ-DTM here means that the DTM ought to restore the terrain surface as exactly as possible, both in geometry and geomorphology. Once the acquired primary data are available, a rigorous consideration of geomorphological information which represents the terrain relief in form of distinctive points (e.g. pits, peaks and saddle points), breaklines and ridge or drainage lines becomes then the first essential of generation of a HQ-DTM. It means that the HQ-DTM should keep this information as good as given. A possible way to achieve this is to triangulate the given sets of points and lines. However, triangulating points and lines (so-called constrained triangulation, a common issue in surface modelling) is quite difficult to be implemented with respect to algorithms and computationally even quite time-consuming in case where a large number of constraint edges exists (Lee/Lin, 1986; Wang/Schubert, 1987; de Floriani/Puppo, 1988). A question arises:

(1) Can the constraint edges be considered in a simple manner so that the computational complexion of the constrained triangulation can be improved to some extent ?

Digitized contours are usually used as primary data for DTM generation. However, generating a HQ-DTM from contours depends to a great extent on the availability of the geomorphological information (Clarke et al., 1982; Christensen, 1987; Ebner/Tang, 1989; Aumann et al., 1990; Tang, 1991, 1992). Actually, it is quite difficult to acquire this kind of information in a direct manner since e.g. it is usually not contained in a topographic map explicitly. Nevertheless, geomorphological elements such as peak and pit regions, saddles, ridges and valleys find their expression in contours and can even possibly be derived from contours (e.g. Finsterwalder, 1986; Tang, 1991). Another question is here of interest:

(2) How can geomorphological elements or at least their approximations be derived from contours automatically and then be used for HQ-DTM generation ?

Answers to this question as well as the first one will be given in following sections.

## 2. A RASTER-BASED TRIANGULATION

To answer the question (1) in the last section a raster-based triangulation was developped and will be described in the following.

### 2.1. Basic idea

In surface modelling, object surface can be described by a triangulated irregular network (TIN), which consists of planar, nonoverlapping, and irregularly shaped triangular facets on the given data sets. In this way the 3-dimensional problem of surface description is then reduced to a 2-dimensional one, i.e. establishing the adjacency relationships among the given data points only on the x-y plane. This is the task of a triangulation as well. Among various approaches (e.g. Lawson, 1977; Watson/Philip, 1984; Preparata/Shamos, 1988) the Delaunay triangulation is the one which is often used for (terrain) surface modelling (e.g. Christensen, 1987; de Floriani/Puppo, 1988; Tang, 1991).

The Delaunay triangulation of a set of points is usually defined in term of another geometric structure, the Voronoi diagram (sometimes called Thiessen polygons or Dirichlet tessellation). Definitions of these two dual graphs can be found in (e.g. Preparata/Shamos, 1988). Therefore, once the Voronoi diagram of the given set of points is constructed, the Delaunay triangulation of the same data set can be derived directly from it.

The construction of the Voronoi diagram can take place either in a vector (Lee/Lin, 1986; Wang/Schubert, 1987; Preparata/Shamos, 1988) or in a raster world (Borgefors, 1986; Gottschalk, 1988; Ebner et al., 1989; Tang, 1989, 1991). For reasons of distinction the Voronoi diagram in raster is referred to as quasi Voronoi diagram (QVD). The difference between the QVD and the Voronoi diagram lies essentially in the raster geometry. This fact leads to the following idea: Mapping the given set of points onto a suitable raster and constructing the QVD there, a vectorial TIN is then derived from the QVD by using certain raster operations. That is, the task of a triangulation, establishing the adjacency relationships among the given points, is accomplished here in a raster world by means of raster operations. This triangulation is called the raster-based triangulation.

