A Simple and Exact DEM Interpolation Procedure to Support Monoplotting

John Stuiver

Wageningen Agricultural University

Department of Landsurveying and Remote Sensing
and
Centre for Geographical Information Processing (CGI)

The Netherlands

Commission IV

ABSTRACT

Mapping from single photographs (monoplotting) requires height information for each measured point. Linking up height information from digital elevation models to measurements on aerial photographs always involves an interpolation procedure. The procedures most often used in practice are of an iterative nature. In the PC digitizing environment, this procedure makes it difficult to transfer measurements from the photograph immediately to geo-referenced coordinates, due to the mathematical relationships of an aerial photograph.
At the Wageningen Agricultural University, a simple and exact interpolation method has been developed to fit the on-line digitizing requirements and speed up batch coordinate transfer operations when working with aerial photographs.

INTRODUCTION

In the field of digital mapping when working from single aerial photographs (MONOPLOTTING), one has always had to deal with the problem of interpolating height information before getting a satisfactory result. At the Wageningen Agricultural University (WAU), monoplotting in a PC environment has been used in several research projects to get basemap material for these projects and to acquire primary data for GIS program packages.

Many typical photogrammetric steps in the monoplotting process are considered by non-photogrammetric users to be a problem. The main photogrammetric problems for these users are:
- getting ground control points for the photo block adjustment.
- carrying out and computing the photo block adjustment.
- measuring and computing a sufficiently accurate digital elevation model (DEM).
- interpolating the height from the DEM with sufficient accuracy in all terrain relief types.

At the Department of Landsurveying and Remote Sensing of the WAU programs have either been made or bought to assist the non-photogrammetric user in carrying out and solving the first three problems. However the last problem caused extra data handling, relatively long computation times and problems in areas with extreme terrain relief. In this paper the mathematical approach and some of the applications will be explained of the developed interpolation.

BASIC MATHEMATICAL RELATIONSHIPS

To understand this interpolation approach, knowledge of the principles of the relationship of a single aerial photograph and a given georeference is necessary.

When a photograph is made [5], a ray of light reflected by a terrain object P, goes through the atmosphere and the camera's lens system to hit the film at the image point p (see Fig. 1). To describe the relationship between terrain object P and the image point p, we assume they are connected by a straight line through the optical centre O of the camera's lens system [2,3]. This assumption is sufficient for most applications for which the monoplotting system has been designed.

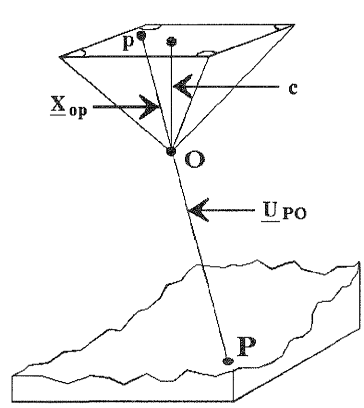

Fig. 1

Because of the assumption that the terrain object, the perspective centre and the image point are on one straight line, we find the following equations:

$$\underline{U}_{PO} = \underline{U}o - \underline{U}p = \begin{vmatrix} Uo-Up \\ Vo-Vp \\ Wo-Wp \end{vmatrix} \quad \text{(Georeference coordinate system)}$$

$$\underline{X}_{OP} = \underline{X}o - \underline{X}p = \begin{vmatrix} Xo-Xp \\ Yo-Yp \\ c \end{vmatrix} \quad \text{(Camera coordinate system)}$$

So: $\underline{U}_{PO} = \mu p \cdot R \cdot \underline{X}op$ *[1]*

μp = length factor
R = rotation matrix [3x3] (camera orientation)
C = cameraconstant

The length factor μp takes care of the difference between the light ray's path from P to O as expressed in $\underline{U}_{PO}$ and the light path from O to P as expressed in R·$\underline{X}$op.

The problem of monoplotting is finding the terrain coordinates U_P, V_P, W_P when the camera coordinates X_p, Y_p of the image points are known. It is clear from the derivation of relationship *[1]* that this is possible only if the position of the perspective centre is known in both the camera system and the georeference system. Furthermore, the camera orientation (R) and the length factor μp should be known.

The position of the perspective centre and the camera's orientation is easily computed by means of a numerical restitution. These parameters of the numerical restitution can only be found if at least three image points for which the vector $\underline{X}$op is known and their corresponding georeferenced coordinates (ground control points) in position and height are known.

When R and Uo, Vo, Wo are known, the problem of monoplotting is to find the coordinates Up, Vp, Wp for each point for which Xp and Yp are known. From equation *[1]* follows :

$$\begin{vmatrix} Up \\ Vp \\ Wp \end{vmatrix} = \begin{vmatrix} Uo \\ Vo \\ Wo \end{vmatrix} - \mu p \cdot R \begin{vmatrix} Xp\text{-}Xo \\ Yp\text{-}Yo \\ C \end{vmatrix} \quad [2]$$

For each measured image point there are three equations with four unknown quantities:
- the georeferenced coordinates of the image point (Up,Vp,Wp)
- the length factor μp.

