THE OPTIMUM PHOTOTHEODOLITE POSITIONS THAT MAXIMIZE THE ACCURACY OF THE OBJECT POINTS

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Abstract

Ground points are necessary in close range photogrammetry , especially for camera calibration and for orientation of the object stereomodel.New mathematical formulae for estimating the required accuracy of object point co - ordinates achieved from using a phototheodolite were developed . The optimum phototheodolite positions that maximize the accuracy of these object points were obtained . A comparative study involving the phototheodolite , the camera and the theodolite was undertaken in order to achieve the required accuracy of the object points.

A set of analysis of results, concluding remarks and recommendations was achieved.

1. INTRODUCTION

The phototheodolite is a terrestrial survey instrument combining a camera and theodolite in which the relationship between the camera axis and the line of collimation of the theodolite can be measured . The accuracy achieved from using a phototheodolite is equal to the accuracy achieved when we used a camera and theodolite together at the same stations and with the same co-ordinate system . To achieve the best accuracy a combination of the formulae developed by the author in 1989 and the formulae developed by Abdel-Aziz (1982) for estimating the optimum theodolite positions that maximize the accuracy of the object points has been used to obtain the optimum positions of the phototheodolite in two cases of photography : (1) normal, and (2) convergent .

The total mean accuracy of the target co-ordinates achieved from both the camera and the theodolite together (phototheodolite) can be obtained as follows :

$$\sigma_{Xmt}^2 = \sigma_{Xmc}^2 + \sigma_{Xmth}^2 \qquad \text{..............(1)}$$

$$\sigma_{Ymt}^2 = \sigma_{Ymc}^2 + \sigma_{Ymth}^2 \qquad \text{...............(2)}$$

$$\sigma_{Dmt}^2 = \sigma_{Dmc}^2 + \sigma_{Dmth}^2 \qquad \text{...............(3)}$$

$$\sigma_{Rmt}^2 = \sigma_{Rmc}^2 + \sigma_{Rmth}^2 \qquad \text{...............(4)}$$

where :

$\sigma_{Xmc}, \sigma_{Ymc}, \sigma_{Dmc}$: the mean accuracy of the object points obtained from the camera .

$\sigma_{Xmth}, \sigma_{Ymth}, \sigma_{Dmth}$: the mean accuracy of object points obtained from the theodolite .

$\sigma_{Xmt}, \sigma_{Ymt}, \sigma_{Dmt}$: the mean accuracy obtained from both camera and theodolite together (phototheodolite).

σ_{Rmt}: the positional error of the camera and theodolite together

2. NORMAL CASE OF PHOTOGRAPHY

From the equations developed by Abdel - Aziz in 1982 and by the author in 1989, we can obtain

$$\sigma_{Xmt}^2 = (\sigma_{th}^2 / B^2 D^2).\,[\,W^6/224 + (8D^2 - B^2)W^4/160 - (2B^2D^2/3 + B^4/24)W^2/4 + B^6/32 + B^4D^2/4\,] + (\sigma_{ph}^2/f^2 + \sigma_{th}^2).(D^2W^2/6B^2 + D^2/2) \qquad \text{.........(5)}$$

$$\sigma_{Ymt}^2 = (((H^2 - 3HE_{th} + 3E_{th}^2).\sigma_{th}^2)/3B^2D^2).[W^4/40 + (4D^2/3 + B^2/6).W^2/4] + \sigma_{th}^2[W^2/24 + B^2/8 + (H^2 - 3HE_{th} + E_{th}^2)/3 + (((H - E_{th})^5 + E_{th}^5)/10WDH).(\tan^{-1}((0.5W+0.5B)/D) - \tan^{-1}((-0.5W+0.5B)/D))] + (D^2/2).(\sigma_{ph}^2/f^2 + \sigma_{th}^2) + (2D^2/3B^2).[\sigma_{ph}^2H^2/f^2 + \sigma_{th}^2(H^2 - 3HE_{th} + E_{th}^2] \qquad \text{..............................(6)}$$

$$\sigma_{Dmt}^2 = (\sigma_{th}^2/B^2).[\,W^4/40 + W^2(B^2 + 4D^2/3)/4 + (B^4/8 + B^2D^2 + 2D^4)] + 2\sigma_{ph}^2 D^4 / D^2F^2(1-T)^2 \qquad \text{.............................(7)}$$

where :

