## THE SIMULTANEOUS ADJUSTMENT FOR SPOT IMAGE USING ORBIT DATA AND A FEW CONTROL POINTS

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### ABSTRACT:

In this paper, the construction of the orbit data for SPOT image is described and the method using the data is discussed. The basic mathematical model for the simultaneous adjustment of SPOT imagery using the orbit data and a few control points are presented. The method can avoid the high correlation between the orientation elements during adjustment processing and decrease the number of necessary control points. Finally, the results of experiments are given. The results have proved that the solution of unknown points and orientation elements obtained by the mathematical model in this paper are of sufficient accuracy. An adjustment without ground control points sows that it is possible to determine positions of ground points using SPOT orbit data directly

KEY WORLDS: SPOT orbit data, Imagery, Orientation elements, Adjustment

### 1. THE CONSTRUCTION OF THE ORBIT DATA FOR SPOT SCENE

Each SPOT CCT consists of five digital files. They are volume directory, leader, Image, trailer and null volume directory file. In which, the leader file is very useful for Photogrammetric processing.

There are six records in the leader file. two records (header record and ephemeris-attitude ancillary record) can be employed for photogrammetric adjustment. The information included in these data are the following.

#### 1.1 Scene parameters

Scene center geographic location (latitude, longitude) and corresponding image coordinates (scanline number, pixel number).

Four scene corners geographic location and corresponding image coordinates.

Scene orientation angle: complement of the angle between the center scene line of the raw scene and meridian through the center of the raw scene.

Angle of incidence of SPOT sensor.

Scene center time (universal time).

#### 1.2 Ephemeris and attitude data

Ephemeris data include the geocentric coordinates (Xc,Yc,Zc) of SPOT sensor position, velocity vector of the satellite movement corresponding to an instant and universal time. Generally, the coordinates for 8 or 9 sensor positions are given, the interval of time is 60 seconds. The Xc axis is in the equatorial plane corresponding to Greenwich meridian. The Yc axis is perpendicular to the Xc axis in equatorial plane. The Zc axis is along the earth polar axis.

Attitude data include drift rate of attitude Pitch, Roll and Yaw for 72 or 73 points and corresponding scanline number, The interval of time is about 0.125 second. The Roll axis corresponding to the  $\varphi$  axis is the direction of SPOT orbit. The Pitch axis corresponding to the  $\omega$  axis is the scanning direction of sensor. The Yaw axis corresponds to the  $\kappa$  axis.

## 2. THE PHOTOGRAMMETRIC SOLUTION FOR SPOT SCENE

#### 2.1 Coordinates system

The basic coordinate system employed in this paper

are:

px, py are image coordinates(IMC). they are expressed in image center coordinate system. The origin of the system is the scene center. In which px is expressed as the pixel number in a scanline and py is expressed as the line number of scanlines.

t is the time interval with respect to the scene center . The interval can be calculated from the py-coordinate,

$$t_i = (py_i - py_o) t_s \tag{1}$$

Where  $py_i$  and  $py_o$  are the coordinates of an image point and scene center respectively;  $t_s$  is the time interval needed to made a scanline.

X,Y,Z are local tangent plane coordinates (LTC). The origin is the satellite position corresponding to a scene center. LTC are employed during bundle adjustment. The geographic or Gauss -Krueger or UTM coordinates should be translated into the system.

