ONE PHOTOGRAM RASTER-PHOTOGRAMMETRY FOR INDUSTRIAL PURPOSES. DEVELOPMENT OF A COMPUTER METHOD.

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One photogram raster-photogrammetry is highly suitable for industrial purposes. In the present paper the technology, an industrial application and the software developed are described. In this method a reticule is projected onto the object by a metric projector and a photogram is taken by a metric camera. The reticule acts as a second photogram. The metric projector was developed by AGIP from the project by E. Baj. The projector is a high precision instrument and his interior orientation is known with the same accuracy as that of the metric camera. A computer program has been developed to correct the relative orientation of camera and projector and evaluate the point coordinates. The method is based on coplanarity equations and does not require control points. A 3 dimensional graphic interface based on the CAD CATIA (Dassault Systèmes) has been developed. This measurement technology has been applied to the dimensional control of prefabricated pieces. In particular results from a structural joint survey are described.

KEYWORDS: Algorithm, CAD-CAM, Close-range, Industrial, Photogrammetry.

INTRODUCTION.

The present paper is the completion of a previous one referring to the rasterphotogrammetry employing one photogram and a metric projector (Baj, Rampolli, 1990). The previous paper was dealing with the photography of a structural joint of an offshore structure made of cylinder tubes of different sizes, fig. 1. This paper deals with the same subject, but it develops the analytical restitution of the points and the software involved.

STRUCTURAL JOINTS OF OFFSHORE PLATFORMS.

General.

The structural joint is the part of an offshore platform where differently inclined beams converge. From the structural point of view it is very important because it must withstand high concentrations of tensions due to the transmissions of forces through the beams. From the construction point of view it is a complex part because many beams converge in it. For these reasons special analyses are carried out during the design phase. The joint is checked for punching shear and fatigue and a suitable geometry is chosen for an easy assembling. The joint is constructed in a workshop using special very thick steel, with thermal treatment and accurate dimensional controls. In the Agip platform building procedure a single joint is prefabricated including the ends of the beams which converge in it. The main pipe is called can, the secondaries ones stubs.

Dimensional Tolerance.

If the prefabricated joint geometry is not compliant with the project specifications, it may cause structural and construction problems. The structural problems may not be easily measurable, because the amount of the geometry deviation is generally small and does not influence the tension flow in the jacket reticular structure. The assembly however can have serious problems because it can be difficult to match the prefabricated parts. In this case the structure tensional state may be changed locally, due to beams not well aligned, shrinkage with jacks or improvised welding procedures. Discrepancies of the structural joint are generally due to the cylinders ovalization and to the angles between can and stubs. Ovalization originates from imperfect bending of the iron plates or from shrinkage phenomena caused by welding. It is more common in the cans where the diameter/thickness ratio is greater. Angular deformations originate from complex geometry of the manufactured part. Accurate dimensional control of the various cylinder lengths is not necessary. In fact, this difference, if not too big, may matter little, because the beams inserted during the assembly are cut after the gap measurement. To avoid these problems, international standards set dimensional tolerance levels. These do not generally specify the joint because the construction method may not include the prefabrication of this element. However the limits are defined for the elements which depend on the joints for the geometry, i.e. the horizontal and side planes.

Dimensional Control.

The constructor carries out dimensional controls of the joints by means of traditional topographic methods or with a surface gauge. In the former case, due to the high precision required, optical instruments such as theodolites are used. These are used for forward intersection from a minimum of 2 stations to define bolted points of 2 or 4 directrices for each cylinder. Recently wave distance meters have also been used, these can measure reflecting signals with a precision of less than 1 mm. The surface gauge consists of a large rectified plane along which a sensor moves. The position of this sensor is read analytically or analogically. Agip dimensional control is carried out by photogrammetry. With this method it is possible to define each cylinder surface with continuity and with an average precision of 0.5 mm (Bonora, Rampolli, 1990). Today rasterphotogrammetry can be used for these controls.

