THE "AERIAL" TRIANGULATION AND THE PLOTTING OF THE EXTERNAL WALLS OF THE COLISEUM IN ROME

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ABSTRACT

A particular procedure of "aerial" triangulation on the external walls of the Coliseum is necessary to insert in an unique reference system their plotting.

A paper on this subject was presented at the 1989 CIPA Symposium in Rome, but it was not published, due to the inactivity of the CIPA organisation. The scientific and operative setup of this aerial triangulation, based on the projective bundles approach, is now reported here; it presents some interesting peculiarity, due to the variable elliptic curvature of the wall. Its results, characterized by a sub-centimetric accuracy, are also reported.

The procedure is then described to plot the whole wall on an unique reference plane, using a dedicated cylindric reference system. A table with a consistent portion of the plotting is finally annexed.

The present research is a part of the **PROGETTO COLOSSEO** of the University of Rome.

KEY WORDS: **Coliseum, Aerial Triangulation, Plotting.**

1. - Foreword

In a paper presented at the XVI Congress of the ISPRS in Kyoto [Birardi et a., 1988], the general setup of the photogrammetric survey of the Coliseum in Rome was introduced. We stated what follows:

1.1 - The plants of the external Northern wall E1, and of the first internal Southern wall E2 are perfect ellipses (see Table 1).

This assumption was based on the graphical measurement of a number of points on the external borders of the walls, taken from a good pre-existing aerophotogrammetric map, scale 1:500. With the program ELLIPSE, based on "best fitting" techiques, we obtained hence reliable values of the parameters of the said ellipses, and of the adjustment residuals.

Nevertheless, for a further confirmation of this assumption we have recently re-computed the best fitting ellipses using a good number of control points, determined on the walls by conventional terrain procedures. We report here below the comparison between the two sets of parameters (schedule 1).

The little differences between the parameters of E1 and E2 mainly depend on the fact that the map measurements were done on the external border of the higher cornices, while the terrain measurements are effected on control points however placed on the walls.

Furthermore, the mean residual of the terrain E2 ellipse are almost halved as to the E1; in fact the surface of the E2 wall is devoid of capitals, cornices etc., while the E1 wall has plenty of them, on which the control points were located.

What above fully confirms the elliptic hypothesis. We hope that with this the *vexata quaestio* of the form of the Coliseum's plant is finally defined.

1.2 - To get a plane representation of the whole wall on an unique cartographic plane, it is opportune to use a special

Schedule 1

Ellipse	Adj. pts (nr.)	Maj.semiax A (m)	Min.semiax B (m)	Excentr. e^2	Perim. L (m)	Mean adj. residual (m)
E1 (map)	27	95.505	78.910	0.317	549.23	0.30
" (terr.)	71	94.631	77.545	0.329	542.14	0.30
E2 (map)	34	80.681	63.844	0.374	455.53	0.38
" (terr.)	45	80.162	63.258	0.377	452.16	0.17

reference (**shq**), which we called "mean cylindric reference", or shortly "Coliseum reference" (fig. 1).

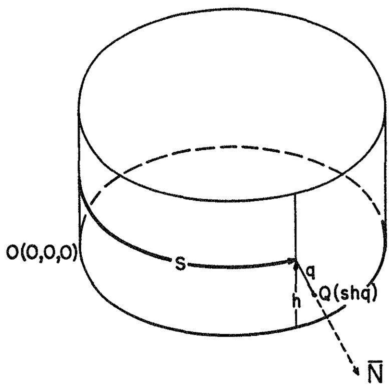


fig. 1

The reference mean cylinder is an elliptic right vertical cylinder, whose plant is the first or the second of the above said ellipses E1 and E2.

The transformation of the local cadastrial coordinates (**ENH**) into Coliseum coordinates (**shq**) is fully described in the Kyoto paper; we will not bring here the subject again.

1.3 - We obtained a very good topographic frame for the whole monument, with a local surrounding triangulation net.

This fundamental net consists of 8 points, placed on sound pillars with autocentering devices; the points were determined with millimetric accuracy by conventional procedures (Table 1, Annex 1).

Some other restricted densification nets, and the control points for the "aerial" triangulation on the walls, were derived frm this net. A sub-centimetric accuracy was obtained everywhere.

The operations for the nets, and their results are described in the Kyoto paper. We shall see later on, in the aerial triangulation adjustment (para 2.3), a reliable check of the controls' accuracy.