As mentioned in section 1 the problem that is of interest here concerns the constraint edges in a triangulation. There are some vectorial algorithms proposed for the constrained (or generalized) Delaunay triangulation (e.g. Lee/Lin, 1986; Wang/Schubert, 1987; de Floriani/Puppo, 1988). The computational time of using them obviously depends above all on the number of constraint edges in the data sets. Since the constraint edges are the known edges in the TIN regardless of the Delaunay criterion they act as boundaries and divide the Voronoi diagram into parts. In this case the Voronoi diagram is called the bounded Voronoi diagram (Wang/Schubert, 1987). Hence, how this kind of QVD can be achieved is the main issue of the raster-based triangulation.

For further descriptions remarks on terminology used in the following are given here. A raster can be described by an array mathematically. In accordance with terms in digital image processing an array element is called a pixel in the 2-dimensional case. In an array the pixels that represent geometric elements (e.g. points, lines and areas) are defined as feature pixels and the rest background pixels. A pixel that is not on the border of the array has two horizontal and two vertical neighbours, which are called the 4-neighbours. In addition, the pixel has four diagonal neighbours. Both these and the 4-neighbours are called 8-neighbours of the pixel. Provided N can be 4 or 8. Two pixels are N-adjacent if they are the N-neighbour to each other. A N-path is a sequence of distinct pixels among which the two neighbouring pixels are N-adjacent.

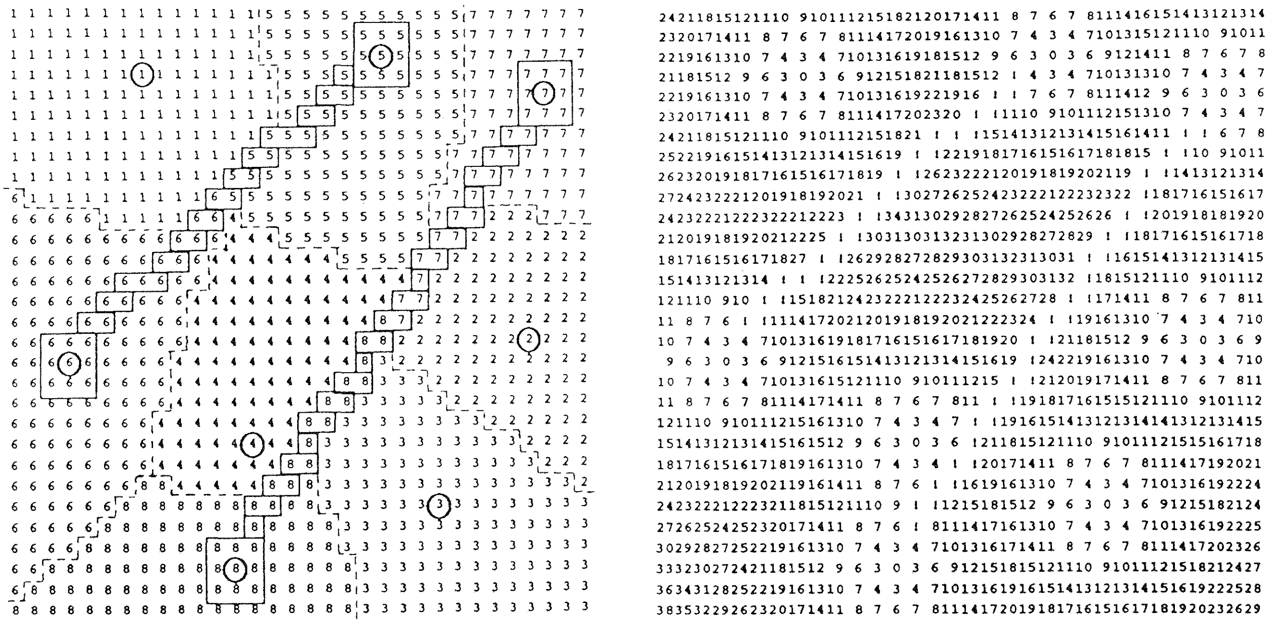
## 2.2. Creating the QVD of points and lines