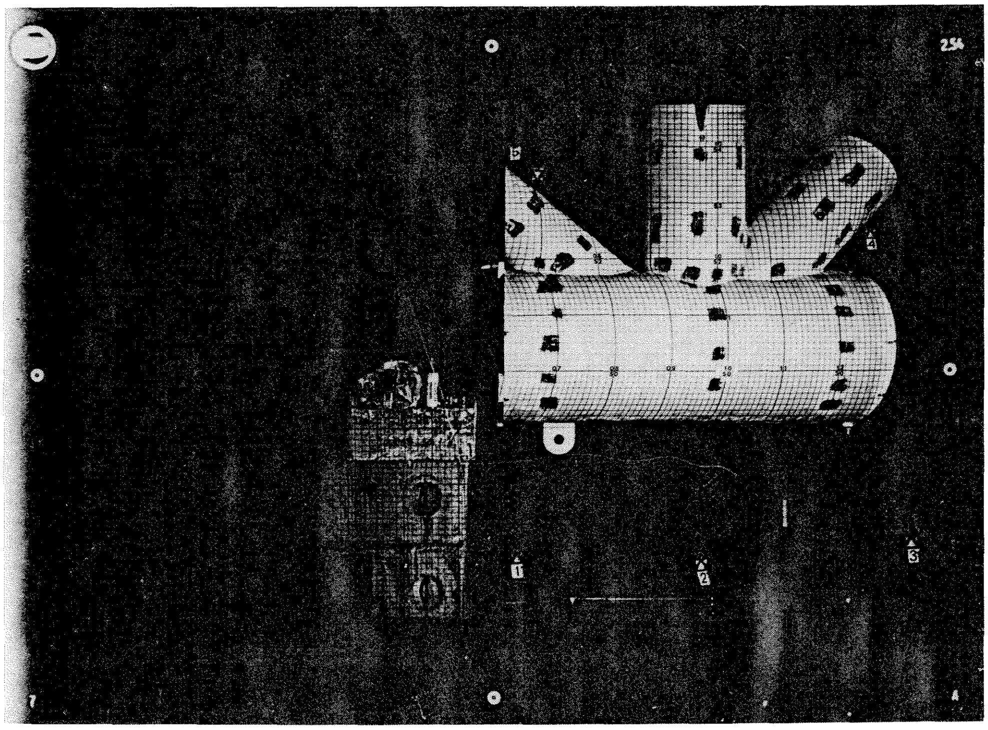

Fig. 1: The Bouri DP4 structural joint.

PHOTOGRAPHIC EQUIPMENT.

The equipment available and employed was:

1) 2 large-format wide angles universal UMK 10/1318 Jenoptik-Jena cameras
2) a syncronizer allowing simultaneous photography
3) the metric projector which projects a reticule onto the object to be surveyed.

The main parts of this prototype of metric projector are (fig. 2):

1) A Durst CLS high power projector without its lens, bellows, slide carrier, featuring 2000 W cold-light bulb and with air cooling system.
2) A Vinten support block consisting of a rugged tripod fitted with wheels and mounted with a special Durst swivelling head.
3) A Jenoptik/Jena UMK 10/1318 camera body fitted with wide-angle lens, with 100. mm focal length. On the back of the camera body fiducial marks are visible. The camera is supported by an alidade, and the whole assembly is mounted on a tripod to be orientable and positioned according the needs.
4) A Jenoptik/Jena adjusting device which is backmounted on the UMK camera-body with 3 screws.
5) An optically flat glass plate on which there is the image of a reticule obtained by photoreproduction. The optical glass is connected to the adjusting device, the reticule will be called pseudo-photogram because it carries the same information as a photogram in one-photogram rasterphotogrammetry.

The pattern of the reticule is shown in fig. 3. At present it has not been decided the final pattern of the reticule and how thin the lines will be. Actually these decisions are related to the device that will be chosen for the automatic reading of the plate coordinates in the rasterphotogram. In fig. 3 the thinnest lines are of 0.04 mm . The study of the

