NOTES ON THE DIRECT PROJECTIVE TRANSFORMATION OF GENERAL STEREO PAIRS INTO THE RIGOROUS NORMAL CASE BY IMAGE CORRELATION

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Abstract: *The problem of direct projective transformation from the general to the normal case of stereophotogrammetry is treated by means of image correlation. Therefrom result* *<u>linear</u>* *equations containing optimal approximate values of relative orientation, which are to be introduced into a post-adjustment because of the redundancy of this method. The resulting error propagation is discussed and finaly an example for a digital stereo pair is given*

KEY WORDS: Projective transformation, normal case, image correlation, digital stereo images.

0. INTRODUCTION

In Vol.12, No.1(1990) of the photogrammetric journal of Finland *H. Haggren* and *I. Niini* published a method for the 2-D projective transformation of general stereo pairs into the strictly normal case of photogrammetry. Their method is based on the correlation of two overlapping projectivities of a spatial object (*Thompson* 1968), from which the parameters of transformation can be derived. Since the correlation refers to metric images, its effect corresponds to the method of linearization by redundant observations, because <u>eight</u> homologous points are needed. This method is already known from (*Rinner* 1963) as "unconditional conjunction of successive images" and delivers two components of the base (b_2, b_3) and three rotations of the second image.
The goal of the transformation to the normal case is to obtain parallel epipolar lines in order to facilitate the automatic search for homologous points in the reconstruction of the object from digital stereo pairs (*Kreiling* 1976). Thus the parameters of Rinner's method are not very useful, because the normal case does not arise directly therefrom. In contrast to this, the other possibility of relative orientation, i.e. the use of rotations only (*Brandstätter* 1991), delivers the convergency and consequently the parameters of the desired transformation.

1. THEORETICAL ASPECTS

1.1 <u>Condition of intersection and projective transformation</u>

Using the analytical quantities

$\mathbf{R} = [\mathbf{i}, \mathbf{j}, \mathbf{k}]$	matrix of orientation (reconstruction)
$\mathbf{E}$	unit matrix ($\mathbf{R}^T\mathbf{R} = \mathbf{E}$)
$\mathbf{x}^T = (x, y, -c)$	vector of centered image coordinates
$\mathbf{p} = \mathbf{R}\,\mathbf{x}$	projector in the model space
$\mathbf{X}_0^{(i)}$	center of projection
$\mathbf{b}^T = (b_1, b_2, b_3)$	stereo base ($\mathbf{b} = \mathbf{X}_0'' - \mathbf{X}_0'$)
λ	scalar coefficient (stretching factor)

the reconstruction of a point $\mathbf{X}$ of the model space from the coordinates $\mathbf{x}'$ and $\mathbf{x}''$ of the two images P' and P" (condition of intersection) reads

$$\mathbf{X} = \mathbf{X}_0' + \lambda'\mathbf{R}'\mathbf{x}' = \mathbf{X}_0'' + \lambda''\mathbf{R}''\mathbf{x}'' \qquad (1.1.1)$$

and the coordinates in one of the two images arise from the projection

$$\lambda\mathbf{x} = \mathbf{R}^T (\mathbf{X} - \mathbf{X}_0). \qquad (1.1.2)$$

If $\mathbf{R}$ does not yet contain the elements of absolute orientation, its parameters Φ', K', Ω'', Φ'', K'' (Ω'' =difference of lateral tilts) represent only the relative orientation. The desired normal case (defined by the unit matrix $\mathbf{E}$) results analogously to (1.1.2) from

$$\lambda_N\mathbf{x}_N = \mathbf{E} (\mathbf{X} - \mathbf{X}_0). \qquad (1.1.3)$$

Introducing $\mathbf{X}$ from (1.1.1) this relation converts to

$$\lambda_N\mathbf{x}_N = \mathbf{E} (\mathbf{X}_0 + \lambda\mathbf{R}\mathbf{x} - \mathbf{X}_0) = \lambda\mathbf{R}\,\mathbf{x}$$

and the direct projective transformation to the normal case is given by

$$\tau\mathbf{x}_N = \mathbf{R}\,\mathbf{x}\,, \quad \tau = \lambda_N/\lambda \qquad (1.1.4)$$

or after elimination of the unknown coefficient τ by formation of the quotients $-x_N/c$ and $-y_N/c$

$$x_N = -c\,\frac{i_1x+j_1y-k_1c}{i_3x+j_3y-k_3c} = -c\,\frac{\mathbf{e}_1.\mathbf{x}}{\mathbf{e}_3.\mathbf{x}},$$
$$y_N = -c\,\frac{i_2x+j_2y-k_2c}{i_3x+j_3y-k_3c} = -c\,\frac{\mathbf{e}_2.\mathbf{x}}{\mathbf{e}_3.\mathbf{x}}, \qquad (1.1.5)$$

wherin the $\mathbf{e}_i$ (i = 1,2,3) are the rows of $\mathbf{R}$. These equations correspond, of course, to the equations of (*Kreiling* 1976) but also to those of (*Haggren and Niini* 1990), disregarding the formal discrepancy that there the last number of the denominator equals 1. The aim of this method is therefore, to find the unknown orienations of the two images.