$\sigma_{Xmt}, \sigma_{Ymt}, \sigma_{Dmt}$: the mean accuracy obtained from the phototheodolite

f :the principal distance of the camera

F : the format size

T : the overlap ratio

B : the base distance

D : the object distance

W	: the object width
H	: the object height
E_{th}	: the theodolite elevation
σ_{th}	: the theodolite accuracy
σ_{ph}	:the accuracy in measuring the image co-ordinates

The optimum theodolite positions as given by Abdel-Aziz (1982) are as follows :
the optimum base distance ($\acute{B}$)= 1.4L or 0.7W ;
the optimum object distance ($\acute{D}$) =0.26L or 0.13W ;
the optimum theodolite elevation ($\acute{E}_{th}$) =0.5H
The optimum camera positions may be achieved when the object distance (D) is a minimum and the base distance (B) is a maximum .

In the normal case of photography B and D are related together by this formula
B=D.F.T/f.......................................(8)
where:
B: the base distance
D: the object distance
F: the format size
T: the overlap ratio
f: the principal distance

The optimum base distance $\acute{B}$ and the optimum object distance $\acute{D}$ are chosen to minimize the value of σ_{Rmt} (equation 4) .Different values of D are assumed and the corresponding values of B are calculated from equation (8) . The optimum values of B and D which give the minimum value of σ_{Rmt} are $\acute{B}$=0.31W and $\acute{D}$=0.35W .

The theodolite elevation E_{th} affects only the value σ_{Ymt} (equation 6) , and it has no effect on the values of σ_{Xmt} and σ_{Dmt} .

The optimum theodolite elevation $\acute{E}_{th}$ is chosen to give the minimum value of σ_{Ymt} (i.e $\partial\sigma_{Ymt}/E_{th}=0$)

$$\partial\sigma_{Ymt}/E_{th} =[W^4/40 + (4D^2/3 + B^2/6).(W^2/4) + 2D^2((2E_{th}-H)/ B^2D^2))] + (2E_{th}-H) + [((H-E_{th})^4-E_{th}^4)/2HDW] [\tan^{-1}(0.5W+0.5B)/D . \tan^{-1}(-0.5W + 0.5B)/D] =0 \quad \text{......(9)}$$

From equation (9) the estimated value of $\acute{E}_{th}$ =0.50 H

2.1 Analysis of Results

1 the expected mean accuracy of all the object points (σ_{Xmt},σ_{Ymt},σ_{Dmt}) for any phototheodolite positions can be obtained from equations (5,6 and 7) .

2 the accuracy of object points obtained either from a camera mounted on a theodolite (phototheodolite) or from a special case of photography (i.e when we used the same stations and the same co-ordinate system for the camera and theodolite together) can be maximized in the normal case of photography if the base distance (B) is taken as 0.31W , the object distance (D) is taken as 0.35W and the theodolite elevation E_{th} is taken as 0.5H .

3 the accuracy of the object points is a non-linear function of the object distance (D) and theodolite elevation (E_{th}) .

3. CONVERGENT CASE OF PHOTOGRAPHY

From the equations developed by Abdel - Aziz in 1982 and by the author in 1989, we can obtain

$$\sigma_{Xmt}^2 = (\sigma_{ph}^2/D^2f^2) [(W^4/10 + W^2B^2 + B^4/2).\sin^4\phi/8 + (W^2+ B^2) . B . D . \sin^3\phi . \cos\phi/2 + D^2 . \sin^2\phi . \cos^2\phi (W^2+3B^2)/2 + 2 . B . D^3 . \cos^3\phi.\sin\phi + (D^4.W^2/6.B^2+D^4/2).\cos^4\phi] + (\sigma_{th}^2/B^2D^2).[W^6/224 + W^4(8D^2-B^2)/160 + W^2(2D^4/3-2B^2D^2/3-B^4/24)/4 + B^6/32 + B^4D^2/4 + D^4B^2/2] \quad \text{......(10)}$$