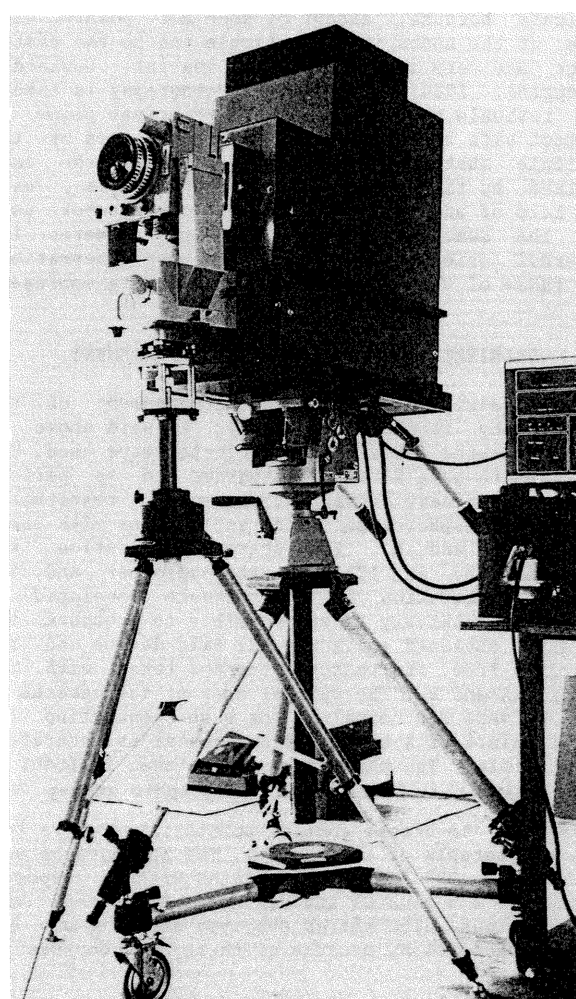

Fig. 2: The metric projector.

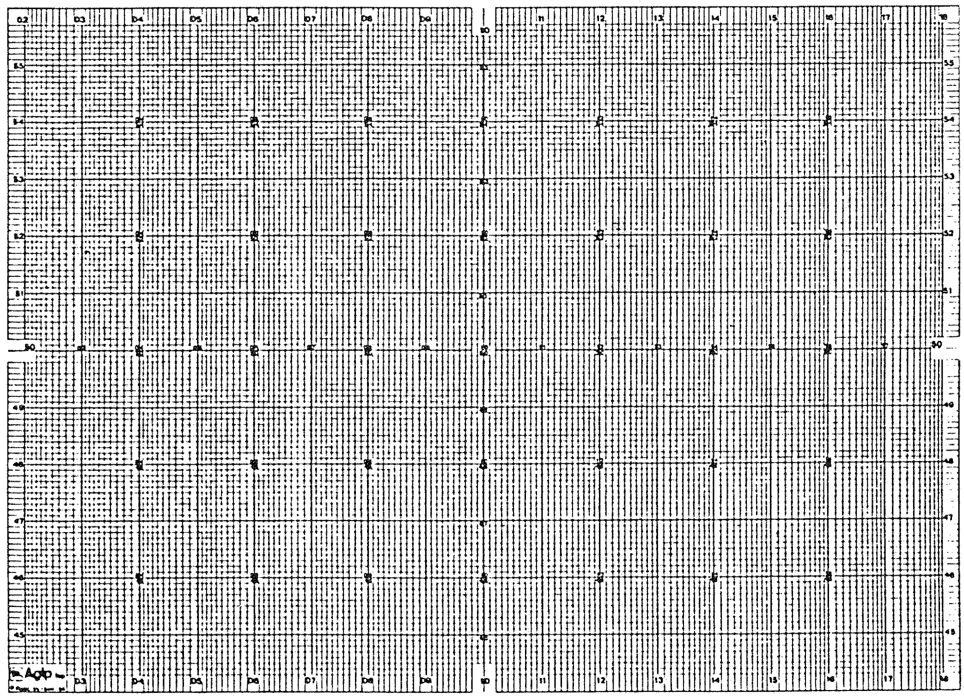

Fig. 3: Reticule (pseudophotogram).

reading method can be carried on by using the reticule because, except of some hot points, the size of the nodes in the reticule and in the raster image are very similar (Baj, Casalini, Lombardi, Tonazzini, 1984). Before the photography is taken, the reticule must be put in the principal plane in contact with the fiducial marks and the axes of the reticule must be put in coincidence with the axes defined by the fiducial marks of the camera , with the help of an eyepiece. The metric projector used has the same prerogative of a metric camera: its internal orientation is known and the aberrations are those of the body of the metric camera employed.

PLANIMETRIC AND AXONOMETRIC ARRANGEMENT.