Knowing $\mathbf{x}_N$, the quotient τ can be determined from

$$\tau^2\mathbf{x}_N{}^T\mathbf{x}_N = (\mathbf{R}\mathbf{x})^T(\mathbf{R}\mathbf{x}) = \mathbf{x}^T\mathbf{R}^T\mathbf{R}\mathbf{x} = \mathbf{x}^T\mathbf{x}$$

regarding $p^2 = p_1{}^2+p_2{}^2+p_3{}^2 = x^2+y^2+c^2 = \mathbf{x}.\mathbf{x}$ ($\mathbf{x}.\mathbf{x}$ is equivalent to $\mathbf{x}^T\mathbf{x}$), as

$$\tau = \frac{\sqrt{\mathbf{x}.\mathbf{x}}}{\sqrt{\mathbf{x}_N.\mathbf{x}_N}} = \frac{p}{p_N}\,, \qquad (1.1.6)$$

the ratio of the two distances from the common center of projection to the points $\mathbf{x}$ (original) and $\mathbf{x}_N$ (transformed).

1.2 <u>Orientation from image correlation</u>

Using $\mathbf{b}$, the condition of intersection (1.1.1) can also be written as

$$\lambda'\mathbf{p}' = \mathbf{b} + \lambda''\mathbf{p}'' \qquad (1.2.1)$$

from which follows after vector multiplication by $\mathbf{b}$ and scalar multiplication by $\mathbf{p}''$ because of

$(\mathbf{p}''\times\mathbf{b}).\mathbf{p}'' = 0$ the condition of coplanarity

$$(\mathbf{p}' \times \mathbf{b}).\mathbf{p}'' = 0. \qquad (1.2.2)$$

The vector product is equivalent to $\mathbf{p}'\times\mathbf{b}=\mathbf{p}'^T\mathbf{B}$, if

$$\mathbf{B} = \begin{bmatrix} 0 & -b_3 & b_2 \\ b_3 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{bmatrix},$$

and by means of $\mathbf{p} = \mathbf{R}\,\mathbf{x}$ (1.2.2) converts to

$$\mathbf{p}'^T\mathbf{B}\mathbf{p}'' = \mathbf{x}'^T\mathbf{R}'^T\mathbf{B}\mathbf{R}''\mathbf{x}'' = \mathbf{x}'^T\mathbf{C}\mathbf{x}'' = 0. \qquad (1.2.3)$$

It contains the matrix **C** of correlation as it is used in (*Rinner* 1963) and put into projective relationships by (*Thompson* 1968). A more detailed structure may be obtained from

$$\mathbf{C} = \begin{bmatrix} \mathbf{i}^T \\ \mathbf{j}^T \\ \mathbf{k}^T \end{bmatrix}' \mathbf{B} \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \end{bmatrix}'' =$$

$$= \begin{bmatrix} (\mathbf{i}''\times\mathbf{i}').\mathbf{b} & (\mathbf{j}''\times\mathbf{i}').\mathbf{b} & (\mathbf{k}''\times\mathbf{i}').\mathbf{b} \\ (\mathbf{i}''\times\mathbf{j}').\mathbf{b} & (\mathbf{j}''\times\mathbf{j}').\mathbf{b} & (\mathbf{k}''\times\mathbf{j}').\mathbf{b} \\ (\mathbf{i}''\times\mathbf{k}').\mathbf{b} & (\mathbf{j}''\times\mathbf{k}').\mathbf{b} & (\mathbf{k}''\times\mathbf{k}').\mathbf{b} \end{bmatrix}, \qquad (1.2.4)$$

which shows the connexion with the unit vectors of the two camera systems.

C has two important properties (*Thompson* 1968):

1. From

   $$\mathbf{C}^T\mathbf{x}_0' = 0 \quad \text{and} \quad \mathbf{C}\,\mathbf{x}_0'' = \mathbf{0} \qquad (1.2.5)$$

   (rank(**C**) = 2) result the coordinates $\mathbf{x}_0$ of the epipoles.

2. The (dualistic) transformations

   $$\mathbf{h}' = \mathbf{C}\,\mathbf{x}'' \quad \text{and} \quad \mathbf{h}'' = \mathbf{C}^T\mathbf{x}' \qquad (1.2.6)$$

   deliver the coefficients of the epipolar lines

   $\mathbf{h}'.\mathbf{x}'=0$ and $\mathbf{h}''.\mathbf{x}''=0$, i.e.

   the geometric loci of homologous points.

Due to the homogeneity of (1.2.3) only a matrix

$$\mathbf{Z} = (1/c_{32})\mathbf{C} \quad (z_{32}=1) \qquad (1.2.7)$$

can be calculated (*Rinner* 1963), where c_{32} is the probably biggest component, but it can be used instead of **C** without any limitation,since (1.2.5) is homogeneous too and the **h** of (1.2.6) contains coefficients of homogeneous equations,where common factors do not have any influence. As for further considerations of this paper, the calculation of the coordinates of the epipoles is of main interest.
One restriction must be obeyed, which results from possible linearities among the rows of the (8x8)-matrix for the determination of the eight components of **Z**. In order to avoid such singularities, in space the points of correlation should not coincide with planes passing three other points. Thus the model should be clearly spatial and the points well-distributed.