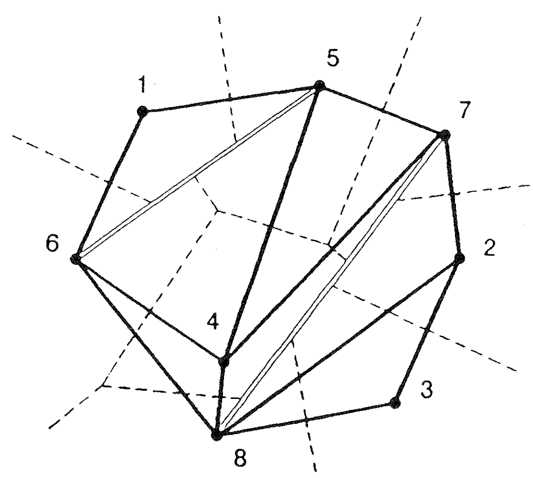
Once a given set of points is mapped onto arrays by a vector-raster conversion the QVD can be constructed by means of a distance transformation (DT). A DT is an operation that converts an array, consisting of feature and background pixels, to a new one where each pixel has a value corresponding to the distance to the nearest feature pixel. Different distances can be used for a DT. The one of interest here is the chamfer(3, 4) which approximates the Euclidean distance quite well and requires a local distance mask of 3x3 pixels only. Detailed descriptions on this topic can be found in (Borgefors, 1986).

For creating a QVD two arrays are necessary: a distance array for carrying out the DT and a feature array for forming the QVD. The given points are mapped onto the two arrays as feature pixels. The distance array is initially two valued: zero for feature pixels and infinity (i.e. a suitably large integer value) elsewhere. In the feature array the corresponding point number is assigned to each feature pixel and a unique value to background pixels. While the DT is applied to the distance array, the background pixels in the feature array are overwritten by feature ones. As a result, the QVD is formed.

In order to take account of constraint edges in the raster-based triangulation a method of encoding feature pixels in both the distance and the feature array was developped. In the following, a feature pixel that corresponds to a point in the given data sets is called a point pixel (P-pixel) and a feature pixel that comes into being by mapping a line segment is referred to as a line pixel (L-pixel). The encoding method aims at achieving a bounded Voronoi diagram. Therefore, the following criteria are applied for encoding feature pixels in both the distance and the feature array:

- Each P-pixel is assigned the initial value zero in the distance array and the corresponding point number in the feature array. In addition, the 8-neignbours of each P-pixel which belongs to the subset of nodes of constraint edges get the local distance values (e.g. 3 for the 4-neighbours and 4 for the rest in case of chamfer(3, 4) distance) in the distance array and the code of the P-pixel in the feature array.





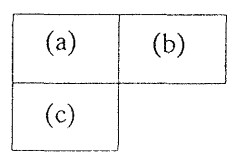




Figure 1. Principle of the encoding method for creating the QVD of points and lines.

(a) The QVD - the feature array after the DT;

(b) The distance array after the DT using the chamfer(3,4) distance;

(c) The constrained Delaunay triangulation and the bounded Voronoi diagram.

- The constraint edges are represented by 4-paths in both arrays. All L-pixels have a unique code in the distance array. In the feature array the L-pixels of each edge are cut in half and get the code of the nearest P-pixel.

In this way the constraints are still retained and the operation will, however, take place regardless of any constraint. That means, the computational time here remains as the same as the case that only points are triangulated. Figure 1 demonstrates the principle of the encoding method.

## 2.3. Deriving a TIN from QVD

Once the QVD of the given sets of points and lines is available, a TIN can be derived from it by using a suitable operator. Two strategies can be applied to forming the TIN:

- constituting triangular edges individually by locating Voronoi edges;
- constituting triangles individually by locating Voronoi nodes.

