

## Chapter 6: ANALYTICAL METHODS AND INSTRUMENTS

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**Abstract:** The paper constitutes Chap 6 (analytical methods and instruments - concepts and procedures) of the book "History of Photogrammetry" to be published by the ISPRS. Starting with definitions, the fundamentals and precursors, all pre- and post World War II developments are elaborated. This is followed by a broad discussion on more recent advancements, conventional and unconventional. Instrumental developments are discussed with regard to acquisition, processing and presentation of the data. Selected references are appended.

**Key words:** History, Analytical Photogrammetry, Concepts, Procedures, Instruments.

## 6.1 INTRODUCTION

In photogrammetry the word "Analytical" has been used synonymously with "computational", where the solutions are obtained by mathematical methods as against "Analog", where solutions are obtained by analogy or similitude developed through optical-mechanical procedures. The backbone of analytical methods consists of various mathematical and procedural concepts to represent relations between points in the object, their corresponding images and operational procedures to solve specific problems.

Analytical photogrammetric procedures may be considered along three operational stages, each involving specific instruments (Fig. 6.1), viz., those used for acquisition of image data (mensural), those used for data-processing and analyses (computational) and those used for display or presentation of the results.

In view of the above, we would study the historical developments firstly with regard to the concepts and next with regard to the instruments and their potentials for the future.

A mathematical model, in expressing the relevant concept, provides insight into the underlying chain of events. There is no mystery about the way in which this insight is achieved. The mathematical models have no scientific value unless they have been validated adequately through experience and research. Scientific validation is an openended process. As a mathematical model is successfully tested and used, it becomes established. Otherwise it stands to be changed, modified or simply dropped. We have witnessed this through the historical development of analytical photogrammetry.

Furthermore, photogrammetry being an applied science, it is the content and not the form of the mathematical statement (language) that matters most. Thus we have noticed that mathematical and operational concepts have been adapted to circumstances without really changing the basic contents. The following sections would highlight the conceptual developments without going into personal details.

## 6.2 MATHEMATICAL AND PROCEDURAL CONCEPTS

## 6.2.1 Fundamentals and Precursors

Development of mathematics as a discipline of logic did not exist before about 1000 B.C. The Greek philosopher Aristotle (~350 B.C.) referred to the process of optical projection of images. Leonardo da Vinci explored the disciplines of optics, geometry and mechanics. In 1492 he demonstrated the principles of optical perspective (MacLeish 1977), which provides the foundation of photogrammetry even today. Albrecht Dürer (1471-1528) in 1525 constructed samples of mechanical devices to make true perspective drawings of nature and studio scenes as well as for producing stereoscopic drawings (ASPRS 1980). The German astronomer Johannes Kepler in 1600 gave a precise definition of stereoscopy. Aughtread of England in 1574 developed the first slide rule and soon thereafter John Napier (1550-1617) published tables of logarithms and Blaise Pascal (1623-1662) established the concept of metrology and gave the world a desk calculator. Isaac Newton (1642-1727) and Gottfried von Leibnitz (1646-1716) firmly established the concepts of differential and integral calculus. Concepts of inverse central perspective and space resection of conjugate images were first discussed by J. Henry Lambert (1728-1777) in his book "Freie Perspective" in 1759.

Wheatstone of England presented in 1838 the stereoscope, one most important tool used in photogrammetry. The practice of photogrammetry could be started only after Arago and Niepce announced a "Heliographic Process", based on which Louis J.M. Daguerre (1789-1851) presented to the French Academy of Arts and Sciences in 1837 the photographs which he called "daguerrotypes". The coining of the term "photogrammetry" in 1855 by Kersten with its introduction by Meydenbauer in 1867 to international literature, the first German textbook on photogrammetry by Koppe (1889) and Aimé Laussedat's classic work on French photogrammetry (1898) are some of the milestones of analytical photogrammetry recorded in history (ISPRS 1980).

Hauck (1883) established the relationship between projective geometry and photogrammetry. This should be considered to be the most fundamental geometric concept and the basis of most classic analytical photogrammetric developments.

Ernst Abbé, the cofounder of the German Zeiss Works in 1871 started intense studies and tests for optical elements on the basis of rigorous mathematical analyses. F. Stolze discovered the principle of the floating mark in 1892 while Carl Pulfrich also of the Zeiss group developed a practicable method of measuring and deriving spatial dimensions from stereo-photographic images with floating marks. He presented in 1901 the Zeiss-Pulfrich Stereocomparator by supplementing Eduard von Orel's (1877-1941) first prototype Stereoautograph at the 73<sup>rd</sup> Conference of Natural Scientists and Physicians held at Hamburg. Separately, a similar stereocomparator was invented in 1901 by Henry G. Fourcade (1865-1948) of South Africa. He presented this at the Philosophical Society of Cape Town.