1.4 - For the photogrammetric takings of the walls two strips were planned, a lower one with horizontal photographic axis, and an upper one with inclined axis. We hoped to carry out horizontal takings of the higher strip from an elevator carriage, and this we did at first; but unfortunately the carriage was no longer available, so that in the Northern upper strip we had only inclined takings.

The taking plan, carefully studied by Prof.Carlucci, provided large longitudinal and trasversal overlaps, in order to obtain double alternate strips and a strong aerial triangulation block. The takings were effected at a mean distance of 30 m, with a P31 Wild camera, f= 100 mm, autodiapositive colour film, photographic scale 1:300. Each taking was repeated twice, in the same position and attitude.

2. - The "aerial" triangulation

2.1 - Considering the big number of photogrammetric pairs necessary to obtain the complete coverage of the walls (about 150), we deemed it opportune to determine their control by a particular kind of "aerial" triangulation. I.e., an aerial triangulation in which the strips are the sets of terrestrial consecutive takings around the monument, as if they were taken by an aircraft who flew around it.

So, both in E1 as in E2 we have a block of two strips. Each block has about 80 photograms, with large overlaps and a large number of well determined control points. The fig. 2 shows their distribution on the Northern E1 wall; in E2 it is repeated almost with the same standard.

2.2 - At the beginnings we tried to operate by independent models, taking into account the variations of the **x** distances due to the elliptic curvature of the reference surface. But we were soon convinced that this was not a good approach, and we fell back on the projective bundle solution. In fact this is the only procedure which permits to adjust the E1 or E2 chainings in an unique block, as it does not require an unique reference plane, and accepts however inclined photograms.

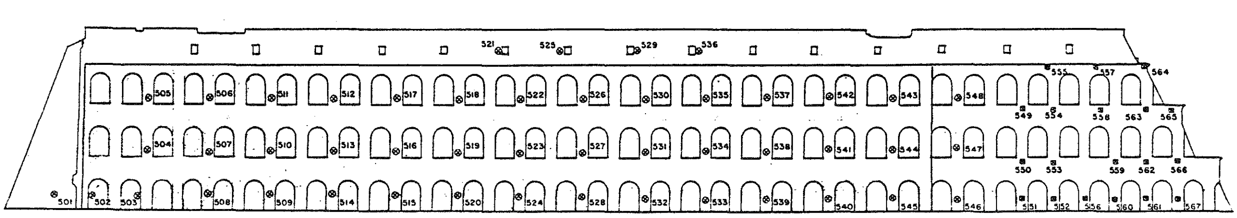


fig. 2

To get an unique block it is opportune to leave the cadastral (**EN**) coordinates and to go onto canonic (**XY**) coordinates, referred to the axes of the mean ellipses. The height over the ground remains as the third **Z** coordinate. A dedicated computation program easily performs this transformation, as the 5 parameters of the ellipses are known; and gives the new **XY** coordinates in the format needed for the block adjustment with bundles.

2.3 - For the chaining observations we used an analytical Kern DSR 11/H plotter. No correction for systematic errors was introduced. The phocal length (Wild P31 camera) is 99.66 cm; the image size 12 x 86 cm.

The adjustment was performed with the excellent program PATB-PC by Ackermann-Klein [1988], with autocalibration and automatic gross error detection. For the E1 and E2 block computations, 25 resp. 14 iterations were effected. The result was superior to any expectation, as it appears from the here below schedule 2.

The Annex 2 reports the results obtained for the control points. We must point out their exceptional quality, which shows the perfection of all the terrain and plant operations (fundamental net, its densification, intersections on the controls, instrumental observations, adjustment computations). The final accuracy of the controls assures a centimetric accuracy in the plotting.

3. - The plotting

3.1 - The above said canonic (**XY**) coordinates cannot be used for the plotting, except for the pairs which correspond to the vertices MN of the mean ellipse (fig. 3). In fact, we need that everywhere the instrumental **Y** axis be normal to the ellipse, so that the Y coordinate of a generic point P permits to deduce its distance to the mean ellipse, i.e. its altitude **q** in the Coliseum reference (**shq**).Therefore we must give a rotation **θ** to the canonic coordinates of each pair, before plotting it; we obtain thus the "normal" coordinates (**X'Y'**).