In the QVD a Voronoi edge can be of one of the following three shapes: (a) a horizontal line; (b) a vertical line; (c) a terraced line (cf. Figure 2). Therefore, an operator which is suitable for edge detection can be used to locate the triangular edges in the QVD.

```
1 1 1 1 1 1      1 1 1 2 2 2      1 1 1 1 2 2
1 1 1 1 1 1      1 1 1 2 2 2      1 1 1 2 2 2
1 1 1 1 1 1      1 1 1 2 2 2      1 1 2 2 2 2
2 2 2 2 2 2      1 1 1 2 2 2      1 1 2 2 2 2
2 2 2 2 2 2      1 1 1 2 2 2      1 2 2 2 2 2
2 2 2 2 2 2      1 1 1 2 2 2      2 2 2 2 2 2

    (a)              (b)              (c)
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Figure 2. Voronoi edges in the QVD.

The cases where a Voronoi node can appear in the QVD are illustrated in Figure 3. In order to locate a Voronoi node in the QVD, an operator of 2x2 pixels (cf. Figure 4) is used. Positioning this operator on a pixel in the QVD, three neighbours of the pixel are involved in it. Hence, it is called the N3-operator. Using it, the procedure for locating Voronoi nodes in the

QVD and thus for constituting triangles as well can then easily be realized (cf. Tang, 1991).

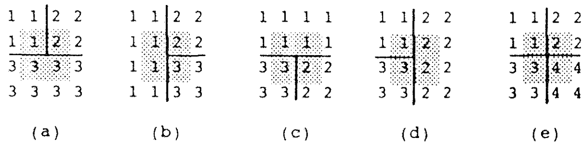


Figure 3. Possible cases of a Voronoi node in the QVD.

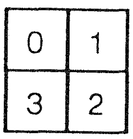


Figure 4. The N3-operator for locating a Voronoi node in the QVD.

Examples for demonstrating the efficiency of the raster-based triangulation will be given in section 4.

## 3. DERIVATION OF GEOMORPHOLOGICAL ELEMENTS FROM CONTOURS

To answer the question (2) in section 1 an approach for automatically deriving geomorphological elements from contours was developped and will be described in the following.

### 3.1. Basic idea

Looking at the Figure 5, one can find out that the dashed lines are horizontal triangular edges which do lead to an incorrect description of the terrain surface. As a matter of fact, it is hardly to gain a satisfactory result for the terrain surface description in this case

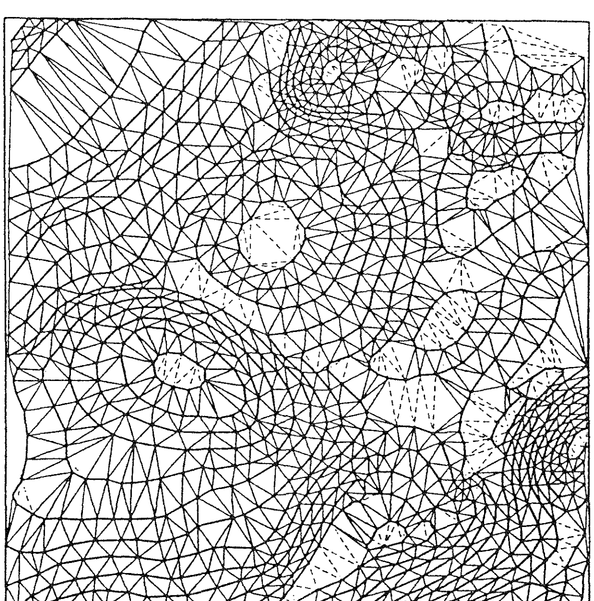

Figure 5. A TIN constructed from contours.

where contours serve as primary data only, though various approaches beside a triangulation can be used for terrain modelling (Clarke et al., 1982; Christensen, 1987; Ebner/Tang, 1989; Aumann et al., 1990; Tang, 1991).

The regions where the dashed lines appear can be referred to as critical regions. They mark, however, the places where certain geomorphological elements exist, too (cf. Figure 6). The geomorphological elements here mean peaks, pits, saddle points, ridge and drainage lines, which are essential for generation of a HQ-DTM from contours. In principle, they can be