Sebastian Finsterwalder (1862-1951) in a series of publications during 1899 to 1937 established a very strong foundation for analytical photogrammetry. In these he brought about the geometric relations which govern resection and intersection as well as relative and absolute orientations. He predicted the future possibility of nadir point triangulation

and the application of photogrammetry to astro-geodetic measurements. He also formulated the basic laws of error propagation in long strip triangulations. He was probably the first person to use vector terminology in photogrammetry literature (Finsterwalder 1899, 1932).

Eduard Dolezal (1862-1955) of Vienna, Austria provided great international driving spirit as he became the founding President of the International Society for Photogrammetry in 1909. He also created the International Archives of photogrammetry.

### 6.2.2 Pre World War II Base Developments

Although organized civil aviation in the early 1900s and the dirigible airships (like Zeppelins and Parsivals) and balloons opened up new explorations, there was a serious setback by the outbreak of World War I in 1914. However, the period between the two World Wars and extended through WW II (1918 to 1945) witnessed tremendous developments in establishing sound mathematical bases and computational tools to provide the necessary foundation for analytical photogrammetry.

During this period developments in analytical photogrammetry were rather infrequent and mainly limited to countries and individual organizations with certain international interlocking involvements and implications. The following would give the highlights of basic developments during this period. One would notice some developmental efforts started during this period extended into the postwar years, as well as certain disappointing events to slow down possible progress.

Reinhard Hugershoff (1882-1941), a professor at the Technical University of Dresden, Germany introduced in 1921 the Autocartograph, the first universal photogrammetric plotter and later at the Second ISP Congress in 1926, the Aerokartograph, a lighter instrument of the universal type which incorporated capabilities of control extension and phototriangulation. Here is an interesting story, which the author learned from Prof. Schermerhorn during his stay at ITC, The Netherlands, indicating one of many undesirable hurdles in the progress of analytical procedures. In 1920 the Dutch government contracted the German Luftbild GmbH (supported by Zeiss) for mapping several islands and a stretch of the Netherlands coast line. Hugershoff applied the pyramid method by using oblique photographs (Hugershoff 1919) and Luftbild applied a method developed by Fischer (1921). The ground control being inadequate, both the results were very unsatisfactory giving scale errors of up to 10 per cent and azimuth errors up to 7 degrees. These obviously created furor in Europe against any further practical application of analytical triangulation in mapping for almost twenty years.

Otto von Gruber (1884-1942), a professor and scientific collaborator of Carl Pulfrich became famous for his landmark publication "Single and Double Point Resection in Space" (1924). His lectures in the Vacation Courses on Photogrammetry at Jena, first published in 1930 (with English reprints published in 1942) provided pioneering theoretical concepts (von Gruber 1942) on differential formulas of projective relations between planes. It was von Gruber who observed in strip triangulation the influence of errors in  $\phi$  on scale and on height. Based on experiments at his initiative, two important facts were emphasized,

viz., the usefulness of auxiliary data and instruments in order to avoid propagation of systematic errors in strip triangulation and the practical advantage of using wide-angle cameras.

Heinrich Wild (1877-1951) presented in 1926 at the Second International Congress at Berlin his modified plotter prototype known as Police Autograph. Subsequently he founded a factory in Switzerland (Wild Heerbrugg Ltd.) where hundreds of well-known and widely used optomechanical autographs, comparators and (now) analytical plotters have been developed and manufactured.

The Kern Co. (now in the Leica group) of Aarau, Switzerland joined the photogrammetric industry in 1930 and continued its contribution up to this date. Umberto Nistri (1895-1962) of Rome, Italy and Ermenegildo Santoni (1896-1970) of Florence, Italy also contributed essentially in designing and manufacturing instruments of various kinds as also in developing numerous corresponding mathematical concepts. Nistri patented in 1919 a method of spatial aerotriangulation. This method, however, was practically applied for the first time to the use of the Multiplex equipment around 1932-33. Georges Poivilliers (1892-1967) of France, Edgar H. Thompson (1910-1976) of the UK are credited with numerous analytical contributions and with designing instruments and stereo-comparators. Thompson's finest contributions were in analytical (matrix algebra) developments, réseaux techniques and aerotriangulation by the method of independent models. Thompson edited the British journal "The Photogrammetric Record" for 14 years. Martin Hotine (1898-1968) of the UK War Office, although primarily a geodesist, published two landmark articles on photogrammetry concepts, "Stereoscopic Examination of Air Photographs" (1927) and "Calibration of Surveying Cameras" (1929).

Willem Schermerhorn (1894-1977) of The Netherlands who became a professor at Delft in 1926, began systematic tests of aerotriangulation in 1932 and applied these ideas to uncharted lands in the East Indies. In close cooperation with Otto von Gruber he contributed much to the understanding of error sources and error propagations in phototriangulation. He was also the initiator of the ISP journal "Photogrammetria". A post World War II Prime Minister of The Netherlands, he was also the founder (in 1950) of the International Training Center (ITC) for Aerial Survey at Delft (now located at Enschede; Schermerhorn 1964). His life was dedicated to the promotion of photogrammetry.