#### 2.2 Error equations for SPOT imagery

Considering some errors due to the movement of satellite and sensor as well as the change of inner orientation parameters, Collinearity equations for SPOT imagery can be expressed as:

$$\begin{bmatrix} px \\ 0 \\ -f \end{bmatrix} + \begin{bmatrix} dx_e \\ 0 \\ 0 \end{bmatrix} = k R \begin{bmatrix} X - X_s \\ Y - Y_s \\ Z - Z_s \end{bmatrix} + \begin{bmatrix} dx_f \\ dy_f \\ 0 \end{bmatrix} + \begin{bmatrix} dx_v \\ dy_v \\ 0 \end{bmatrix}$$
(2)

i.e.

$$x = -f \frac{\overline{X}}{\overline{Z}} - dx$$
$$0 = -f \frac{\overline{Y}}{\overline{Z}} - dy$$
$$x = px + dx_{e}$$
$$dx = dx_{f} + dx_{v}$$
$$dy = dy_{f} + dy_{v}$$

Where, k represents the scale factor between image and ground points,  $R = R_t R_s$ ,  $R_t$  is a rotation matrix corresponding to the rotation of SPOT sensor,  $R_s$  is a constant matrix corresponding to the angle of sensor incidence. The vector  $[dx_f dy_f 0]^t$ represents the correction for the error due to the change of focal length, the vector  $[dx_v dy_v 0]^t$ expresses the correction for the error due to the distortion of scene,  $dx_v$  represents the correction of radial distortion in scanning direction,  $dy_v$  is introduced to compensate for the distortion due to the possible misalignment of scene,  $dx_e$  represents the correction for the error due to rotation of the earth, the correction will be discussed below.

The extra orientation elements in formula (2) can be expressed as:

$$\begin{bmatrix} \varphi_{i} \\ \omega_{i} \\ \kappa_{i} \end{bmatrix} = \begin{bmatrix} \varphi_{o} \\ \omega_{o} \\ \kappa_{o} \end{bmatrix} + t_{i} \begin{bmatrix} \varphi_{r} \\ \omega_{r} \\ \kappa_{r} \end{bmatrix}$$
(2--1)  
$$\begin{bmatrix} X_{si} \\ Y_{si} \\ Z_{si} \end{bmatrix} = \begin{bmatrix} X_{o} \\ Y_{o} \\ Z_{o} \end{bmatrix} + t_{i} \begin{bmatrix} X_{r} \\ Y_{r} \\ Z_{r} \end{bmatrix}$$
(2-2)

The number of unknown parameters in formula (2) is 26 for a stereopair. The fact that the view field of the sensor is very narrow ( $4.13^{\circ}$ ), and the SPOT orbit reaches into the height 820 kilometers results in intensive correlation between the angular and linear elements in the same direction. In order to reduce the correlation, the constraint conditions for the position of sensor corresponding to the center scanline and the components of average velocity of the satellite movement are employed. The constraints can be driven from the ephemeris data in SPOT CCT.

The standard condition equations based on the discussion above are expressed as:

$$\mathbf{A}\mathbf{v} = \mathbf{B}\mathbf{u} + \mathbf{I} \tag{3}$$

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix}^{\mathsf{t}} \tag{3-1}$$

$$\mathbf{v}_1 = [v_{x1} \ v_{y1} \ \dots \ v_{xn} \ v_{yn}]^t$$
 (3-1-1)

$$\mathbf{v}_2 = [v_{X1} \ v_{Y1} \ v_{Z1} \dots v_{Xk} \ v_{Yk} \ v_{Zk}]^{\mathrm{I}} \quad (3-1-2)$$

 $\mathbf{v}_3 = [\mathbf{v}_{XS} \mathbf{v}_{YS} \mathbf{v}_{ZS} \mathbf{v}_{Xr} \mathbf{v}_{Yr} \mathbf{v}_{Zr}]^{t}$  (3 -1-3) Where n is the number of observations, k is the number of control points, and l is the residual vector corresponding to vector v.

$$\mathbf{B}\mathbf{u} = \begin{bmatrix} \mathbf{I}_{1} & \mathbf{I}_{2} & \mathbf{I}_{3} & \mathbf{I}_{4} & \mathbf{I}_{5} & \mathbf{I}_{6} \\ 0 & 0 & 0 & 0 & \mathbf{G} \\ 0 & 0 & \mathbf{P}_{s} & 0 & 0 & \mathbf{G} \\ 0 & 0 & 0 & \mathbf{P}_{r} & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{u}_{1} \\ \mathbf{u}_{2} \\ \mathbf{u}_{3} \\ \mathbf{u}_{4} \\ \mathbf{u}_{5} \\ \mathbf{u}_{6} \end{bmatrix}$$
(3 -- 2)