$$\sigma_{Ymt}^2 = (\sigma_{ph}^2/D^2f^2).[D^2W^2\sin\phi^2/12 + D^2B^2\sin\phi^2/4 + H^2W^2\sin\phi^4/36 + H^2B^2\sin\phi^4/12 + D^3 B \sin\phi \cos\phi + H^2 D B \sin^3 0.\cos\phi/3+D^2.H^2.\sin\phi^2.\cos\phi^2/3+(2.D^4.H^2/3.B^2 +D^4/2).\cos\phi^2]+(\sigma_{th}^2(H^2-3HE_{th}+E_{th}^2)/3B^2D^2) [W^4/40 + W^2(4D^2/3+B^2/6)/4 + 2D^4)] + \sigma_{th}^2 [W^2/24 + B^2/8 + D^2/2 + (H^2-3HE_{th}+3E_{th}^2)/3 + (((H-E_{th})^5+E_{th}^5)/ 10WDH).(\tan^{-1}((0.5W+0.5B)/D - \tan^{-1}((-0.5W+0.5B)/D))] \quad \text{......(11)}$$

$$\sigma_{Dmt}^2 = (2 \sigma_{ph}^2/B^2f^2(1+\tan\varnothing^2)^2).[D^4 + 2D^3 B \tan\varnothing + \tan\varnothing^2 (D^2W^2/2+3D^2B^2/2) +\tan\varnothing^3(D W^2 B/2 + DB^3/2) + \tan\varnothing^4(W^4/80 + W^2B^2/8 + B^4/16)] + (\sigma_{th}^2/B^2).[W^4/40 + (B^2 + 4D^2/3).(W^2/4) + (B^4 /8 + B^2D^2 + 2D^4)] \quad \text{......(12)}$$

The optimum base distance ´B and the optimum object distance ´D at different values of convergence angle (Ø) are chosen to minimize the values of σ_{Xmt} and σ_{Dmt} for

$\sigma_{Xmt}=\sigma_{Dmt}$ (i.e $\sigma_{Xmt}^2 - \sigma_{Dmt}^2 = 0$)......(13)

Putting $\sigma_{Xmt}^2 - \sigma_{Dmt}^2 = 0$, we obtain one equation with two unknowns , B and D . The values of ´B and ´D at each value of convergence angle (Ø) are the values which satisfy the condition $\sigma_{Xmt} = \sigma_{Dmt}$ at the minimum values of σ_{Xmt} or σ_{Dmt} .

Table 1. gives the values of the convergence angle (Ø) in column 1 , the base distance (B) in column 2 , the corresponding object distance (D) which satisfies equation (13) in column 3 and the expected value σ_{Dmt} in column 4 .

Table 2. gives the values of the convergence angle (Ø) in column 1 , the optimum base distance (´B) in column 2 , the corresponding optimum object distance (´D) in column 3 , and the optimum theodolite elevation ($\acute{E}_{th}$) which satisfies equation (9) in column 4 .
Figure 1. gives the ratios of (´D/W) , (´B/W) and ($\acute{E}_{th}$/H) against the value of convergence(Ø) . Having the values of the object width (W) and the object height (H) , we can estimate the optimum object distance (´D) , the optimum base distance (´B) and the optimum theodolite elevation ($\acute{E}_{th}$) required to achieve the best accuracy in the case of using a phototheodolite at different values of camera convergence angle (Ø) .

3.1 Comments on Results

1 The expected mean accuracy of all the object points (σ_{Xmt},σ_{Ymt},σ_{Dmt}) for any phototheodolite positions can be obtained from equations (10,11 and 12) respectively .
2 The accuracy can be maximized if B , D and E_{th} are taken according to each value of convergence angle (Ø) as mentioned in Table 2 or as shown in Figure 1.
3 The accuracy of object points is a non-linear function of the base distance(B) , the object distance (D) , the convergence (Ø) and the theodolite elevation (E_{th}) .

4. A COMPARISON BETWEEN , A PHOTOTHEODOLITE,A CAMERA AND A THEODOLITE

In our comparison we chose a Wild P32 metric camera mounted on a T2 one second theodolite as a phototheodolite,a Wild P32 metric camera and a Wild T2 theodolite . The comparison was between the mean positional error achieved from the theodolite (σ_{Rmth}), the camera (σ_{Rmc}) and the phototheodolite (σ_{Rmt})at different values of object and base distances .

4.1 Normal Case of Photography

In the normal case of photography the base distance (B) in both the camera and phototheodolite depends on the object distance (D) , but in the theodolite it does not .

By assuming different values of D we can calculate the corresponding values of B from equation 8 . According to the values of B and D and by applying the equation developed by the author, we can obtain the mean positional error for the camera and the phototheodolite . In the case of the theodolite we can assume different values of B for each value of D, and by applying equation developed by Abdel - Aziz we can obtain the mean positional error of the theodolite .