We can assume a mean **θ** between the nadiral points of the two photos of the pair. This is not quite rigorous, as we should assume a different **θ** for each point P; but, as we shall see later, the normal (**X'Y'**) coordinates are only transition coordinates to arrive to the Coliseum coordinates, and a little variation of **θ** has no influence on the final result. In fact, the purpose of the transition (**XY**) -> (**X'Y'**) is only to obtain that the Y instrumental mouvement be quasi-normal to the monument's surface in the whole pair.

A dedicated program TRASFH1 computes the **θ** rotation for the central point of the pair, and effects the transformation from the canonic onto the normal coordinates. The normal coordinates thus obtained for the control points are used for the absolute orientation of the pair; its whole plotting results therefore expressed in its particular normal system. The observation is done in the usual way, and does not require special modalities.

3.2 - The last step is to transform the normal coordinates into Coliseum coordinates, i.e. to get the transition (**X'Y'Z'**) -> (**shq**).

This is effected by the program EL1, which:

- gives to the normal coordinates (**X'Y'**) a counter-rotation **-θ**, thus reporting them to canonic coordinates (**XY**);
- transforms these coordinates (**XY**) into Coliseum coordinates (**sq**).

The third coordinate, i.e. the height of the point on the ground, remains unchanged in the three references (**Z = Z' = h**).

The (**shq**) coordinates are the final ones, which are employed for the graphical or numerical representation of the whole wall, reported onto an unique reference

Schedule 2

Block	Sigma naught Image (microns)	Sigma naught Terrain (m)	Controls Horiz. (nr)	Controls Vert. (nr)	Mean resid. on the controls Horiz. (m)	Mean resid. on the controls Vert. (m)
E1	5.88	0.001	71	71	0.003	0.003
E2	6.71	0.001	58	58	0.004	0.003

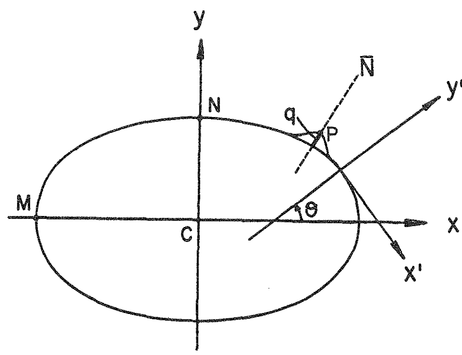


fig. 3

plane. Their efficiency is emphasized by the following considerations:

- Table 2,a) is a direct plotting in the original normal coordinates (X'Y'Z'); here a protruding capital appears strongly twisted. After the transformation (X'Y'Z') -> (**shq**) the Table 2,b) is obtained, in which the capital's distortion is completely eliminated.

- Table 3 shows the plotting of some very far pairs (about 50 m), reported to the Coliseum system. The perfect alignment of borders and cornices demonstrates the perfect coplanarity of the pairs' representations. The distance d = 47.8 m between 2 control points, measured on the map, exactly corresponds to the value Ds = 47.84 m obtained from the computation.

After all, the Coliseum (**shq**) coordinates appear to be quite reliable.

3.3 - The Table 4 reports the complete AUTOCAD plotting of 4 pairs, which cover the so called "Valadier's rampart" and its surroundings.

The plotting was obtained from the relevant numerical files; the control points, and their residuals, are reported on the drawing. But we deemed it unsuitable to transcribe the heights **q** - which are implicit in the numerical data of each line - and to draw the contours. If necessary, it is always possible to draw them on a separate transparency; but we guess that the numerical files are the only useful item utilizable in any research. Starting from these files, in a separate paper Paoluzzi and Rosina present at this Congress the compressed computer graphics representation of these pairs; while the static situation of the Valadier's rampart is described in another paper by Croci et a.

3.4 - If it is possible to get adequate funding, the plottings of the whole E1 and E2 walls will be collected in an album, format A3, scale 1:100; the relevant numerical data will be reported in floppies, annexed to the album.

The remaining items - i.e. the horizontal map, the internal survey, plants, sections, etc - will follow, always depending on the available funds.

All the above geometry constitutes the basis for any further collateral study on the Coliseum's architecture, statics, materials, archaeology, etc. Something is already presented in this Congress; we hope that much more may be carried out in the next future.

Rome, April 1992

ACKNOWLEDGEMENTS

The research was funded by the University of Rome (Rector G.Tecce, Dean A. Misiti), who appropriated a moderate sum for the "**PROGETTO COLOSSEO**", so giving it the possibility of taking off.

The planning and the direction of the photogrammetric takings were effected by Eng. R.Carlucci; the takings were carried out by Geom. E. Ferri.