1.3 <u>Reconstruction of the model</u>

Regarding **R** = **E** , from the two formulas (1.1.1) of reconstruction results their difference

$$\lambda_N''\mathbf{x}_N'' - \lambda_N'\mathbf{x}_N' = -\,\mathbf{b} \quad |.\mathbf{y}_N'' \quad |.\mathbf{y}_N'$$

and the successive scalar multiplications by a vector $\mathbf{y}_N^T=(c,0,x_N)$ yield therefrom because of $\mathbf{x}_N^{(i)}.\mathbf{y}_N^{(i)} = 0$ the expressions

$$\lambda_N' = \frac{\mathbf{b}.\mathbf{y}_N''}{\mathbf{x}_N'.\mathbf{y}_N''} \quad \text{and} \quad \lambda_N'' = -\frac{\mathbf{b}.\mathbf{y}_N'}{\mathbf{x}_N''.\mathbf{y}_N'} \qquad (1.3.1)$$

for the stretching factors, depending only on the base and the image coordinates of the normal case. If rotational relative orientation is to be used, the base takes the form $\mathbf{b}^T = (1,0,0)$ and the formulas of (1.3.1) change by means of $\mathbf{b}.\mathbf{y}_N=c$, $\mathbf{x}_N'.\mathbf{y}_N''=c(x_N'-x_N'')$, $\mathbf{x}_N''.\mathbf{y}_N'=c(x_N''-x_N')$ to

$$\lambda_N = \lambda_N' = \lambda_N'' = \frac{1}{x_N'-x_N''}, \qquad (1.3.2)$$

that is the reciprocal of the x-parallax. λ_N approaches infinity, if $x_N'=x_N''$, indicating parallel projectors, or in other words, images of points in infinity.
Knowing λ_N, from (1.1.3) arises the simple formula of reconstruction

$$\mathbf{X} = \mathbf{X}_0 + \lambda_N\mathbf{x}_N, \qquad (1.3.3)$$

which delivers the coordinates of the model. The well-known effect of double determination from P' and P" enables the check of calculation and from (1.3.3) results analogously to (1.1.6) the expression

$$\lambda_N = \frac{\sqrt{(\mathbf{X}-\mathbf{X}_0).(\mathbf{X}-\mathbf{X}_0)}}{\sqrt{\mathbf{x}_N.\mathbf{x}_N}} \qquad (1.3.4)$$

as a final test of the reconstruction from the normal case.

2. DETERMINATION OF THE PARAMETERS OF TRANSFORMATION

2.1 <u>The rotational relative orientation</u>

This procedure is well-known from analog photogrammetry and is executed in such a way that the base remains unchanged, that is $\mathbf{b}^T=(1,0,0)$, the left image P' is moved only by tip Φ' and swing K', the right image P" by tilt Ω'', tip Φ'' and swing K''. Thus the movement of P" is to be described by the orientation matrix (*Wolf* 1974, p. 533)

$$\mathbf{R}'' = \begin{bmatrix} \cos\Phi\cos K & -\cos\Phi\sin K & \sin\Phi \\ \sin\Omega\sin\Phi\cos K+\cos\Omega\sin K & -\sin\Omega\sin\Phi\sin K+\cos\Omega\cos K & -\sin\Omega\cos\Phi \\ -\cos\Omega\sin\Phi\cos K+\sin\Omega\sin K & \cos\Omega\sin\Phi\sin K+\sin\Omega\cos K & \cos\Omega\cos\Phi \end{bmatrix} \qquad (2.1.1)$$

and the movement of P'(Ω'=0) by

$$\mathbf{R}' = \begin{bmatrix} \cos\Phi\cos K & -\cos\Phi\sin K & \sin\Phi \\ \sin K & \cos K & 0 \\ -\sin\Phi\cos K & \sin\Phi\sin K & \cos\Phi \end{bmatrix}'. \qquad (2.1.2)$$

The correlation matrix (1.2.4) results now because of $b_2=b_3=0$ in

$$\mathbf{C} = \begin{bmatrix} i_3'i_2''-i_2'i_3'' & i_3'j_2''-i_2'j_3'' & i_3'k_2''-i_2'k_3'' \\ j_3'i_2''-j_2'i_3'' & j_3'j_2''-j_2'j_3'' & j_3'k_2''-j_2'k_3'' \\ k_3'i_2''-k_2'i_3'' & k_3'j_2''-k_2'j_3'' & k_3'k_2''-k_2'k_3'' \end{bmatrix} \qquad (2.1.3)$$

and contains only the second and third components of the **i, j, k.**

2.2 <u>Computation of the parameters</u>

First of all it is to be assumed that the coordinates x_0', y_0' and x_0'', y_0'' of the epipoles are already calculated from

$$\mathbf{Z}^T\mathbf{x}_0' = \mathbf{0} \quad \text{and} \quad \mathbf{Z}\,\mathbf{x}_0'' = \mathbf{0}.$$

They are the images of the base given by

$$\lambda_0'\mathbf{x}_0' = \mathbf{R}'^T\mathbf{b} \quad \text{and} \quad \lambda_0''\mathbf{x}_0'' = -\mathbf{R}''^T\mathbf{b},$$

or

$$\lambda_0'\begin{bmatrix} x_0' \\ y_0' \\ -c \end{bmatrix} = \begin{bmatrix} \cos\Phi'\cos K' \\ -\cos\Phi'\sin K' \\ \sin\Phi \end{bmatrix} \quad \text{and} \quad \lambda_0''\begin{bmatrix} x_0'' \\ y_0'' \\ -c \end{bmatrix} = -\begin{bmatrix} \cos\Phi''\cos K'' \\ -\cos\Phi''\sin K'' \\ \sin\Phi'' \end{bmatrix},$$

from which independently from Ω'' follow

$$\tan K = -\frac{y_0}{x_0} \quad \text{and} \quad \tan\Phi = \frac{-c}{\sqrt{x_0^2+y_0^2}}. \qquad (2.3.1)$$

for both images. By means of these parameters $\mathbf{R}' = [\mathbf{i}',\mathbf{j}',\mathbf{k}']$ is known.
The still missing parameter Ω'' of $\mathbf{R}''$ may be calculated now from any component of (2.1.3). The best way is to use the third column