4.1.1 Comments and Analysis of Results

From the results achieved it is clear that:
1 in descending order ,the best accuracy of object points was obtained from the theodolite, then from the camera and lastly from the phototheodolite .
2 above the ratio D/W=0.3 the accuracy achieved from a theodolite is about 56% better than that obtained by camera and is about 62% better than that obtained by phototheodolite .
3 above a certain ratio (D/W) the positional accuracy achieved from a camera mounted on a theodolite (phototheodolite) becomes nearly equal to the positional accuracy achieved from both a camera and theodolite separately .

4.2 Convergent Case of Photography

In this case the choice of B and D is independent , so we can assume different values of B for each value of D and by applying equation 4 we can obtain the best accuracy achieved at this object distance (D).

4.2.1 Comments and analysis of results

From the results obtained, we can see that :
1 when (B) is a constant and (D) variable or when (B) is variable and (D) constant the best positional accuracy achieved was obtained from a theodolite ,secondly from a camera and lastly from a phototheodolite .
2 at certain ratios for (D/W) and (B/W),the accuracy achieved from a camera mounted on a theodolite(phototheodolite) is nearly equal to the accuracy achieved from both the camera and theodolite separately .

5. OBSERVATIONS,CONCLUDING REMARKS AND RECOMMENDATIONS

1 With the developed formulae relating the object space co-ordinate standard errors to the system parameters (B,D,Ø,W and H) , it has been illustrated that the standard errors obtained from a given close range photogrammetric system can be sufficiently estimated . It is , therefore , possible to arrive at a decision as to whether , using a given system , a certain required accuracy is achievable or not . But more importantly , the formulae provide a convenient tool with which we can choose the parameters and ,in turn , the components , of a system which will yield the required accuracy for a proposed project .
2 This study confirms that the highest accuracy of object points for close range photogrammetry could be obtained by using a theodolite.
3 Above certain ratios for (D/W) and (B/W) in the normal and convergent cases of photography the accuracy achieved from using a phototheodolite will be nearly equal to the accuracy achieved if we used the camera and the theodolite separately. In this study since the smallest difference in the accuracy achieved between the phototheodlite and the camera theodolite combination was 0.6 um in the normal case of photography and was o.10 um in the convergent case of photography (Ø = 30°) , in simple tasks when high accuracy is not required and the time is significant , the use of the phototheodolite is more satisfactory.
4 A more further study between the theodolite , the camera and the phototheodolite will be needed with different types of theodolites , cameras and phototheodolites and for different configurations.

References

Abdel - Aziz , Y.I, 1982. The accuracy of control points for close range Photogrammetry. International Archieves of photogrammetry , 24(5) : 1 - 11.

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TABLE 1

The base distance B , the corresponding object distance D that satisfies $\sigma Xmt = \sigma Dmt$ and the values of σDmt at different values of convergence angle Ø .

Convergence Angle (Ø) 1	Base Distance B (m) 2	Object Distance D (m) 3	σDmT (mm) 4
5.0	0.1W	0.295W	0.0200
	0.2W	0.288W	0.0100
	0.3W	0.110W	0.0046
	0.4W	0.121W	0.0039
	0.5W	0.223W	0.0042
	0.6W	0.286W	0.0045
	0.7W	0.328W	0.0047
	0.8W	0.363W	0.0048
	0.9W	0.335W	0.0044
	1.0W	0.290W	0.0039
	1.1W	0.247W	0.0037
	1.2W	0.174W	0.0035
	1.3W	**0.122W**	**0.0034**
	1.4W	0.156W	0.0036
	1.5W	0.178W	0.0038
10.0	0.1W	0.289W	0.0200
	0.2W	0.281W	0.0102
	0.3W	0.271W	0.0070
	0.4W	0.256W	0.0054
	0.5W	0.201W	0.0041
	0.6W	0.266W	0.0045
	0.7W	0.306W	0.0047
	0.8W	0.251W	0.0040
	0.9W	0.225W	0.0037
	1.0W	0.196W	0.0036

TABLE 1 (Cntinued)