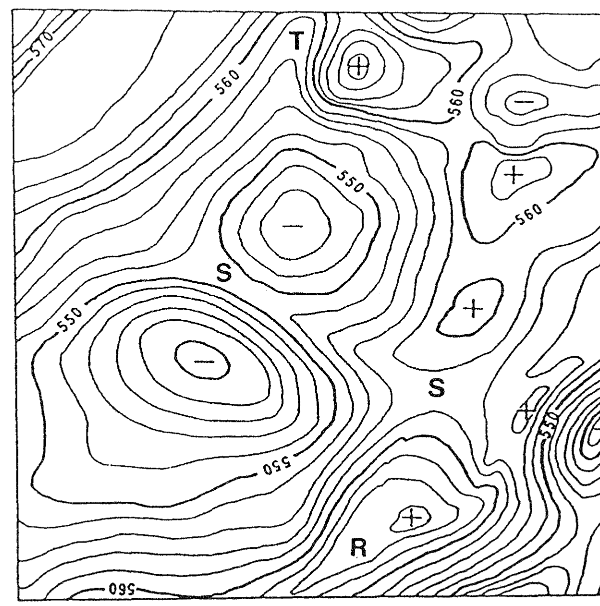


Figure 6. Geomorphological structures reflected by contours.

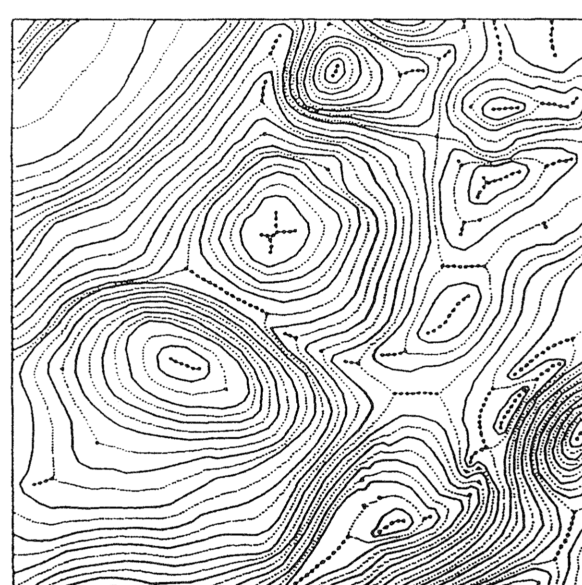

Figure 7. Medial axes of contours.

derived from the given contours since they represent structures on the terrain surface, that are reflected by shapes of contours as well (Finsterwalder, 1986; Ebner/Tang, 1989; Aumann et al., 1990; Tang, 1991, 1992b).

Medial axis is a descriptor of shape (Blum, 1967) and finds a lot of applications in computer vision, graphics and image processing as well as computational geometry. Intuitively, the medial axis of a 2-dimensional object is the set of points within the object figure that are medial between the boundaries. The corresponding operation is called medial axis transformation (MAT), which has different synonyms such as symmetric axis transform, skeletonization or thinning according to purposes and customs. The MAT can take place either in a vector (e.g. Lee, 1982) or in a raster world (e.g. Arcelli, 1981). In the latter case the criterion of local maxima of distances to boundaries can be applied to locating the medial axis or skeleton pixels (Arcelli, 1981).

Treating contours as boundaries of shapes, the medial axes between them can be obtained then by a MAT algorithm. Figure 7 shows an example. There are two types of medial axes to be distinguished: the medial axes that lie between neighbouring contours and the ones that are found between two parts of the same contour or between different contours of the same elevation. The former and the latter are called the normal and the special medial axes, respectively. Comparing Figure 7 with Figures 5 and 6, one can find out that the special medial axes just occur in the critical regions and approximate geomorphological elements to a certain extent. This leads to the idea that special medial axes should be used for tracing geomorphological elements. To realize this idea two procedures are necessary: locating geomorphological elements and assigning elevations to them.

### 3.2. Locating geomorphological elements

The MAT algorithm based on the local maxima criterion can locate medial axes between the given contours, but is not able to distinguish them. As mentioned above, only the special medial axes are of interest. So an algorithm based on QVD was proposed for locating special medial axes since Voronoi edges are medial axes as well (Tang, 1991).