$$(i_2'\cos\Omega'' + i_3'\sin\Omega'')\cos\Phi'' = c_{32}z_{13}$$
$$(j_2'\cos\Omega'' + j_3'\sin\Omega'')\cos\Phi'' = c_{32}z_{23}$$
$$- k_3'\sin\Omega''\cos\Phi'' = c_{32}z_{33}$$

and to eliminate $\cos\Phi''$ by

$$\cos\Phi'' = \frac{c_{32}z_{33}}{-k_3'\sin\Omega''}.$$

Therefrom the <u>two</u> symmetric possibilities

$$\tan\Omega'' = \frac{z_{33}\,i_2'}{z_{13}\,k_3' - z_{33}\,i_3'} = \frac{z_{33}\,j_2'}{z_{23}\,k_3' - z_{33}\,j_3'} \qquad (2.3.2)$$

arise for the determination of Ω'', which result from the fact that the transcendental problem of orientation has been linearized by more observations than necessary. Moreover, **C** is calculated irrespective of the conditions of rectangularity and normalization of the unit vectors **i, j, k,** so that an iterative post-processing must take place in order to get an algebraically and stochastically consistent set of parameters.

2.3 <u>Adjustment</u>

The rotation matrices of section 2.2 undoubtedly will be very close approximations **(R)** to the most probable solutions **R.** Hence small additional rotations **dR** will give the final position of the images according to

$$\mathbf{R} = \mathbf{dR}(\mathbf{R}) = (\mathbf{E}+\mathbf{dA})(\mathbf{R}),\ \mathbf{dA} = \begin{bmatrix} 0 & -dK & d\Phi \\ dK & 0 & -d\Omega \\ -d\Phi & d\Omega & 0 \end{bmatrix}.$$

By means of a vector $\mathbf{v}^T=(v_x,v_y,0)$ of the residuals of coordinate measurement and by neglecting quantities of second order, (1.2.3) turns to

$$(\mathbf{x}'+\mathbf{v}')^T\{(\mathbf{E}+\mathbf{dA}')(\mathbf{R}')\}^T\mathbf{B}\{(\mathbf{E}+\mathbf{dA}'')(\mathbf{R}'')\}(\mathbf{x}''+\mathbf{v}'')=$$
$$=\mathbf{x}'^T(\mathbf{C})\mathbf{x}''+\mathbf{v}'^T(\mathbf{C})\mathbf{x}''+\mathbf{x}'^T(\mathbf{C})\mathbf{v}''+(\mathbf{p}')^T\mathbf{dA}'^T\mathbf{B}(\mathbf{p}'')+$$
$$+(\mathbf{p}')^T\mathbf{B}\mathbf{dA}''(\mathbf{p}'')=0,$$

wherein $(\mathbf{C})=(\mathbf{R}')^T\mathbf{B}(\mathbf{R}'')$ and $(\mathbf{p})=(\mathbf{R})\mathbf{x}$. Because of

$$\mathbf{dA}'^T\mathbf{B} = \begin{bmatrix} 0 & -d\Phi' & -dK' \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{B}\mathbf{dA}'' = \begin{bmatrix} 0 & 0 & 0 \\ d\Phi'' & -d\Omega'' & 0 \\ dK'' & 0 & -d\Omega'' \end{bmatrix}$$

and using the substitutions $\delta p=\mathbf{x}'^T(\mathbf{C})\mathbf{x}''$(=parallax), $\mathbf{v}'^T(\mathbf{C})\mathbf{x}''=\mathbf{v}'^T(\mathbf{h}')$, $\mathbf{x}'^T(\mathbf{C})\mathbf{v}''=(\mathbf{h}'')^T\mathbf{v}''$ (according to equ. (1.2.6)), one linearized coplanarity equation (without round brackets at **h** and p_i) reads

$$\delta p + \mathbf{h}'.\mathbf{v}' + \mathbf{h}''.\mathbf{v}'' = p_1'p_2''d\Phi' + p_1'p_3''dK' -$$
$$- (p_2''p_2'+p_3''p_3')d\Omega'' - p_1''p_2'd\Phi'' - p_1''p_3'dK''$$

and represents formally the general case of least squares adjustment, i.e. conditions with unknowns. But as the residuals of one equation do not appear in any other equation (Tab. 2.3), the procedure can be simplified by introduction of the fictitious residuals (*Wolf* 1968, p.105, *Rinner* 1972, p.402)

$$w = h_1'v_x'+h_2'v_y'+h_1''v_x''+h_2''v_y''$$

and the related weights

$$\frac{1}{g} = \frac{h_1'^2}{g_x'} + \frac{h_2'^2}{g_y'} + \frac{h_1''^2}{g_x''} + \frac{h_2''^2}{g_y''},$$

i	Residuals $v_x'v_y'v_x''v_y''$	$v_x'v_y'v_x''v_y''$	......	$v_x'v_y'v_x''v_y''$	Unknowns $d\Phi'$	dK'	$d\Omega''$	$d\Phi''$	dK''	Par. δp
1	■ ■ ■ ■				■	■	■	■	■	δp_1
2		■ ■ ■ ■			■	■	■	■	■	δp_2
:					:	:	:	:	:	:
:					:	:	:	:	:	:
:					:	:	:	:	:	:
8				■ ■ ■ ■	■	■	■	■	■	δp_8