Where submatrices I are coefficients of error equations for image observations.  $I_1$ ,  $I_3$  and  $I_2$ ,  $I_4$  correspond to vector  $\mathbf{u}_1$ ,  $\mathbf{u}_2$ , and  $\mathbf{u}_3$ ,  $\mathbf{u}_4$  which represent the corrections for sensor position, attitude and their linear rate respectively.  $I_5$  and  $I_6$  correspond to vector  $\mathbf{u}_5$  and  $\mathbf{u}_6$  which express the corrections for scene distortion and calculated coordinates of ground points. Submatrices  $\mathbf{P}_s$  and  $\mathbf{P}_r$  are the coefficients of constraint equations for sensor position and linear rate.

For a single point,

$$\mathbf{I}_{1} = \begin{bmatrix} \overline{Z} \mathbf{f} - \overline{X} \mathbf{x} & \overline{Y} \mathbf{x} & -\overline{Y} \mathbf{f} \\ \\ -\overline{Y} \mathbf{x} & -\overline{Z} \mathbf{f} - \mathbf{f} \frac{\overline{Y}^{2}}{\overline{Z}} & \overline{X} \mathbf{f} \end{bmatrix}$$
(3--2--1)

$$\mathbf{I}_{3} = \begin{bmatrix} \mathbf{f} & \mathbf{0} & \mathbf{x} \\ \mathbf{0} & \mathbf{f} & -\mathbf{f} \, \overline{\overline{\mathbf{Y}}} \\ \mathbf{0} & \mathbf{f} & -\mathbf{f} \, \overline{\overline{\mathbf{Y}}} \end{bmatrix}$$
(3--2--2)

$$\mathbf{I}_{5} = \begin{bmatrix} -\frac{\overline{\mathbf{X}}}{\overline{\mathbf{Z}}} & -\mathbf{x}^{2} \mathbf{f} \frac{\overline{\mathbf{X}}}{\overline{\mathbf{Z}}} \\ -\frac{\overline{\mathbf{Y}}}{\overline{\mathbf{Z}}} & -\mathbf{x}^{2} \mathbf{f} \frac{\overline{\mathbf{Y}}}{\overline{\mathbf{Z}}} \end{bmatrix}$$
(3--2--4)  
$$\mathbf{I}_{2} = \mathbf{t}_{i} \mathbf{I}_{1} \qquad \mathbf{I}_{4} = \mathbf{t}_{i} \mathbf{I}_{3} \qquad \mathbf{I}_{6} = -\mathbf{I}_{3}$$
$$\mathbf{A} = \begin{bmatrix} \overline{\mathbf{Z}} & \mathbf{0} \\ \mathbf{0} & \overline{\mathbf{Z}} \end{bmatrix}$$
(3--2--5)

The coordinates of ground control points should be considered as weighted observations. Submatrix G, corresponding to the control points, is a unit matrix.

In the submatrices  $P_s$  and  $P_r$  are same form as G :

$$\mathbf{P}_{s} = \mathbf{P}_{r} = \mathbf{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(3--2--6)

Through solving equations (3) using the weighted least square method, 26 unknown parameters for a stereopair are obtained. The weights for several equations can be valuated according to the variances of residuals of the equations.

# 2.3 <u>The correction for the error caused by earth</u> rotation

Firstly, we discuss the displacement in the scanning direction. As shown in figure 1, while the SPOT sensor scans the surface of the earth from point  $P_o$  to point  $P_b$ , the displacement  $D_x$  is caused due to the earth rotation. This error is symmetric obviously and must be removed before adjustment.