As described in section 2, given contours are at first mapped onto the two arrays as feature pixels. By way of contrast, contour edges are represented here by 8-paths and nodes of each continuous contour line are labelled by successive numbers. In addition, all L-pixels get the same initial value zero in the distance array as P-Pixels. In this way, contour pixels will propagate themselves in every direction during the DT in both arrays, and as a result a desired QVD is obtained.

Voronoi edges in this QVD can be classified into three groups: a Voronoi edge between (a) two nodes of a contour edge, (b) two different contours and (c) two parts of the same contour. The differentiation of them is realized by checking the codes which are involved in the edge detection (cf. Figure 2). For example, if the code difference is equal 1, the Voronoi edge belongs to the group (a), else it is a member of the group (b) or (c). Obviously, the Voronoi edges in group (a) are neither the normal nor the special medial axes and should be left out of consideration. In order to distinguish the special from the normal medial axes among the rest two groups, elevation should be taken into account, i.e. if the involved codes indicate the contour points which have the same elevation, the Voronoi edge is the special medial axis, else the normal one.

There are three types of special medial axes: they occur (1) in peak or pit regions, (2) in saddle regions and (3) in ridge or valley regions. To differentiate them Voronoi nodes are used. Two kinds of Voronoi nodes are of interest, i.e. the ones that are shared by the special and the normal medial axes and the ones to that only the special medial axes are incident. In the following, the former is referred to as SN-node and the latter as SO-node. While a SN-node indicates the connection to the neighbouring contour, a SO-node relates only with the contour from which the special medial axis comes into being. According to the node status of each special medial axis, it can be classified into a certain geomorphological element: it is of type (1) if both nodes are SO-nodes; it is of type (2) if both nodes are SN-nodes; it is of type (3) if one node is a SN-node and the other a SO-node. Connecting each special medial axis with the corresponding contour points the geomorphological elements are then located (cf. Figure 8).

### 3.3. Elevation Assignment

For further uses, e.g. for terrain modelling, appropriate elevations should be assigned to points of located geomorphological elements. Depending on types, different strategies are used for the elevation assignment. In the following, the elevtion of the contour from which the special medial axis comes into being is denoted as H1 and the elevtion of the neighbouring contour as H2.