Tab. 2.3: Scheme of the linearized equations of coplanarity

which convert to

$$1/g = \sigma^2 \; (h_1'^2 + h_2'^2 + h_1''^2 + h_2''^2), \qquad (2.3.1)$$

if the a priori variances $\sigma_x^2 = \sigma_y^2 = \sigma^2$ of the measured coordinates are equivalent. Adjustment and error computation correspond therefore to the rules of customary adjustment of weighted observation equations. In this way, also more than eight points can easily be used for image correlation <u>without</u> adjustment of the calculation of **Z** where the condition det(**Z**)=0 must be obeyed (*Haggren and Niini* 1990). Thus **Z** can only deliver approximate values of relative orientation.
The results of the adjustment will be the solutions

$$\mathbf{da}'^T=(d\phi' dK'), \quad \mathbf{da}''^T=(d\Omega'' d\phi'' dK'')$$

and the matrix of dispersion

$$\mathbf{S_a} = \sigma^2\mathbf{Q} = \sigma^2 \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{12}^T & \mathbf{Q}_{22} \end{bmatrix}, \qquad (2.3.2)$$

containing instead of the estimate s^2 the known a priori variance σ^2 and the submatrices $\mathbf{Q}_{11}$ belonging to P' and $\mathbf{Q}_{22}$ belonging to P". $\mathbf{Q}_{12}$ indicates the stochastic correlation between the images, which influences the reconstruction of the model but not the transformations into the normal case. Hence the dispersion of the rotation P'-->P_N' will be

$$\mathbf{S_a}' = \sigma^2 \begin{bmatrix} 0 & 0 & 0 \\ \mathbf{0} & & \mathbf{Q}_{11} \end{bmatrix} = \sigma^2\mathbf{Q_a}' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{\phi\phi} & \sigma_{\phi K} \\ 0 & \sigma_{\phi K} & \sigma_{KK} \end{bmatrix}'$$

and of the rotation P"-->P_N"

$$\mathbf{S_a}'' = \sigma^2\mathbf{Q}_{22} = \sigma^2\mathbf{Q_a}'' = \begin{bmatrix} \sigma_{\Omega\Omega} & \sigma_{\Omega\phi} & \sigma_{\Omega K} \\ \sigma_{\Omega\phi} & \sigma_{\phi\phi} & \sigma_{\phi K} \\ \sigma_{\Omega K} & \sigma_{\phi K} & \sigma_{KK} \end{bmatrix}''.$$

3. NORMAL CASE

3.1 <u>Transformation</u>

By means of the calculated elements of relative orientation, the transformation (1.1.4) will yield image coordinates $\mathbf{x_N}$ of the normal case. Using now eight points $\mathbf{x_N}$ for a correlation of the <u>transformed</u> images, the result must be, because of **R'=R"=E**, the easily predictable matrix

$$\mathbf{Z_N} = \mathbf{C_N} = \mathbf{E\,B\,E} = \mathbf{E} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{E} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

as a global check of the whole procedure. The detailed test may be performed by the inverse transformation $\mathbf{x} = \tau\mathbf{R}^T\mathbf{x_N}$ from the normal case to the real situation or analogous to (1.1.5)

$$x = -c\,\frac{\mathbf{i}.\mathbf{x_N}}{\mathbf{k}.\mathbf{x_N}} \quad \text{and} \quad y = -c\,\frac{\mathbf{j}.\mathbf{x_N}}{\mathbf{k}.\mathbf{x_N}}. \qquad (3.1.1)$$

These formulas will be needed also for the inevitable transformation of pixels from the normal to the original images in connexion with the interpolations of grey levels by resampling.
The search for homologous points (pixels) is to be executed now in P' along the epipolar line $\mathbf{h}'.\mathbf{x}'= 0$ with

$$\mathbf{h}' = \mathbf{C_N}\mathbf{x}'' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x'' \\ y'' \\ -c \end{bmatrix} = \begin{bmatrix} 0 \\ c \\ y'' \end{bmatrix},$$

i.e. $\quad cy' - cy'' = 0 \qquad (3.1.2)$

or $y'=y''=y$ and in P" along the epipolar line $\mathbf{h}''.\mathbf{x}''= 0$ with

$$\mathbf{h}'' = \mathbf{C_N}^T\mathbf{x}' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ -c \end{bmatrix} = -\begin{bmatrix} 0 \\ c \\ y' \end{bmatrix},$$

i.e. $\quad - cy'' + cy' = 0, \qquad (3.1.3)$

hence $y'=y''=y$ too. This implies that, of course, all homologous points are situated at identical parallel epipolar lines in the very same plane (*Haggren and Niini* 1990).

3.2 <u>Propagation of errors concerning transformation</u>

The influence of small variations onto (1.1.4) is implicitely given by

$$d\tau\mathbf{x_N} + \tau d\mathbf{x_N} = \tau d\mathbf{A}\mathbf{x_N} + \mathbf{R}d\mathbf{x}$$

or in scalar notation after regrouping

$$\tau(dx_N + y_N dK + c d\phi) = \mathbf{e}_1.d\mathbf{x} - x_N d\tau$$
$$\tau(dy_N - x_N dK - c d\Omega) = \mathbf{e}_2.d\mathbf{x} - y_N d\tau$$
$$\tau(x_N d\phi - y_N d\Omega) = \mathbf{e}_3.d\mathbf{x} + c d\tau.$$