Convergence Angle (Ø) 1	Base Distance (B) (m) 2	Object Distance (D) (m) 3	σDmT (mm) 4
10.0	**1.1W**	**0.155W**	**0.0034**
	1.2W	0.735W	0.0101
	1.3W	0.805W	0.0110
	1.4W	0.875W	0.0120
	1.5W	0.104W	0.0037
20.0	0.1W	0.273W	0.0200
	0.2W	0.263W	0.0105
	0.3W	0.259W	0.0076
	0.4W	0.265W	0.0064
	0.5W	**0.293W**	**0.0061**
	0.6W	0.347W	0.0065
	0.7W	0.414W	0.0073
	0.8W	0.485W	0.0082
	0.9W	0.556W	0.0091
	1.0W	0.626W	0.0101
	1.1W	0.696W	0.0111
	1.2W	0.766W	0.0122
	1.3W	0.835W	0.0131
	1.4W	0.904W	0.0142
	1.5W	0.973W	0.0152
30.0	0.1W	0.254W	0.02080
	0.2W	0.245W	0.01110
	0.3W	0.248W	0.00830
	0.4W	**0.271W**	**0.00740**
	0.5W	0.314W	0.00741
	0.6W	0.372W	0.00790
	0.7W	0.437W	0.00860
	0.8W	0.505W	0.00950
	0.9W	0.574W	0.01050

TABLE 1 (Continued)

Convergence Angle (Ø) 1	Base Distance (B) (m) 2	Object Distance (D) (m) 3	σDmT (mm) 4
30.0	1.0W	0.643W	0.01150
	1.1W	0.712W	0.01250
	1.2W	0.781W	0.01350
	1.3W	0.849W	0.01460
	1.4W	0.918W	0.01560
	1.5W	0.986W	0.01670
40.0	0.1W	0.238W	0.0214
	0.2W	0.230W	0.0117
	0.3W	0.241W	0.0091
	0.4W	**0.274W**	**0.0083**
	0.5W	0.324W	0.0084
	0.6W	0.383W	0.0089
	0.7W	0.448W	0.0096
	0.8W	0.514W	0.0104
	0.9W	0.582W	0.0113
	1.0W	0.650W	0.0122
	1.1W	0.718W	0.0132
	1.2W	0.786W	0.0142
	1.3W	0.855W	0.0152
	1.4W	0.923W	0.0162
	1.5W	0.991W	0.0173

TABLE 2

The optimum base distance 'B , the optimum object distance 'D and the optimum theodolite elevation Éth at different values of convergence angle Ø

Convergence Angle (Ø) (Degrees) 1	Optimum Base Distance 'B (m) 2	Optimum Object Distance 'D (m) 3	Optimum Theodolite Elevation (Éth) (m) 4
5.0	1.3W	0.122W	0.50H
10.0	1.1W	0.155W	0.51H
20.0	0.5W	0.293W	0.51H
30.0	0.4W	0.271W	0.50H
40.0	0.4W	0.274W	0.50H

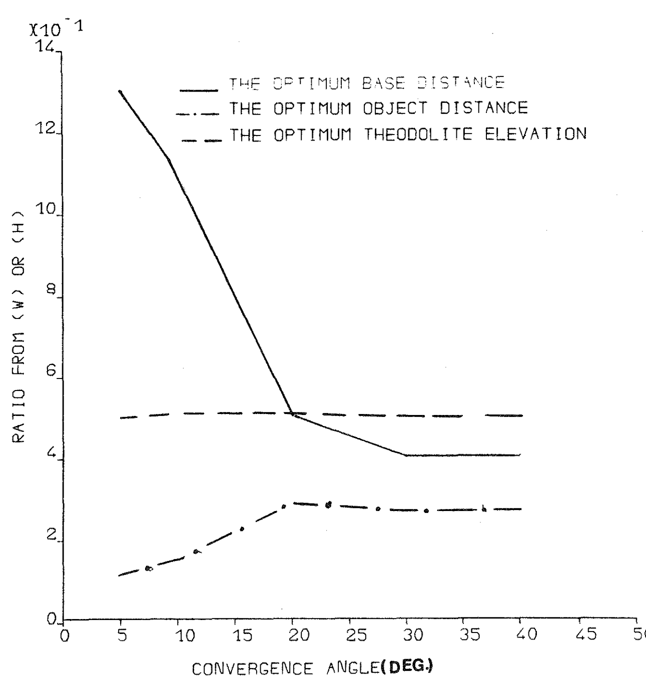

FIG. 1. The curves give the expected ratios É/H , 'B/W and 'D/W at convergence angle (Ø) for an object of width (W) and height (H).