In figure 1, We represents the angular velocity of earth rotation, r is the radius of the parallel through  $P_b$ , B represents the latitude of point  $P_b$ , N expresses the radius of the prime vertical through point  $P_b$ . The displacement  $D_x$  can be given as:

$$D_{x} = r W_{e} t_{b}$$
$$r = N \cos B$$
$$N = \frac{a}{\sqrt{1 - e^{2} \sin^{2} B}}$$

where a represents the long radius of earth, e is the eccentricity of earth,  $t_b$  represents time interval from point  $P_o$  to point  $P_b$ .



Figure 1 the displacement in the scanning direction

Considering the scene orientation angle A between the orbital direction and Y axis of the local coordinate system, the displacement at a instant corresponding scene center can be expressed as:

$$d_{x} = \frac{a \cos B W_{e} t_{b}}{\lambda \sqrt{1 - e^{2} \sin^{2} B}} \cos A$$
(4)

$$\sin A = \frac{\cos I}{\cos \phi} \tag{4--1}$$

Where, I represents the angle of SPOT orbit incidence, and  $\phi$  expresses the geocentric latitude of point P<sub>b</sub>. Generally, B and  $\phi$  for an image point can be calculated by interpolation according to the latitude and geocentric latitude of scene center and four corner points.

Earth rotation also causes the variance of scale factor in y-direction of scene (i. e. SPOT orbit direction). The variance is no effect on time serial of scene. Therefore it will not be discussed in this paper.

#### 3. THE POSSIBILITY OF SPOT SCENES ADJUSTMENT WITHOUT GROUND CONTROL POINTS

As described above, the number of GCP needed to make the adjustment can be reduced obviously due to the application of ephemeris data. If the geographic coordinates of scene corner and center as well as attitude data are employed, it is possible to take the adjustment without GCP for a particular level of accuracy.

In SPOT CCT, pitch ( $\omega$ ), roll ( $\varphi$ ), and yaw ( $\kappa$ ) drift rates are supplied at a 0.125 second interval. The value of the drift rate at instant t<sub>i</sub> can be calculated by linear interpolation. Then, by integrating these rates from the instant to for scene center to t<sub>i</sub>, the pointing variance of sensor at instant t<sub>i</sub>, with respect to scene center instant t<sub>o</sub> can be obtained.

$$\alpha_{i} = \int_{t_{0}}^{t_{i}} v \, dt = \sum_{i=1}^{n} v \left( Py_{i} - Py_{i-1} \right) t_{s}$$
(5)

where n represents the number of points included from instant  $t_i$  to  $t_o$ .

The averaged velocities  $\varphi_r$ ,  $\omega_r$ ,  $\kappa_r$  of variances for  $\varphi$ ,  $\omega$ ,  $\kappa$  can be calculated in formula (5) according to 72 or 73 attitude data respectively. In this case, the averaged velocity in formula (2--1) become known constants. Therefore, the unknown parameters in formula (2) are reduced to 20.

Excluding attitude data, the geographic coordinates of corner and center of scene can also be employed as pseudo control conditions. The image coordinates corresponding to the pseudo control points are constants and can be obtained from SPOT CCT.

Using the data above, the adjustment without GCP for SPOT scenes can be realized.

#### 4. TEST AND ANALYSIS

A group of programs for simultaneous bundle adjustment for SPOT scenes using orbital data and a few ground control points has been developed. The flow chart of the programs is shown in figure 2.