For type (1): At first, H2 should be found out by searching the neighbouring contour since it is not included in the SO-nodes. Then, if $H1 < H2$ holds the geomorphological element contains a pit else a peak. For selection of the pit or peak point on the medial axis the criterion of symmetry is applied, i.e. the middle point. Since no additional information is available in the concerned region it is quite reasonable to assign to the middle point an elevation of (H1 - the half equidistance) in case of a pit or (H1 + the half equidistance) in case of a peak. Other points on the medial axis get adjusted values between the middle point and the concerned contour point.

For type (2): The selection of the saddle point follows the symmetry criterion, too. The saddle point gets then the mean value of H1 and H2. Other points get adjusted values as described above.

For type (3): If H1 < H2 holds the geomorphological element represents a ridge line else a drainage line. The elevation assignment by a linear interpolation between H1 and H2 is a simple and also widely accepted way.

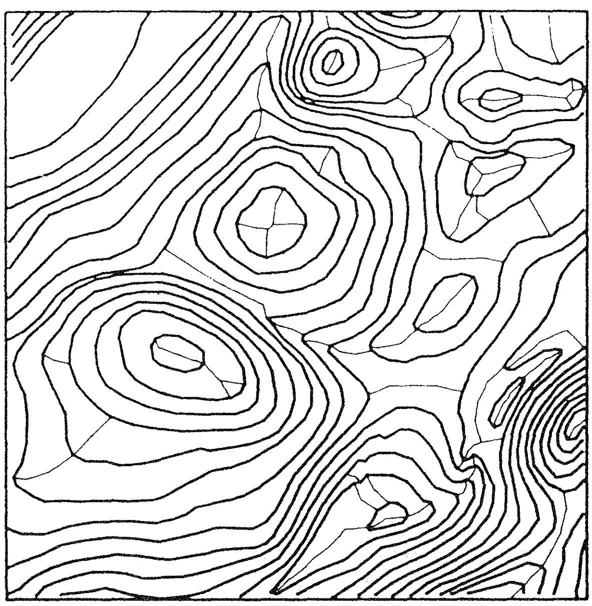

Figure 8. Geomorphological elements from contours.

## 4. DIGITAL TERRAIN MODELLING USING RASTER ALGORITHMS

Contours are often used as reference data for DTM generation. A HQ-DTM from contours means that the given contours ought to be restored from it on the one hand and the intermediate contours derived from it should reasonably be shaped on the other hand. The former can be guaranteed by the constrained triangulation, and the latter requires, however, geomorphological elements for assistance.

At first, geomorphological elements are derived by means of the medial axis approach described above. A TIN-DTM is then generated by the raster-based triangulation of the given contours and the derived geomorphological elements. Finally, various follow-up products can be derived from the TIN-DTM.

To evaluate the presented approaches two practical examples are given in the following:

Example "Thalham": It covers an area of 400x400 square meters on the ground. The contours with a constant equidistance of 2 meters were manually digitized from a topographic map of a scale of 1:10000. The total number of contour points amounts to 1168. The given contours and the derived geomorphological elements are shown in Figure 8. Figure 9 is the TIN-DTM.