A planimetric and axonometric arrangement of the photography is shown in fig. 4. As said above, 2 metric cameras and a metric projector were used. The aim was to have 2 rasterphotograms and to perform both an ordinary stereophotogrammetric restitution and 2 rasterphotogrammetric restitutions with the 2 cameras A and B. The stereo restitution was performed by an AC1 Wild stereoplotter and the raster restitution by the software developed in Agip. In that way it was possible to compare the results obtained with the AC1 Wild device and the results from the rasterphotograms taken with the cameras A and B. The optical axes of the cameras A and B were set normal to the base connecting the nodal points of A and B and horizontal as accurately as possible. The photographs were taken in front of and behind the object and a topographic survey was performed to create control points in order to link the photographs of the 2 sides. The photographs were taken using ORWO MO1 14 DIN plates with an exposure time of 2-3 minutes and diaphragm 8 and they were simultaneous. The plates employed are suitable for raster photography because of their high contrast.

PLOTTING.

Two plotting methods were considered. First the usual restitution of stereo raster-photograms with

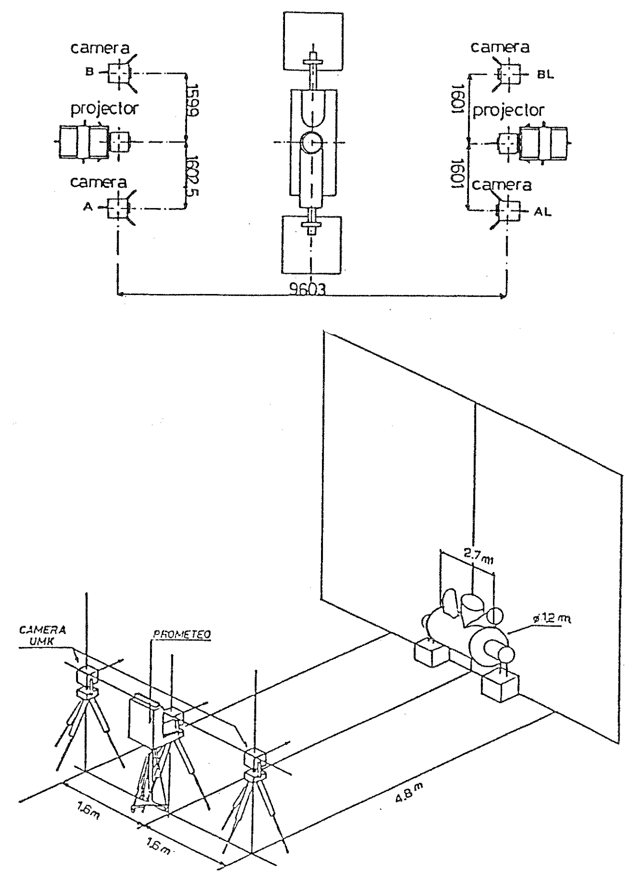

Fig. 4: Arrangement of the photography.

the AC1 Wild stereocomparator was performed. Some problem was found to observe the raster pair of photograms because the ratio distance/base was less than 2. Despite that the average error of the control point coordinates was less than 0.3 mm. The results obtained with this procedure have been used as a comparison term to check the quality of the results obtained from the monoscopic procedure. The monoscopic analytical procedure will be shown in the next paragraph.

COMPUTER METHOD FOR THE ANALYTICAL RESTITUTION.

The method developed calculates the 3 dimensional coordinates from the 2 dimensional plate coordinates and the reticule code of each reticule node. The method uses the coplanarity equations to correct the relative position of camera and projector. Lenses optical distorsion can be accounted for.

As described above in the rasterphotogrammetric technique a reticule is projected on to the object and a photograph is taken. At each cross point of the reticule a couple of 2 dimensional coordinates are given by the plate coordinates and another couple are given by the 2 reticule identification codes. The identification codes give the 2 coordinates which in the stereoscopic photogrammetry are given by the 2 nd plate. The method developed accounts for possible differences between the theoretical relative position of camera and projector and the actual relative position. The camera position is assumed correct and a correction to the projector coordinates is evaluated. The distance between camera and projector is not modified to conserve the model scale. The free variables of the problem are:

```
y     : projector nodal point coord along y axis
z     : projector nodal point coord along z axis
omega : projector rotation angle around x axis
k     : projector rotation angle around y axis
fi    : projector rotation angle around z axis
```

where: the local reference system x,y,z has the origin in the projector nodal point, the x axis is defined by the horizontal straight line normal to the camera focal axis, the y axis is the focal axis, the z axis is upward and normal to x and y.