Additional information is needed for monoplotting:
- the height Wp.

BASIC PRINCIPLES OF THE INTERPOLATION METHOD

When the first prototype of the monoplotting program package was developed in 1986 at the WAU, height information had to be introduced by means of the computer's keyboard for every measured point. This cumbersome operation did not give too many problems in flat or hilly terrain with medium and small scale photography. However it could not be denied that an automated procedure to find the height from a pre-determined DEM would reduce mistakes during measuring, speed up the whole monoplotting process and make this process more accessible to non-photogrammetric users. To achieve this, an interpolation procedure needed to be developed according to the mathematical equations of monoplotting.

As stated earlier, every vector goes through the perspective centre. One vector is orthogonally defined to the georeference of the ground control points. This vector is parallel to the height axis (W) and perpendicular to the plain defined by the U,V axis (see Fig 2).

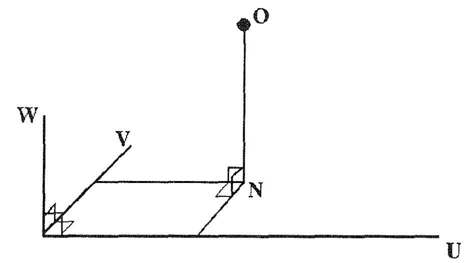

Fig.2

This means that a perpendicular plain to the U,V axis of the georeference can always be defined for every image point except for the image point which lies in the extension of vector ON. This image point is also referred to as the nadir point (see Fig. 3).

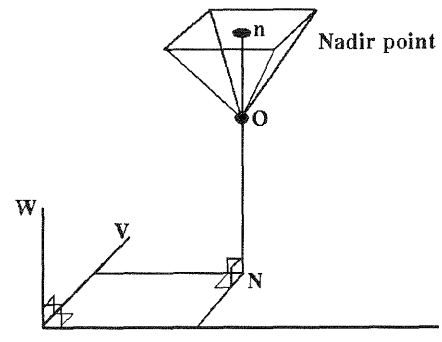

Fig. 3

Using the reference height (W=0) of the georeference's origin and the image point's X,Y coordinates, the U,V position of the image point can be determined by using equation *[2]*. This means that every image point except the nadir point defines a plain which pivots around the vector ON (see Fig. 4).

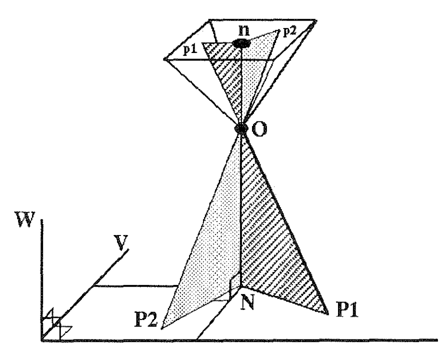

Fig. 4

The correct height needed to compute the corresponding U,V coordinates of an image point can be found on the vector OP. When introducing a DEM into the computation, the minimum and maximum heights of the DEM are used to speed up the computation because the correct height and position lie between the two points P_{max} and P_{min} on the vector OP (see Fig. 5).

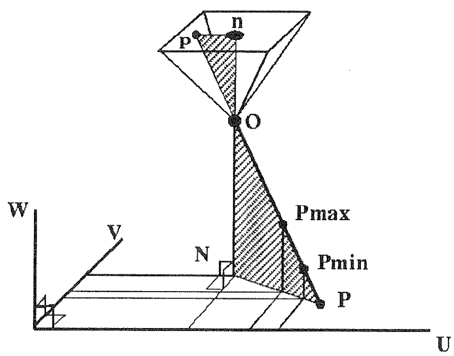

Fig. 5

The vector $P_{max}P_{min}$ also defines at the same time a cross-section along the surface of the DEM between the points N and P.(see Fig. 6).

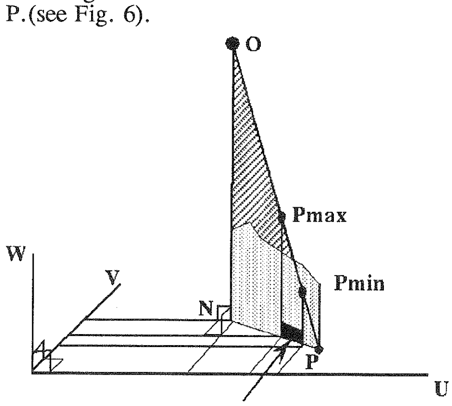

Fig. 6

The cross-section is built up of one or more vector segments. The correct U, V, W coordinates of a given image point are found simultaneously by intersecting the vector $P_{max}P_{min}$ with the vector segment of the cross-section where the U,V coordinates lie between the starting and finishing points of one of the given vector segments (see Fig. 7).