The observations for the determination of the controls were effected by Dr. G. Maruffi, and Technicians O. Evandri and M. Gaeta.

The plottings were done by the firm GEOGRAFICA of Florence. Eng. Carlucci finally cared for the AUTOCAD plotting of the pairs which are presented here.

To them all goes our sincere gratitude and our thanks.

REFERENCES

Birardi, G.; Carlucci, R.; Ferrara, E.; Giannoni, U.; Maruffi, G. (1987)- Il rilievo fotogrammetrico del Colosseo, parte I - Bollettino della SIFET, Società Italiana di Fotogrammetria e Topografia, n. 2, 1987.

Birardi, G.; Carlucci, R.; Ferrara, E.; Giannoni, U.; Maruffi, G. (1988,a) - Il rilievo fotogrammetrico del Colosseo, parte II - Bollettino della SIFET, Società Italiana di Fotogrammetria e Topografia, n. 2, 1988.

Birardi, G.; Carlucci, R.; Ferrara, E.; Giannoni, U.; Maruffi, G. (1988,b) - The photogrammetric survey of the Coliseum in Rome - Paper presented at the XVI Congress of the ISPRS, Kyoto, 1988.

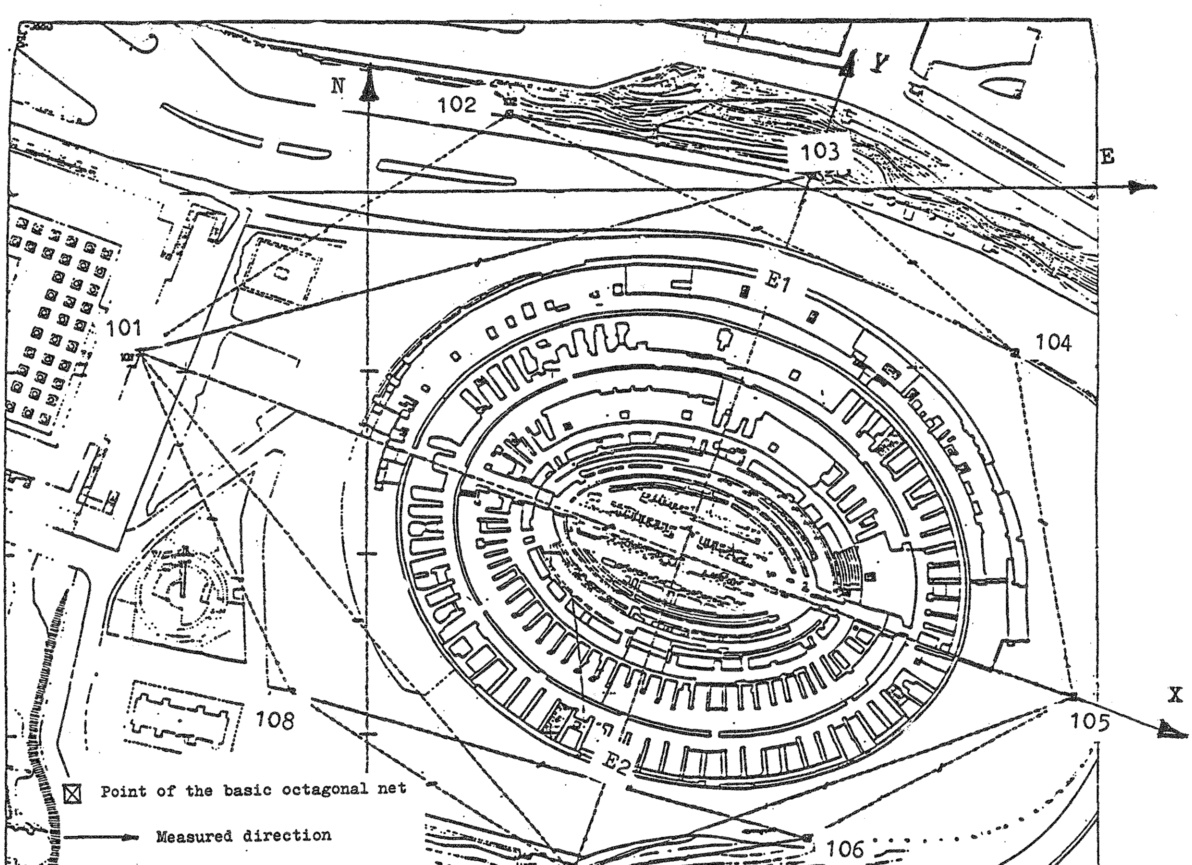
Carlucci, R. (1989) - Un progetto per il rilievo del Colosseo - Rivista del Catasto e dei SS.TT.EE., n. 1, 1989.