The basic idea of the method is to find the values of the problem variables, which minimize a suitable error function. The error function F is defined as:

```
F = SUM(i) d(i)**2                                   (1)
```

where SUM(i) means summation on i index, d(i) are the distances between the 2 straight lines coming out of camera and projector, and the double star ** means exponentiation. The 1 st line passes through the coordinates of point(i) on the plate of the camera and the nodal point of the camera and the second line through the coordinates of point (i) on the virtual plate of the projector and the projector nodal point. The function F can be written as an explicit function of the variables:

```
delta y      = eps(1)
delta z      = eps(2)
delta omega  = eps(3)                                (2)
delta k      = eps(4)
delta fi     = eps(5)
```

If all these variables are small enough, a first order approximation of F(eps(k)) can be taken and F(eps(k)) can be assumed to be linear with respect to the variables eps(k). The minimum problem can now be solved by setting to zero the partial derivatives of F respect to eps(k). This leads to a set of 5 linear equations in 5 unknowns. The equations used to express the distances d(i) are known as coplanarity equations (ASPRS, Non Topographic Photogrammetry, 1989). The details of the equations are given in Appendix A. The algorithm can be applied many times until convergence is reached. A program has been written to apply the algorithm described and to perform if required statistical comparisons with reference data. The program called CCP has been tested by a set of numerical tests and then has been applied to the analysis of 6 series of real rasterphotogrammetric measurements. The measurements were performed in March 1990 at Porto Marghera, Italy, to check the dimensions of a true structural joint of the Bouri platform, Bouri DP4. The Bouri DP4 joint has a can with radius=600. mm and 3 stubs with radii equal to 349. , 399. and 300. mm . Three series of measurements were effected each with 2 cameras set along an axis approximately parallel to the axis of the can, and a projector approximately in the middle point between the 2 cameras. In the first 2 series of photographs the cameras and the projector were set at the 2 opposite sides of the joint, and a 3 rd series of measurements was effected with the joint rotated of 30 degrees. The camera and the projector were about 5. meters far from the joint and the distance between the nodal points of camera and projector was about 1600. mm . In this way for each of the 3 series of measurements it was possible to calculate one series of stereophotogrammetric data and 2 series of rasterphotogrammetric data and a comparison between stereo and raster data was possible. The focal lengths of the 3 cameras used were 101.75 , 101.59 , 102.08 mm . To test the CCP program the characteristics of the true situation were simulated by some numerical tests.

In the numerical tests a small FORTRAN program generated the 2 dimensional plate coordinates of 100 points on the surface of a cylinder. The cylinder had the same radius of the can of Bouri DP4 joint and the program calculated the coordinates on the plates of 2 cameras in the same positions as the camera and the projector used during one of the real measurement of Bouri DP4. The results of the numerical tests can be summarized in the following points:

1) in a case similar to Bouri DP4 with a likely error in the relative position of camera and projector and a likely radial distorsion but no other measurement error the average distance from the exact position of each point calculated by CCP is about 0.02 mm.

2) if an error in the simulated plate coordinates is introduced, it can be seen how the error of the 3 dimensional coordinates depends on the plate coordinates error. In case of a likely projector position error (which had to be corrected by the program) the following results were obtained:

plate error mm	average 3D error mm
0.001	.24
0.005	1.8
0.01	3.6

In the real cases the precision obtained depends on how the bad measurements are treated. Bad measurements can reduce the precision and must be skipped by some kind of filter. CCP skips the points with a d(i) value, eq. (1), greater than the average value plus an input factor by the d(i) standard deviation sigma. Another kind of selection can be performed when a set of reference data is available for a comparison. In this case again only the points can be selected for the comparison whose distance from the reference points is minor than the average distance plus the standard deviation by an input factor.

Six series of rasterphotogrammetric data were effected corresponding to 6 different positions of the couple camera projector:

A3 : front left
B1 : front right
AL3 : back right
BL1 : back left
B6 : front right joint rotated 30 deg
AL4 : back right joint rotated 30 deg

The results obtained in the 6 cases are summarized in tables 1 and 2 which are different just for the point selection criteria. In each case about 2500 points were measured and a comparison was made with the stereophotogrammetric data coming from a Wild AC1 stereoplotter.