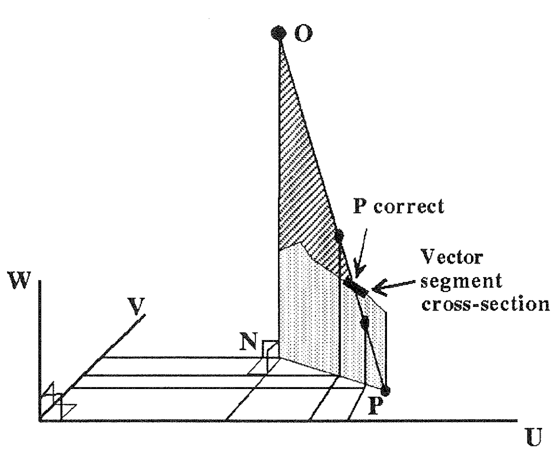

Fig. 7

The intersection of the vector $P_{max}P_{min}$ with the cross-section can however give more than one result as seen in Fig. 8.

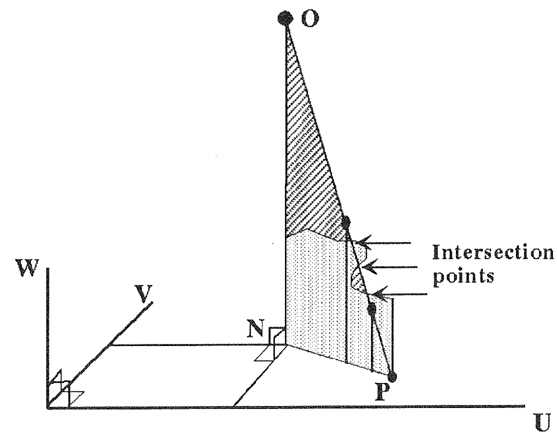

Fig. 8

Using the properties of the central projection of the aerial photograph, the computed point nearest to the perspective centre is the correct one. This point reflects the ray of light during exposure. The other points cannot be seen from this perspective centre. By ordering the vector segments of the cross-section according to their nearest distance over the surface of the DEM to the point N, the correct solution will always be found first (see Fig 9).

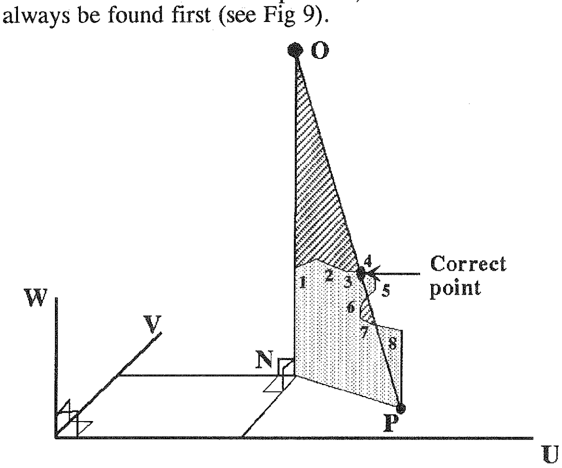

Fig. 9

CHECKS NEEDED BEFORE INTERPOLATION

There are three situations which need to be studied before the intersection can take place. These are :

- when the image point is the nadir point.
- when the vector $P_{max}P_{min}$ begins and/or ends within a triangle or gridcell of a DEM.
- when the selected vector segment of the cross-section coincides with the vector $P_{max}P_{min}$.

Although it is highly unlikely that the measured image point is the nadir point and the vector segment coincides with the vector $P_{max}P_{min}$ will occur, these situations are checked to allow the program to be as robust as possible.

The consistency of the result is also dependent upon the DEM type being used. A TIN DEM is preferred to a grid because the triangles that are formed define a flat surface. Every vector segment passing through a given U, V coordinate will always give the same height coordinate. However when using a grid DEM the four corner points defining a gridcell do not necessarily have to define a flat surface. In this case it is assumed when using a grid DEM that this influence is not significant [6]

As can be seen in Fig. 3, the nadir point's position (N) in U,V is already known. To be able to carry out the computation, at least one vector segment of a cross-section is needed. Therefore every cross-section's vector segment going through this point will give the height coordinate. A vector parallel to the U axis through the point N is used to define the direction of the cross-section (see Fig 10).