$d\tau$ can be eliminated by the third equation and, considering $\tau=-\mathbf{e}_3.\mathbf{x}/c$ from the third component of (1.1.4), the differential relation

$$d\mathbf{x_N} = \mathbf{B_a}\mathbf{da} + \mathbf{B_x}d\mathbf{x}, \qquad (3.2.1)$$

with

$$\mathbf{B_a}\mathbf{da} = \frac{1}{c} \begin{bmatrix} x_N y_N & -c^2+x_N^2 & -y_N c \\ c^2+y_N^2 & -x_N y_N & x_N c \end{bmatrix} \begin{bmatrix} d\Omega \\ d\phi \\ dK \end{bmatrix}$$

and

$$\mathbf{B_x}d\mathbf{x} = \frac{1}{\mathbf{e}_3.\mathbf{x}} \begin{bmatrix} x_N i_3 + c i_1 & x_N j_3 + c j_1 \\ y_N i_3 + c i_2 & y_N j_3 + c j_2 \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix}$$

results, where $\mathbf{B_a}$ contains the well-known coefficients of small rotations and $\mathbf{B_x}$ indicates the influence of small coordinate shifts in the original image. If these differential movements are stochastic quantities, the uncertainty of $\mathbf{x_N}$ results from the expectation $\mathbf{S_N}=E\{d\mathbf{x_N}d\mathbf{x_N}^T\}$ (*Pelzer et al.* 1985) because of $E\{\mathbf{da}d\mathbf{x}^T\}=0$ (**da** and d**x** are independent!) as

$$\mathbf{S_N} = E\{(\mathbf{B_a}\mathbf{da})(\mathbf{B_a}\mathbf{da})^T\} + E\{(\mathbf{B_x}d\mathbf{x})(\mathbf{B_x}d\mathbf{x})^T\} =$$
$$= \mathbf{B_a}E\{\mathbf{da}\mathbf{da}^T\}\mathbf{B_a}^T + \mathbf{B_x}E\{d\mathbf{x}d\mathbf{x}^T\}\mathbf{B_x}^T =$$
$$= \mathbf{B_a}\mathbf{S_a}\mathbf{B_a}^T + \mathbf{B_x}\sigma^2\mathbf{E}\mathbf{B_x}^T =$$
$$= \sigma^2(\mathbf{B_a}\mathbf{Q_a}\mathbf{B_a}^T + \mathbf{B_x}\mathbf{B_x}^T). \qquad (3.2.2)$$

Assuming that the original images are very close to the normal case, i.e. $\mathbf{R} \approx \mathbf{E}$, the second part of (3.2.2) converts, because of $\mathbf{e}_3.\mathbf{x}=-c$ and

$$\mathbf{B_x}d\mathbf{x} = \frac{1}{c} \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} = \mathbf{E}d\mathbf{x},$$

to $\mathbf{B_x}\mathbf{B_x}^T=\mathbf{E}$. In this case, the uncertainties of the coordinate measurement add directly to the uncertainties from relative orientation and its fictitious weights (2.3.1) take the form

$$1/g = 2\,\sigma^2\,(\,y^2 + c^2\,) \qquad (3.2.3)$$

because of (3.1.2) and (3.1.3). It shows the fact that, in the normal case, the weights decrease strictly with y only. As weights do not influence very much results of adjustments, relation (3.2.3) could also be used for images which do not deviate to much from normal position.

3.3 Propagation of errors concerning reconstruction

After relative orientation and transformation to the normal case, the uncertainty of the model will depend on the dispersion $\mathbf{S_N}$ (3.3.2) of the image coordinates $\mathbf{x_N}$. Since small variations of $\mathbf{X}$ read (using the left image P')

$$d\mathbf{X} = d\lambda_N \mathbf{x_N}' + \lambda_N d\mathbf{x_N}', \qquad (3.3.1)$$

as derived from (1.3.3) by differentiation, the uncertainties result again from the expectation $\mathbf{S_M} = E\{d\mathbf{X}d\mathbf{X}^T\}$, i.e.

$$\mathbf{S_M} = E\{d\lambda_N^2\}\mathbf{x_N}'\mathbf{x_N}'^T + \lambda_N[\mathbf{x_N}'E\{d\lambda_N d\mathbf{x_N}'^T\} + E\{d\lambda_N d\mathbf{x_N}'\}\mathbf{x_N}'^T] + \lambda_N^2 E\{d\mathbf{x_N}'d\mathbf{x_N}'^T\}.$$

By means of the differential form

$$d\lambda_N = \frac{dx_N''-dx_N'}{(x_N''-x_N')^2} = \lambda_N^2(dx_N''-dx_N') \qquad (3.3.2)$$

of equ. (1.3.2) the expectations are:

$$E\{d\lambda_N^2\} = \lambda_N^4(\sigma_{x''x''}+\sigma_{x'x'}-2\sigma_{x'x''}),$$

$$E\{d\lambda_N d\mathbf{x_N}'^T\} = \lambda_N^2 \begin{bmatrix} \sigma_{x'x''}-\sigma_{x'x'} & \sigma_{x''y'}+\sigma_{x'y'} & 0 \end{bmatrix}$$

$$E\{d\lambda_N d\mathbf{x_N}'\} = \lambda_N^2 \begin{bmatrix} \sigma_{x'x''}-\sigma_{x'x'} \\ \sigma_{x''y'}-\sigma_{x'y'} \\ 0 \end{bmatrix},$$

$$E\{d\mathbf{x_N}'d\mathbf{x_N}'^T\} = \begin{bmatrix} \sigma_{x'x'} & \sigma_{x'y'} & 0 \\ \sigma_{x'y'} & \sigma_{y'y'} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and regarding $E\{d\mathbf{a}'d\mathbf{x}''^T\}=E\{d\mathbf{x}'d\mathbf{a}''^T\}=E\{d\mathbf{x}'d\mathbf{x}''^T\}=0$, the co-variances of the correlation P'-P" may be taken from $\mathbf{S_N}'' = E\{d\mathbf{x_N}'d\mathbf{x_N}''\}$, i.e.

$$\mathbf{S_N}'' = \begin{bmatrix} \sigma_{x'x''} & \sigma_{x'y''} \\ \sigma_{y'x''} & \sigma_{y'y''} \end{bmatrix} = \sigma^2\,\mathbf{B_a}'\mathbf{Q_a}''\mathbf{B_a}''^T,$$

$$\mathbf{Q_a}'' = \begin{bmatrix} \mathbf{0}^T \\ \mathbf{Q}_{12} \end{bmatrix}$$ and $\mathbf{Q}_{12}$ from (2.3.2).