Carlucci, R.; Fiani, M.; Paradisi, S. (1992) - Restituzione fotogrammetrica per le analisi statiche agli elementi finiti e per la resa qualitativa nelle visualizzazioni computerizzate. Una sperimentazione sul Colosseo. - Bollettino della SIFET, Società Italiana di Fotogrammetria e Topografia, n. 4, 1992.

Klein, H. (1988) - The PAT.M/PC Program - The PAT.B/PC Program - XVI Congress of the ISPRS, Kyoto, 1988.

Kotowski,R.; Peipe, J.; Wester-Ebbinghaus, W. (1988) - Bundle triangulatiion in Architectural Photogrammetry: the Basilica of San Francesco in Siena - Photogrammetric Record, 12 (72), October 1988.

TABLE 1



Annex 1

Results obtained in the adjustment of the octagonal primordial net, with a. - bi-dimensional + height approach (Bencini- Birardi's program); b. - tri-dimensional approach (Ferrara- Giannoni's program); c. - tri-dimensional approach on 8+3 points (id.id).

Point	Approach	Coordinates (m) N	E	H	Rmse (mm) N	E	H
101	a.	-3744.841	3187.501	32.041	0	0	2
	b.	.839	.500	.039	-	2	0
	c.	.839	.500	.040	-	1	0
102	a.	-3678.251	3286.525	25.232	1	1	2
	b.	.250	.526	.231	2	2	1
	c.	.250	.526	.231	1	1	0
103	a.	-3696.259	3371.820	25.283	1	1	2
	b.	.258	.822	.283	2	2	1
	c.	.259	.822	.285	1	1	0
104	a.	-3746.573	3427.516	26.853	1	1	2
	b.	.573	.518	.851	2	2	1
	c.	.574	.517	.852	1	1	0
105	a.	-3841.993	3442.971	25.070	1	0	2
	b.	.994	.973	.068	2	1	0
	c.	.994	.972	.069	2	1	0
106	a.	-3880.317	3370.606	28.201	1	0	1
	b.	.317	.607	.202	2	1	0
	c.	.317	.607	.203	2	1	0
107	a.	-3890.537	3306.043	28.639	0	0	1
	b.	.538	.043	.641	1	1	0
	c.	.537	.043	.642	1	1	0
108	a.	-3838.780	3228.341	22.950	-	-	-
	b.	.780	.341	.950	-	-	-
	c.	.780	.341	.950	-	-	-
201	c.	-3666.574	3266.283	32.0481	1	1	0
202	c.	-3788.266	3179.710	26.991	1	1	0
203	c.	-3744.837	3187,501	32.403	1	2	0