A short analysis was performed to reduce the errors coming from the optical lenses distortion. Two different approaches were followed: an exact approch based on the Jenoptik calibration data for each lens, and an analytical approach to evaluate the optimal radial distortion to minimize the error function of equation (1). In both cases nul or minor improvements were obtained probably because the errors in the 2 dimensional plate coordinates were greater or similar to the distortion errors. The distortion errors according to the calibration documents range about from 1. to 10. micron. These errors are about the same as the 2 dimensional errors which in the numerical tests lead to the precision obtained in the real cases.

The 3 dimensional points evaluated by CCP have been analyzed by a chain of software programs developed at Agip, EDPI, interfaced with the graphic package CATIA (Dassault Systemes). In particular the program VDIM has been used to calculate the beam diameters. VDIM performs a best fit analysis of the calculated points with assigned surfaces or lines in the 3 dimensional space. Different views of the joint can be merged by different techniques both with or without common reference known points. In table 3 the radii and the angles evaluated in a set of 6 cases are reported along with the results of an old set of measures. The first 5 sets have been evaluated by merging the points of a set in the front side of the joint and the points of a set in the back side. The merging was based on the reference known points used for the stereophotogrammetric measures. In the last case: B6AL4B1BL1 two merged sets are merged again to exploit the points measured with the joint rotated of 30 degrees. In this case there are no common known points and the merging is based on the axes of the cylinders calculated by the 2 subsets B6AL4 and B1BL1. A program interfaced with the CAD package CATIA finds the optimal rototranslation to make the corresponding axes to overlap.

Conclusion.

The accuracy of the final results obtained is about 0.1 to 1. mm for the radii and about 0.2 gon for the angles, see table 3. The main error source in the measurements described was probably the reticule

TABLE 1

Bouri DP4 6 Base Cases
EPD=-.1 DSMA=-.1

case	av dist mm (1)	total couples of p	selected couples of p (2)	average relative interval error (3)	total points	selected points (4)	av err mm (5)
A3	1.46E-01	2145	1136	1.29E-03	2268	1432	1.00E+00
B1	1.16E-01	2108	348	4.11E-03	2232	1113	2.51E+00
AL3	1.60E-01	2328	2239	1.45E-03	2416	2345	1.43E+00
BL1	9.53E-02	2368	449	1.37E-03	2429	892	1.05E+00
B6	9.71E-02	1853	532	7.35E-04	1888	885	1.09E+00
AL4	8.62E-02	1772	353	1.25E-02	1903	872	6.30E+00

Notes

1) Average distance between the lines coming out from camera and projector, passing through the plate coordinates and the nodal point.
2) Only couples of points selected both for projector position correction and for best fit rototraslation. For the projector rototranslation are selected the points for which the distance of note (1) is less than the average value plus the standard deviation by an input factor EPD, in these cases EPD=-0.1 . For the best fit are selected the points whose distance from the stereo corresponding point is less than the average value plus the standard deviation by an input factor DSMA, in these cases DSMA=-0.1 .
3) Average relative error for the distance between 2 points: Dx/Dx'-1
4) See note 2 for the points selected.
5) Average distance between the calculated point and the corresponding exact point.

TABLE 2

Bouri DP4 6 Base Cases
EPD=0. DSMA=0.

case	av dist mm (1)	total couples of p	selected couples of p (2)	average relative interval error (3)	total points	selected points (4)	av err mm (5)
A3	1.85E-01	2145	1764	1.73E-02	2268	2001	5.39E+00
B1	1.21E-01	2108	428	3.99E-03	2232	1198	2.28E+00
AL3	1.62E-01	2328	2239	1.45E-03	2416	2345	1.43E+00
BL1	1.21E-01	2368	1019	1.03E-03	2429	1530	1.07E+00
B6	1.05E-01	1853	619	7.42E-04	1888	974	1.14E+00
AL4	9.00E-02	1772	427	1.28E-02	1903	939	6.84E+00

Notes

1) Average distance between the lines coming out from camera and projector, passing through the plate coordinates and the nodal point.
2) Only couples of points selected both for projector position correction and for best fit rototraslation. For the projector rototranslation are selected the points for which the distance of note (1) is less than the average value plus the standard deviation by an input factor EPD, in these cases EPD=0.0 . For the best fit are selected the points whose distance from the stereo corresponding point is less than the average value plus the standard deviation by an input factor DSMA, in these cases DSMA=0.0 .
3) Average relative error for the distance between 2 points: Dx/Dx'-1
4) See note 2 for the points selected.
5) Average distance between the calculated point and the corresponding exact point.