Finally, there results the somewhat long but useful formula

for the uncertainty of a stereoscopically reconstructed model. It is seen that $\lambda_N=1/(x_N''-x_N')$ represents the dominating factor and that the first term of this relation will have the most important influence at the limits of accuracy. Thus quality control of stereophotogrammetric evaluation should focus mainly on this expression in order to avoid regions of insufficient precision.

4. NUMERICAL EXAMPEL

The following page contains a stereo pair (1,2) taken by a Rolleimetric 6006 (c=51.18) in general positions. These two images are to be correlated in order to get their relative orientation. The coordinates of the points of correlation are (in mm):

	P'≡1		P"≡2	
	x	y	x	y
1	-10.620	1.694	-1.851	2.316
2	8.308	0.808	14.936	1.613
3	-16.623	14.596	-7.583	13.604
3	17.472	13.804	22.767	17.806
5	-11.904	-1.314	-15.519	-0.481
6	14.764	-2.293	1.799	-1.931
7	-21.802	6.968	-18.058	6.299
8	-12.778	8.770	-4.346	8.746

Result of computational correlation:

$$\mathbf{Z} = \begin{bmatrix} -0.00391 & 0.26581 & 0.01067 \\ 0.28609 & 0.01536 & -0.99664 \\ -0.00645 & 1.00000 & -0.01313 \end{bmatrix},$$

det(Z) = -0.0001351 ≠ 0 because of neglecting the conditions of orthonormalization.

Provisional epipole in P':
$(x_0') = 192.457 \quad (y_0') = 1.476$

Approximate rotations of P':
$(\Phi') = -16.546 \quad (K') = -0.488$

Provisional epipole in P":
$(x_0'') = -178.264 \quad (y_0') = -0.569$

Approximate rotations of P":
$(\Phi'') = 17.799 \quad (K'') = -0.203$
$(\delta\Omega) = -0.868$

The rotations are given in grads.

Matrix of correlation from
$(\mathbf{Z}) = (1/c_{32})(\mathbf{R}')^T\mathbf{B}(\mathbf{R}'') =$

$$= \begin{bmatrix} -0.00404 & 0.26580 & 0.01111 \\ 0.28548 & 0.01706 & -0.99454 \\ -0.00696 & 1.00000 & -0.01310 \end{bmatrix}.$$

Error equations:

Nr.	dΦ'	dK'	dΩ	dΦ"	dK"	δp	g
1	4.81	-141.38	-2545.03	28.22	-829.52	0.0500	1.0
2	17.77	-1129.97	-2524.54	-0.18	11.25	0.0118	1.0
3	-36.4	132.58	-2731.04	314.63	-1147.66	-1.0271	1.1
4	511.63	-1679.62	-2736.65	-106.82	351.20	-1.1611	1.1
5	-1.71	-73.60	-2359.54	-35.51	-1525.32	-0.0885	1.0
6	-71.63	-1360.76	-2274.17	-29.84	-566.33	-0.1081	0.9
7	-45.25	348.39	-2479.64	224.47	-1731.91	-0.6591	1.0
8	7.05	-41.83	-2608.91	162.05	-963.52	-0.6622	1.0

$$\mathbf{S_M} = \begin{bmatrix} \sigma_{XX} & \sigma_{XY} & \sigma_{XZ} \\ & \sigma_{YY} & \sigma_{YZ} \\ \text{symm.} & & \sigma_{ZZ} \end{bmatrix} = \lambda_N^4(\sigma_{x''x''}+\sigma_{x'x'}-2\sigma_{x'x''}) \begin{bmatrix} x_N'^2 & x_N'y_N' & -x_N'c \\ & y_N'^2 & -y_N'c \\ \text{symmetric} & & c^2 \end{bmatrix} +$$

$$+\lambda_N^3 \begin{bmatrix} 2x_N'(\sigma_{x'x''}-\sigma_{x'x'}) & y_N'(\sigma_{x'x''}-\sigma_{x'x'})+x_N'(\sigma_{x''y'}-\sigma_{x'y'}) & -c(\sigma_{x'x''}-\sigma_{x'x'}) \\ & 2y_N'(\sigma_{x''y'}-\sigma_{x'y'}) & -c(\sigma_{x''y'}-\sigma_{x'y'}) \\ \text{symmetric} & & 0 \end{bmatrix} + \lambda_N^2 \begin{bmatrix} \sigma_{x'x'} & \sigma_{x'y'} & 0 \\ & \sigma_{y'y'} & 0 \\ \text{symmetric} & & 0 \end{bmatrix} \qquad (3.3.3)$$