Annex 2

COORDINATES OF CONTROL POINTS AND RESIDUALS

IN UNITS OF TERRAIN SYSTEM

HORIZONTAL CONTROL POINTS VERTICAL CONTROL POINTS

POINT-NO.	X	Y	Z	RX	RY	RZ
5023	89.363	24.302	26.928	-0.001	0.001	0.002
5033	84.760	33.556	27.144	0.001	0.004	0.005
5043	84.578	35.345	37.428	0.001	0.005	0.002
5053	84.190	35.050	48.960	-0.002	0.003	0.005
5063	76.154	45.911	49.094	0.001	0.005	-0.002
5073	76.515	46.282	37.278	0.000	0.002	-0.001
5083	76.484	46.312	27.170	0.001	0.003	-0.004
5093	66.707	55.762	26.603	0.004	-0.004	-0.002
5103	66.771	55.750	37.168	0.005	0.000	-0.004
5113	66.498	55.310	49.341	0.004	0.002	-0.006
5123	55.386	63.216	48.628	0.001	-0.005	-0.002
5133	55.594	63.623	37.251	-0.001	-0.001	-0.005
5143	55.576	63.579	26.544	-0.001	-0.001	-0.007
5153	43.546	69.820	26.919	-0.004	0.000	-0.004
5163	43.583	69.805	37.519	-0.002	0.001	-0.004
5173	43.329	69.438	48.873	-0.005	-0.002	-0.005
5183	30.588	73.927	48.413	-0.002	0.001	-0.001
5193	30.734	74.376	37.435	-0.003	0.000	-0.001
5203	30.676	74.364	27.033	-0.005	0.005	-0.005
5213	21.985	75.479	63.854	-0.002	-0.003	0.006
5223	17.380	76.788	49.129	-0.004	0.000	0.001
5233	17.415	77.237	37.250	0.003	0.005	-0.003
5243	17.376	77.233	26.020	0.003	0.000	0.007
5253	8.453	77.495	64.213	0.001	0.000	0.005
5263	3.918	78.190	49.238	0.001	-0.002	0.001
5273	3.980	78.622	37.304	0.004	-0.005	0.003
5283	3.897	78.522	26.850	0.005	-0.008	0.000
5293	-7.861	77.503	64.103	-0.004	0.004	-0.002
5303	-10.235	77.882	49.225	0.000	0.001	-0.003
5313	-10.242	78.307	37.444	-0.001	0.000	-0.001
5323	-10.235	78.228	26.571	-0.002	-0.003	-0.002
5333	-23.641	76.250	26.893	-0.001	-0.004	0.001
5343	-23.732	76.274	37.541	0.002	-0.001	0.001
5353	-23.682	75.908	49.278	0.001	-0.003	0.002
5363	-21.693	75.779	63.824	0.004	-0.002	0.006
5373	-36.702	72.293	49.288	0.005	0.021	-0.008
5383	-36.792	72.709	37.514	0.003	-0.003	0.003
5393	-36.818	72.604	26.470	0.003	-0.003	-0.001
5403	-49.385	67.321	26.863	0.008	-0.003	0.002
5413	-49.397	67.361	37.248	0.003	0.000	0.002
5423	-49.277	67.110	49.363	-0.001	0.000	-0.003
5433	-61.038	60.032	49.488	-0.004	0.009	-0.004
5443	-61.168	60.477	37.635	-0.001	0.002	0.002
5453	-61.143	60.353	26.629	-0.004	0.010	0.003
5463	-71.664	51.685	26.640	-0.001	0.015	0.001
5473	-71.759	51.670	37.290	-0.001	0.007	0.000
5483	-71.441	51.389	49.056	0.000	0.005	0.000
5493	-81.361	41.953	45.413	0.008	-0.009	0.010
5503	-81.017	41.713	35.431	0.002	-0.017	0.006
5513	-80.809	41.645	25.941	0.001	-0.014	0.004
5523	-84.554	36.010	26.006	-0.004	-0.010	0.002
5533	-84.842	36.036	35,377	-0.001	-0.014	0.004
5543	-84.703	37.014	45.418	0.002	-0.005	0.005
5563	-87.883	30.102	26.122	-0.003	-0.007	-0.001
5583	-90.118	26.905	45.509	-0.005	-0.003	-0.001
5593	-90.901	23.876	35.599	0.000	-0.002	-0.001
5603	-90.660	23.840	25.961	-0.002	0.000	-0.001
5613	-92.758	17.393	26.010	0.000	0.007	-0.005
5623	-92.917	17.430	35.510	0.000	0.009	-0.003
5663	-94.345	10.737	35.549	-0.006	0.014	-0.008
5673	-94.190	10.740	26.002	0.000	0.005	-0.006
12261	51.303	66.088	70.797	0.003	-0.004	-0.001
12301	77.646	44.718	70.776	-0.010	-0.009	0.008
12341	89.565	23.530	69.272	0.003	0.000	0.009