TABLE 3

Bouri DP4 Radii and Angles.

case	radius Leg mm	radius B1 mm	radius B2 mm	radius B3 mm	angle L-B1 gon	angle L-B2 gon	angle L-B3 gon
previous	600.11	348.60	398.59	299.74	147.22	99.52	155.77
VZA3BL1	601.21	350.41	400.78	301.95	147.49	99.89	155.64
VZA3AL3	601.35	349.28	399.29		147.07	99.53	155.56
VZB1AL3	601.27	347.70	398.89		147.01	99.58	155.56
VZB1BL1	601.32	348.79	399.91	301.57	147.47	99.88	155.61
VZB6AL4	601.09	350.58	399.05	299.88	147.40	99.78	155.94
B6AL4B1BL	601.43	350.07	399.66	302.01	147.47	99.79	156.21
average	601.2783	349.4716	399.5966	301.3525	147.3166	99.74074	155.7537
var	0.108076	1.008884	0.631761	0.866728	0.198942	0.137835	0.243720

Note:
the average value and the variance refer to the six new measurements.

accuracy. That should be proved by a careful check of the old reticule lines or by new measurements with a new reticule.

The new method offers the following advantages over the traditional method:

1) the spatial coordinates can be obtained from a single photogram avoiding the use of expensive apparatus.

2) the 2 dimensional plate coordinates can be read more easily.

3) the method is suitable for an automatic digital reading.

CCP EQUATIONS.

Introduction.

CCP calculates the 3D coordinates of a point from the 2D coordinates read on the plate of a photogrammetric camera and from the identification codes of the reticule projected onto the object. CCP includes an implicit correction to the projector position, supposed slightly different from the theoretical position. The camera position is supposed to coincide with the theoretical one. In such a way just the relative position camera-projector is corrected, while the possible error in the camera position can be corrected by a rototranslation based on the comparison with reference points, if available.

The algorithm finds out the projector rototranslation which minimizes the sum of the square distances of the straight lines coming out from camera and projector. In particular the 1 st line is defined by the 2D coordinates of each point on the plate and by the nodal point of the camera. The 2 nd line is the equivalent line coming out from the projector. The projector freedom degrees are 5, because the distance between camera and projector is supposed fixed. When the optimal projector rototranslation has been evaluated the 3D final coordinates are calculated. The points P1 and P2 are found out along the straight lines coming out from camera and projector for which the distance between P1 and P2 is minimum. As the most likely cross point the arithmetic average of P1 and P2, is taken.

Description of the algorithm to optimize the projector position.

Parametric equations of the 2 straight lines.

```
g:  XG = XC +lambda*ALFA
                                  (1)
h:  XH = XP +     mu*BETA
```

where: capital letters indicate 3D vectors
XC : camera nodal point
XP : projector nodal point

If :

GAMMA = ALFA v BETA

(v: vectorial product)

XC = (0,0,0)

(origin in the camera nodal point)

the distance between the 2 straight lines coming from camera and projector can be written as:

D=XG-XH=(XC+lambda'*ALFA)-(XP+mu'*BETA)

this leads to:

d=ABS(GAMMA*(XC-XP))=ABS(GAMMA*XP) (2)

The function to minimize can be written:

f = SUMMAT(i) d(i)**2 (3)

where:
-i- is the point index
SUMMAT(i) means summation on index -i-

The terms d(i) can now be written by pointing out the problem unknowns, that is the projector rototranslations. It is convenient to choose a reference system with the origin in the camera nodal point, the x axis horizontal and normal to the camera focal axis, the y axis coinciding with the camera focal axis supposed horizontal, the z axis vertical upward.

```
XP = (1,-f2y,hz)                          (4)

ALFA = (x1,y1,z1)/NORMA(X1)
                                          (5)
BETA = (x2,y2,z2)/NORMA(X2)

NORMA(X1) = (x1**2+y1**2+z1**2)**0.5
NORMA(X2) = (x2**2+y2**2+z2**2)**0.5

GAMMA = (gam1,gam2,gam3)

gam1 = N*(y1*z2-z1*y2)
gam2 = N*(z1*x2-x1*z2)                    (6)
gam3 = N*(x1*y2-y1*x2)

N  = 1./(NORMA(X1)*NORMA(X2))
```

where:
x1,y1,z1 : plate coordinates, with z1=-d1 for all the points
d1 : camera focal distance

x2,y2,z2 : new projector coordinates written in the projector reference system

```
x0,y0,z0 : projector coordinates from the grid
           codes, with y0=-d2 for all the
           points, where d2 is the projector
           focal distance
```