The results for tests reported here are from three different stereo scenes: STF (France), STC (China) and STU (USA). The results are shown in Table 1. In test STF, the orbital data are not included. The results demonstrate the practical effect of earth rotation. It is obvious from Table 1 that the fewre the number of GCP, the more important the correction for earth rotation. The test indicates that the error due to earth rotation can not be removed completely if the coordinates of image points are not



Figure 2 Flow chart of programs

Scene	Status	Number of		RMSE of GCP (m)			RMSE Of CHP (m)			S (11-1)
		GCP	CHP	Х	Y	Z	X	Y	Z	ο <sub>ο</sub> (μm)
STF Incedent 25.0 <sup>0</sup> (L) 26.2 <sup>0</sup> (R) B/H=1.0	With the rotation correction	34	50	6.2	8.5	1.5	10.5	15.1	5.0	9
		18	66	5.6	7.1	1.2	11.8	14.7	6.2	10
		9	75	1.9	4.1	1.0	11.4	15.4	6.2	9
	With the rotation correction	34	50	9.9	8.9	1.7	12.7	16.0	9.1	9
		18	66	9.0	7.9	1.6	13.0	16.5	9.4	9
		9	75	5.1	6.2	1.0	16.1	22.1	11.9	8
STC Incedent 14.8 <sup>0</sup> (L) 16.5 <sup>0</sup> (R) B/H=0.5	With the ephemeris data	25	32	5.2	6.1	2.0	13.2	14.3	7.8	11
		15	42	4.8	6.9	1.5	14.5	14.6	7.9	11
		9	48	4.0	6.1	1.1	14.7	15.1	8.1	10
		·5	52	0.8	1.4	0.7	14.6	16.0	8.8	10
	Without the ephemeris data	25	32	4.9	5.7	1.5	13.9	15.5	8.5	9
		15	42	4.7	5.9	1.2	15.7	15.4	8.9	9
		9	48	3.5	4.1	1.2	16.7	17.5	<u>9.9</u>	11
STU Incedent 12.9 <sup>0</sup> (L) 8.9 <sup>0</sup> (R) B/H=0.6	With the ephemeris data	25	23	5.1	8.7	1.7	16.1	16.2	7.1	10
		15	33	4.7	8.8	1.4	15.5	16.1	7.2	11
		9	39	2.2	5.3	1.7	16.4	17.4	8.1	10
		5	43	0.9	1.2	0.4	16.0	17.7	8.9	9
	(Atittude)	0	48				221.5	250.2	36.2	
	Without the ephemeris data	25	23	4.7	7.9	1.5	15.9	18.1	8.2	11
		15	33	4.7	8.8	1.4	17.2	16.3	7.9	10
		9	39	2.0	5.3	0.8	16.0	19.6	13.3	10

Table The results of tests

corrected previously. The reason is that there is unavoidable correlation between the additional parameters for the rotation correction and the element of sensor position during processing.

In tests STC and STU, the orbital data are included. The interesting results of the two tests are useful to demonstrate the importance of application of ephemeris and attitude data during processing. The results show that the accuracy of the solution using ephemeris data is obviously improved. The accuracies obtained from five to nine points are comparable to those from 18 to 34 points. The tests also indicate that the stable solution of exterior elements of scene can be obtained using ephemeris data.

In test STU, the attitude drift rates are employed. The result of the test proves that it is possible to process SPOT scenes without GCP at a special level of accuracy. The accuracy of the solution depends on that of the ephemeris and attitude data.

#### CONCLUSION

The method presented in this paper is a rigorous photogrammetric reconstruction of ground targets from SPOT stereo scenes. The mathematical model and programs can accommodate single stereopairs and aerotriagulation for SPOT scenes. According to the tests, the primary conclusion can be given below:

1. It is necessary that the error due to earth rotation is removed before adjustment. That can separate the symmetric errors from adjustment system rigorously and ensure the reliability of solutions. 2. The improvement effect using ephemeris data is obvious and tends to increase in case where the number of GCPs is small. The application of orbital constraints can avoid the high correlation between exterior elements and get stable solutions. The tests show that the accuracy achieved by using five to nine GCPs is sufficient to support topographical mapping and generation of orthophotos at 1:50,000 - 1:100,000.

3. The tests show that determination of ground targets without GCP is possible and indicate the important interesting of using ephemeris, attitude data and scene parameters in area where no GCPs can be employed. The accuracy of solutions will be improved with the improvement of reliability of orbital data.

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