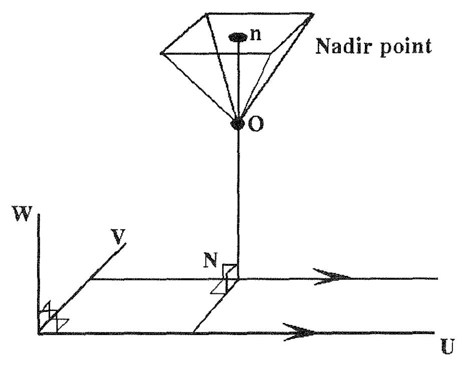

Fig. 10

By intersecting this vector with the first rib of the DEM's triangle or gridcell in positive and negative direction, the required vector segment can be found. (see Fig 11a and 11b).

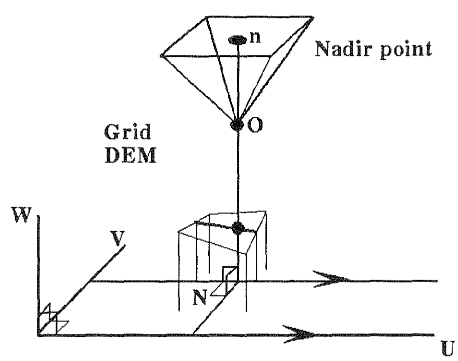

Fig. 11a

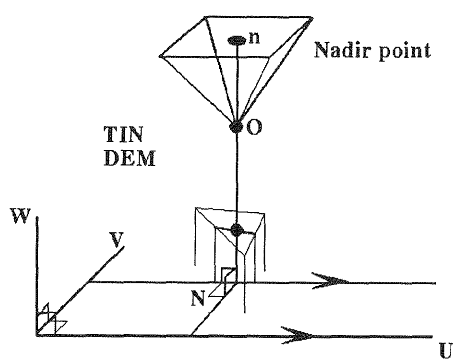

Fig 11b

The computation can now be carried out as previously described.

In the case when the vector $P_{max}P_{min}$ begins and/or ends within a triangle or gridcell of a DEM, these points must also must be extended until a rib of the DEM is intersected. This once again gives the required vector segment to start the interpolation procedure.

In the case of the vector $P_{max}P_{min}$ coinciding with the vector segment of the cross-section, an infinite number of solutions can be found between the points P_{max} and P_{min} (see Fig. 12).

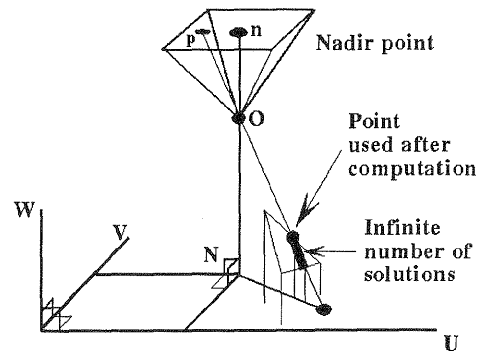

Fig. 12

The ray of light reflected by the terrain during exposure follows exactly the slope of the terrain's surface. In this case the starting point of the vector segment is considered to be the correct point because it is the nearest to the perspective centre.

FINAL REMARKS

A procedure to superimpose georeferenced points only known in U,V has also been developed using the same mathematical principles. The direction of the cross-section needed is determined by the U, V coordinates of the new perspective centre (Uo, Vo). The cross-section begins at this point and ends at the given point. The cross-section must be extended at both ends to find an intersection point with one of the ribs of the DEM. Thereafter height can know be easily interpolated. By using the inverted equation of *[2]*, it is then possible to compute the image coordinates.

The computation time is dependent upon the number of segments of the cross-section used. Naturally in mountainous terrain the computation time will be longer than in flat or hilly terrain types. Tests have proven this interpolation is fast enough to be used during digitizing. However this procedure was implemented in a separate data transfer program instead of integrating it into a digitizing procedure. The reason for this was to not disturb the existing digitizing know-how built up by the people measuring from photographs and maps. By making a seperate data transfer program a wider group of people could be assisted within the existing research projects.

The interpolation procedure is applicable in both a TIN and grid DEM. Extracting a cross-section out of both DEM types are wellknown procedures. The rest of the interpolation procedure remains the same.
Once the image point has been transferred to P_{max} and P_{min}, the rest of the procedure takes place within the georeference coordinate system.

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REFERENCES

1. Albertz, J and W Kreiing. 1980 Photogrammetric Guide. Herbert Wichmann Verslag, Karlsruhe.
2. ASP, 1980, Manual of Photogrammetry, C G Slama (ed.). American Society of Photogrammetry, Falls Church.
3. Makarovic, B, 1973. Digital monoplotters. ITC Journal 1973-4.
5. Molenaar, M and H J Stuiver. 1987. A PC Digital Monoplotting System for Map Updating. ITC Journal 1987-4.
6. Radwan, M M and B Makarovic. 1980. Digital monoplotting system; improvements and tests. ITC Journal 1980-3.