Example "Koralpe": It covers an area of 1480x1620 square meters on the ground. The contours with a constant equidistance of 10 meters were manually digitized from a topographic map of a scale of 1:10000. The total number of contour points amounts to 4252. Figure 10 (a) shows the given contours and the derived geomorphological elements. Following the procedure described above, a TIN-DTM was generated from these data at first. It was then converted into a raster DTM by a planar interpolation of the triangular facets. The 10-meter contours and the 2.5-meter intermediate contours were derived from the raster DTM (cf. Figure 10 (b)). The improvement of the DTM quality is evident.

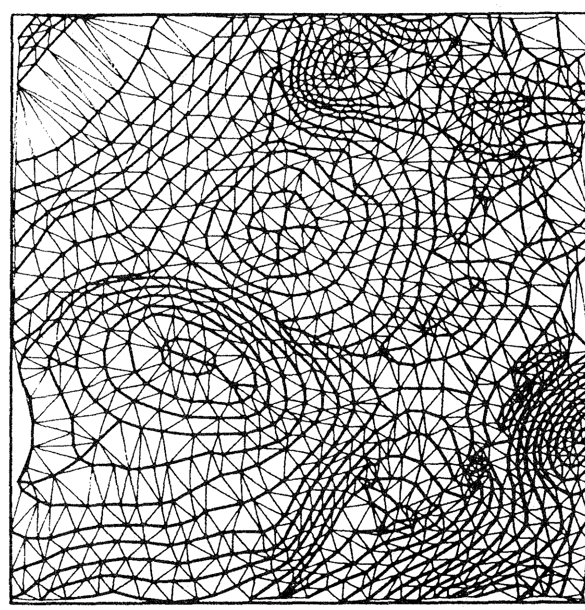

Figure 9. Example "Thalham": the TIN constructed from the given contours and the derived geomorphological elements.

With regards to the computational complexion the raster-based triangulation was compared with an existing vectorial algorithm for the constrained Delaunay triangulation. The comparsion was carried out on a Hewlett Parckard graphics workstation HP 9000/350 CHX, which is equipped with a MC68020/25MHz CPU, a MC68881/20MHz floating point processor and 8 MBytes main memory. In addition, a test on the computational time for a triangulation without con-

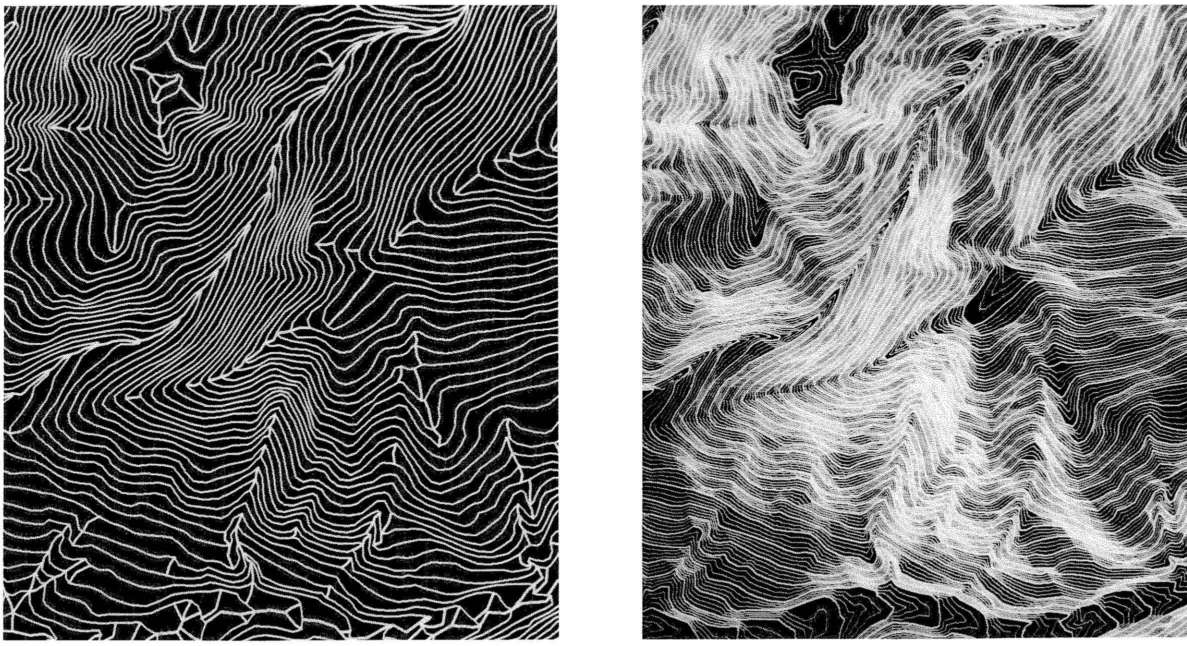


Figure 10. Example "Koralpe": (a) given contours and derived geomorphological elements; (b) contours (thick) and intermediate contours (thin) derived from a TIN-DTM.

straints was made for the example "Koralpe" to see the relative difference. The results are given in Table 1.

Table 1: Computational time for the triangulation.

| example | vectorial (mm:ss) | raster-based (mm:ss) |
|---|---|---|
| Thalham | 2:05 | 0:51 [401x401] |
| Koralpe | 21:28(2:59) | 1:39(1:38)[493x540] |
| remarks | 1) The time in () is valid for a triangulation without the constraints. 2) Digits in [] indicate the size of array for the QVD construction. | |

## 5. CONCLUSIONS

Techniques from raster data processing can lead to quite simple and robust solutions for complicated problems in surface modelling. The raster-based triangulation improves the computational complexion at a great deal in case of a large number of existing constraints. The medial axis approach derives geomorphological elements from the given contours and thus contributes a lot to the improvement of the DTM quality in the case where only contours are available for terrain modelling. The combination of the two approaches leads to an even satisfactory result. So the use of raster techniques for digital terrain modelling is promising.

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