Inverse matrix (units 1.10^{-6}):

$$Q = \begin{bmatrix} 8.484 & 0.902 & 1.076 & 4.334 & -1.887 \\ 0.902 & 1.768 & -0.871 & -4.126 & 1.276 \\ 1.076 & -0.871 & 0.521 & 2.128 & -0.812 \\ 4.334 & -4.126 & 2.128 & 20.392 & -2.079 \\ -1.887 & 1.276 & -0.812 & -2.079 & 1.517 \end{bmatrix}.$$

Standard error of adjustment: $s_0=\pm0.117$
Standard error of measured cordinates: $s=\pm1.6\mu m$
(fictitious weights $g\approx5000$)

Solutions:

$d\Phi'=-0.182 \pm .022$ $\quad dK'=0.025 \pm .010$ $\quad d\Omega=-0.010 \pm .005$ $\quad d\Phi''=-0.237 \pm .034$ $\quad dK''=0.023 \pm .009$

Definitive rotations:

$\Phi'=-16.728, K'=-0.463, \delta\Omega=-0.878, \Phi''=17.561, K''=-0.180$

Definitive matrix of rotation of P'

$$R' = \begin{bmatrix} 0.965449 & 0.007025 & -0.259756 \\ -0.007275 & 0.999974 & 0.000000 \\ 0.259749 & 0.001890 & 0.965674 \end{bmatrix}$$

containing the elements for the transformation of image 1.

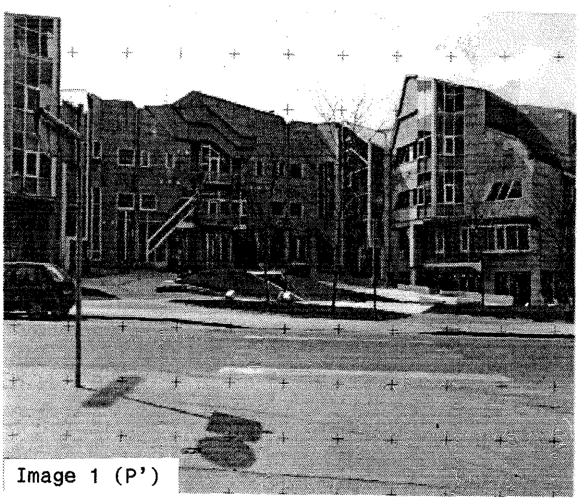
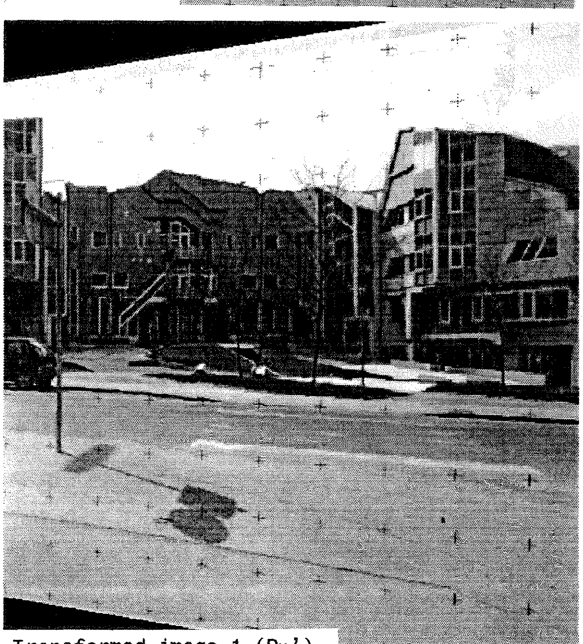


Image 1 (P')



Transformed image 1 (P_N')

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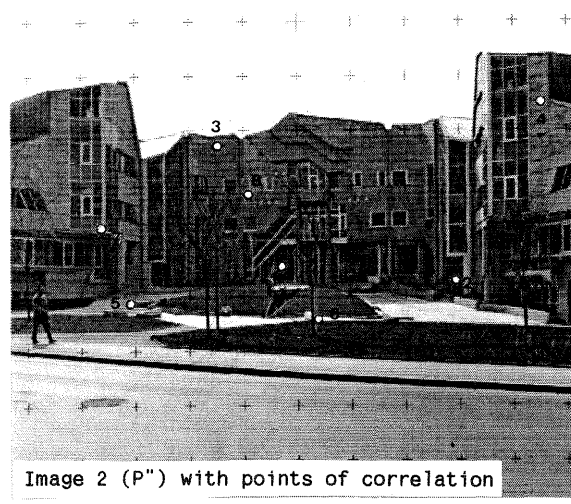
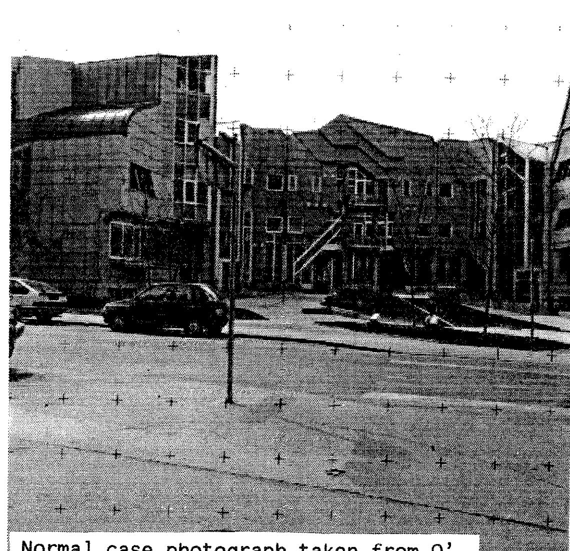


Image 2 (P") with points of correlation



Normal case photograph taken from O'