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Schedule 2

Block	Sigma naught Image (microns)	Sigma naught Terrain (m)	Controls Horiz. (nr)	Controls Vert. (nr)	Mean resid. on the controls Horiz. (m)	Mean resid. on the controls Vert. (m)
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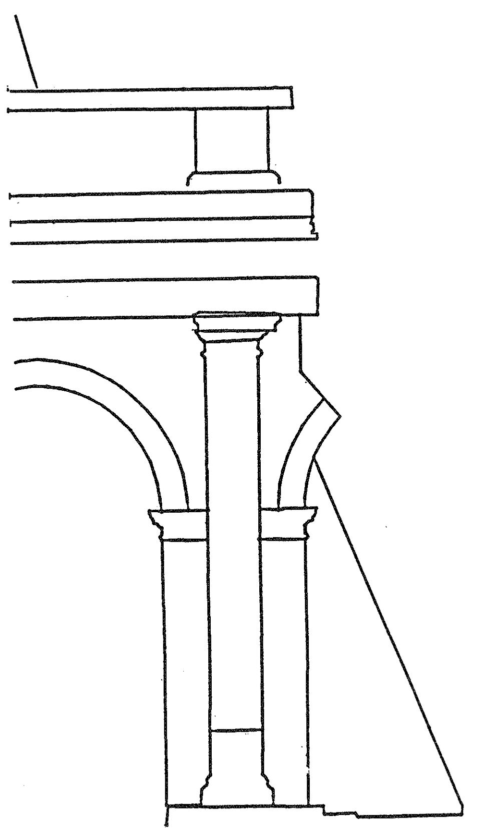
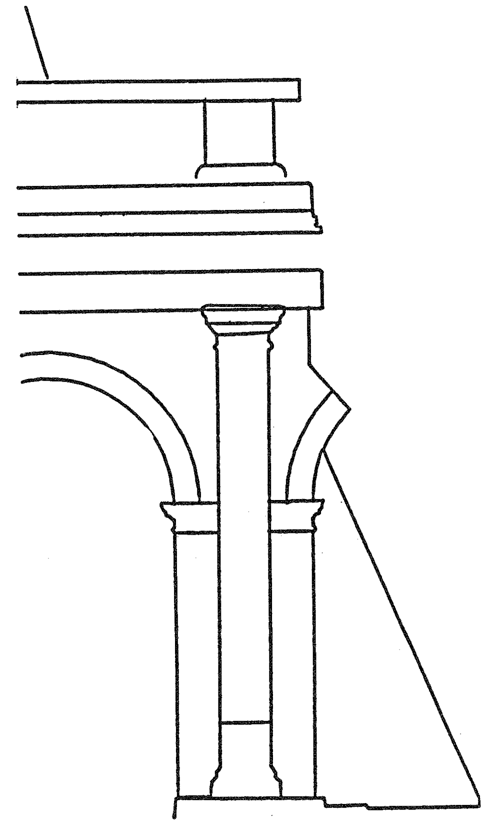
TABLE 2

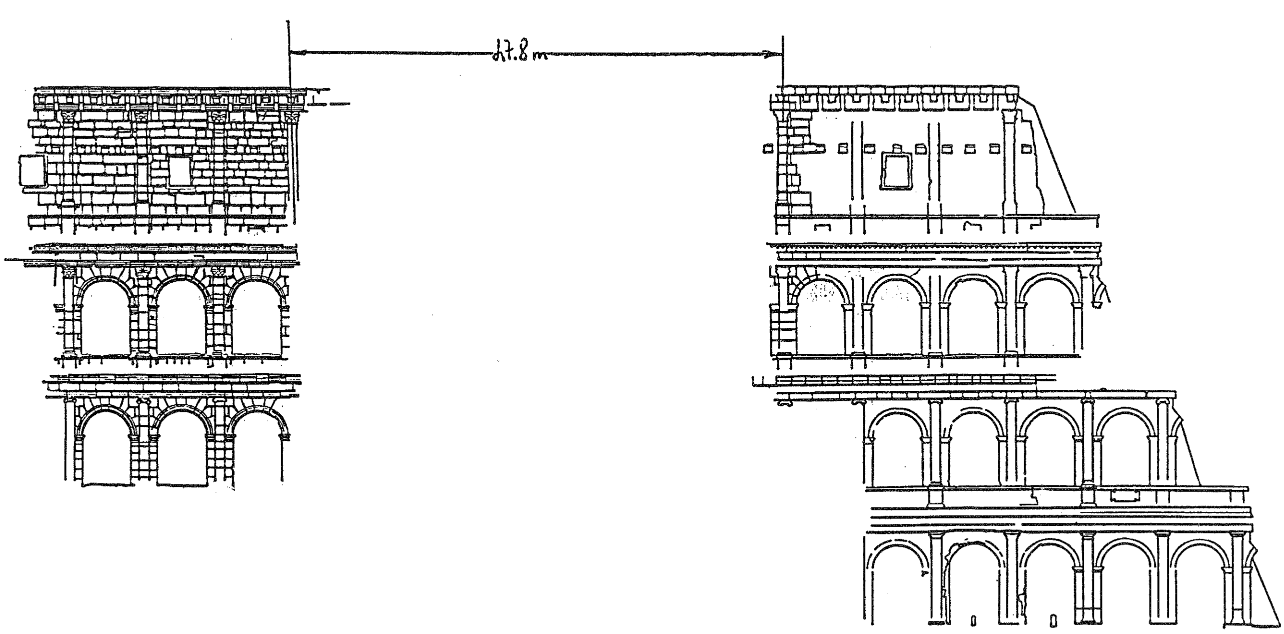


(a)