If the coordinates x2,y2,z2 are substituted in eqs. (6) by their expressions functions of x0, y0, z0 and of the angles of rotation around the axis, eps1,eps2,eps3 , it can be written:

```
gam1 = N*(y1*z0-z1*y0-eps1*(y1*y0+z1*z0)-
       eps2*y1*x0+eps3*z1*x0)
gam2 = N*(z1*x0-x1*z0+eps1*
       x1*y0+eps2*(z1*z0+x1*x0)+eps3*z1*y0)
gam3 = N*(x1*y0-y1*x0+eps1*x1*z0-eps2*y1*z0-
       eps3*(x1*x0+y1*y0))
                                             (7)
```

In the eqs. (7) it is assumed that the angles eps1/2/3 are small enough to allow a 1 st order approximation in the transformation equations of the X2 coordinates by a rotation around the origin. The variables eps1/2/3 are then small variations of the 3 angolar variables omega, k, fi:

```
omega: rotation around x axis
k    : rotation around y axis
fi   : rotation around z axis
```

Let be:

```
GAMMA = GAMM0+delta GAMMA
GAMM0 = (gam10,gam20,gam30)
XP    = XP0  +delta XP
```

then:

```
XP0       = (1,-f2y,hz)
delta XP  = (0,delta y,delta z)              (9)
GAMM0     = N*((y1*z0-z1*y0),(z1*x0-
            x1*z0),(x1*y0-y1*x0))

delta GAMMA = N*((-eps1*(y1*y0+z1*z0)-
              eps2*(y1*x0)+eps3*z1*x0),
              (eps1*x1*y0+eps2*(z1*z0+x1*x0)
              +eps3*z1*y0),
              (eps1*x1*z0-
              eps2*y1*z0-eps3*(x1*x0+y1*y0)))
```

For each point -i- :

```
d(i) = ABS(GAMMA*XP)
     = ABS(GAMM0*XP0+delta GAMMA*XP0+GAMM0*
       delta XP)                             (10)
```

Products in equation (10) can be written explicitly:

```
GAMM0*XP0 = 1*gam10-f2y*gam20+hz*gam30
delta GAMMA*XP0=(eps1*(-1*(y1*y0+z1*z0)-
                f2y*x1*y0+hz*x1*z0)+
                eps2*(-1*y1*x0-
                f2y*(z1*z0+x1*x0)-hz**y1*z0)+
                eps3*(1*z1*x0-f2y*z1*y0-
                hz*(x1*x0+y1*y0)))*N
                                             (11)
```

The scalar product: delta GAMMA*XP0 , can be written in a concise form:

```
delta GAMMA*XP0=eps1*A(1,i)
          +eps2*A(2,i)+eps3*A(3,i)
```

where i: point index

The last term in equation (10) can be written:

```
GAMM0*delta XP=gam20*delta y + gam30*delta z
```

Let

```
A(4,i) = gam20
A(5,i) = gam30
eps(1) = eps1
eps(2) = eps2
eps(3) = eps3
eps(4) = delta y
eps(5) = delta z
```

then

```
f = SUMMAT(i)  d(i)**2
  = SUMMAT(i)  (GAMM0(i)*XP0(i)+
    SUMMAT(k) A(k,i)*eps(k))**2              (12)
```

and

```
d f/ d eps(k) = 2*(SUMMAT(i) A(k,i)*d(i))    (13)
```

where d f/d eps(k) means the partial derivative

The unknowns eps(k) can be derived by setting to zero the partial derivatives (13), which are a system of 5 linear equations in 5 unknowns.

In the CCP program the equations described here are inserted in an iterative scheme, where the rototranslations eps(k) are treated as perturbations to the the projector position. At each iteration the new projector coordinates X2 become the initial projector coordinates X0 for the new iteration, until convergence is reached.

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