(b)

TABLE 3



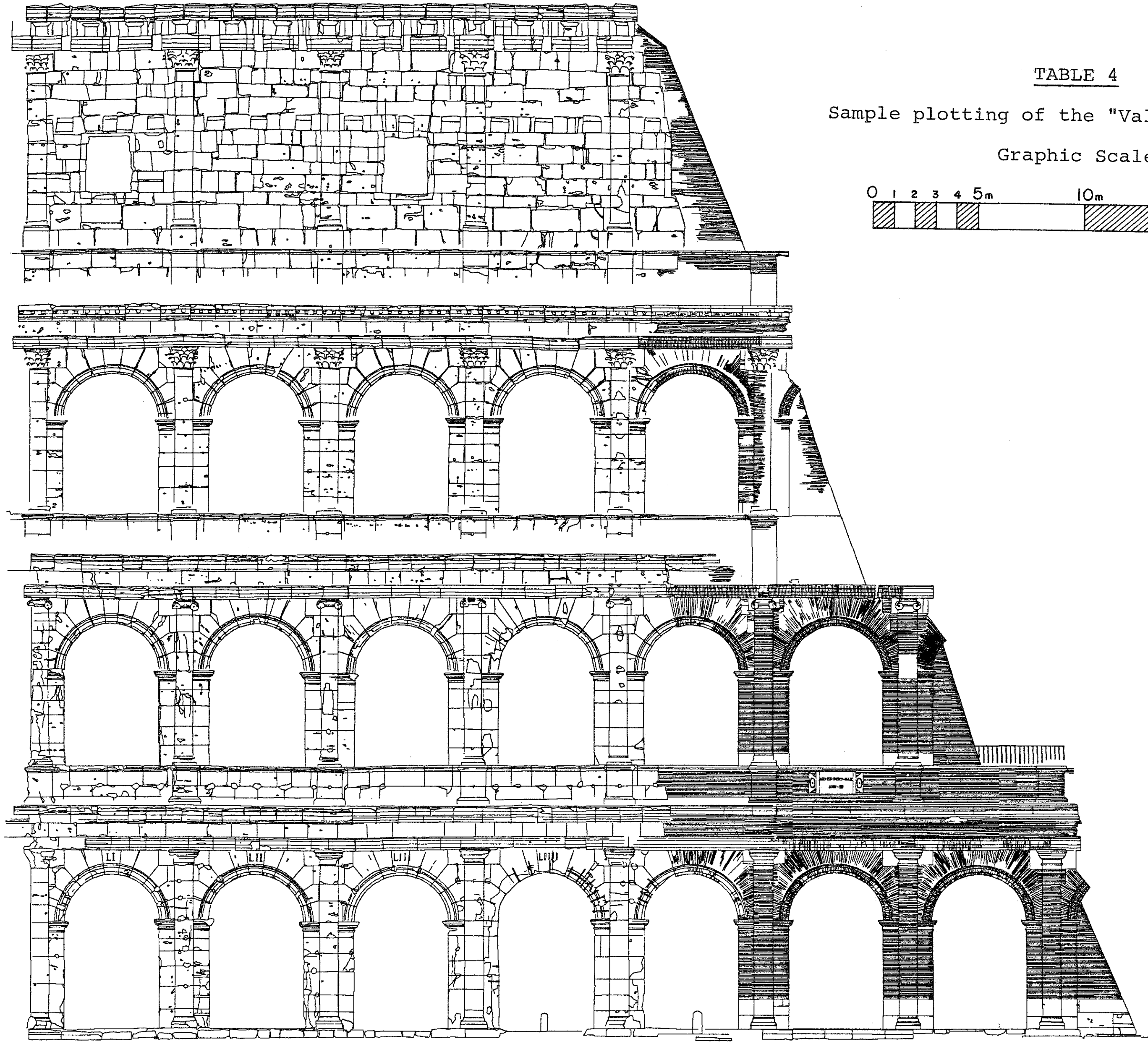


TABLE 4

Sample plotting of the "Valadier's Rampart"

Graphic Scale

0 1 2 3 4 5m 10m 15m 20m