V.P. Nenonen and Y. Vaisälä of Finland developed in 1936 a method of aerotriangulation aided by horizon photographic records and statorscope readings. This initiated the concept of using auxiliary data for phototriangulation.

Bertil Hallert (1910-1971) a professor of the Swedish Royal Technical Institute at Stockholm contributed much in developing numerical relative orientation procedures and establishing the concepts of calibration, and standards for testing cameras, comparators and stereoinstruments. The term "standard error of unit weight" was first used by him.

Starting in the early 1930s through the 1950s, some theoretical and conceptual analytical developments were made at the Federal Institute of Technology, Zurich by Professor Max Zeller and his associates,

some of whom continued their contributions well into the 1970s. Arthur J. Brandenberger and W.K. Bachmann, two outstanding ones of the group made numerous contributions in various orientation concepts. Hugo Kasper, also of Zurich, Switzerland contributed considerably to the general concepts during the 1940s and later.

Earl Church (1890-1956) published a series of 19 articles on computational photogrammetry in the 1930s after the first American Institute of Photogrammetry was established in 1929 under his direction at Syracuse University. The first six articles were bound in a book (Church 1934). Church started a trend in which numerous scientists made significant contributions in the USA for solving problems of space resection, orientation, intersection, etc. One of the approaches developed by Church, on the determination of the camera station (perspective center) coordinates by utilizing an approximate position and an iterative approach, has in effect remained virtually unchanged to this day. He, however, separated the solution for the orientation angles from the camera station coordinates. He also tackled another problem which he called "Determination of scale data" i.e. to compute the dimensions of objects from the photographs without reference to their absolute positions in space. He also turned his attention to the calculations of rectifier settings. He later formalized his procedures by codifying his derivations in the direction cosine notations (Church 1948). The approaches of Church were, however, explicit i.e., with no consideration of redundant observations or data. Also, he never applied any error analysis to his solutions.

### 6.2.3 Post World War II Developments

World War II had a major effect on developments in all countries. Nonetheless, within each European country postwar efforts were somewhat continuation of previous developments. Destruction of manufacturing and service facilities on the European continent had brought the industry to a virtual standstill. However, during the postwar period the centers of evolution were greatly extended to the North American continent where a tremendous relatively high need in mapping and associated control network existed with untapped scientific and industrial resources and capital to support the growth. Real advancements were made with regard to analytical methods only after World War II, although it is recorded (ISP Archives, 1948 and 1952 Congresses) that numerous "experts" would define photogrammetry as the "art of avoiding calculations". Many of them felt that analog plotting machines had achieved sufficient accuracy with regard to detail plotting and contouring. They considered that the only area where further developments were required was aerotriangulation, in which supplementary computational work was always necessary. In this regard, governmental and commercial interests with academic collaboration were successful in establishing steady growths in various aspects in numerous countries. Although these were peace-time efforts, international competitions, national priorities (in planning and developments) and the challenge of outer space provided the stimuli while technological advancements continued to provide the necessary support.

Furthermore, at the beginning of this period, the basic principles of statistics were no novelty to the photogrammetrist. The theory of errors and the method of least squares had served him well.

However, at this stage more and more people started to realize that the modern statistical principles would show how to improve the reliability of various minor operations, analogic or analytic. They even started to realize as to how to plan a job to obtain the maximum amount of information from the number of observations which one can afford to make (due to economic and time considerations) and how to determine the reliability of inference from them.

The units to measure distances remained an unresolved entity to the photogrammetrists. Centuries ago, a foot was defined as the length of 36 barley grains strung end to end and the yard was the distance from the tip of King Edgar's nose to the end of his outstretched hand. Since then we have come a long way. However a confusion did exist internationally, particularly between the two major systems: CGS (Centimeter-Gram-Second) and FPS (Foot-Pound-Second). The CGS system with two variations devised by European Scientists in the 1800s was unified in the early 20<sup>th</sup> century into the MKS (Meter-Kilogram-Second) system. Then in 1954, at the X General Conference of Weights and Measures held at Sèvres, France the Ampere (A) being chosen as the fourth base unit, this system was referred to as the MKSA system. Finally in 1960 at the XI General Conference of Weights and Measures, the system of units proposed in 1954 was officially entitled "Système International d'Unités" with its abbreviation being SI. This being a coherent system, it is now used by over 80 per cent of the people of the world. Although ISPRS encourages this system, its full official implementation encounters difficulties.

As with the units (meter, foot, etc.), there existed a confusing multitude of systems of coordinates. After numerous deliberations finally an International System of Coordinates was accepted in 1956 (ISP Archives, 1956 and 1960 Congresses). Its universal implementation, however, remains yet to be fulfilled.

In view of computational-analytical approaches, the photogrammetrist started to understand in the early 1950s the necessity and importance of items like "random sampling", "test of hypotheses" or "degrees of freedom" at even minor stages of operations involving also such effects as film shrinkage, lens distortion or temperature variation. Questions related to consideration of for example "weight" and "correlation", "observation equations" against "condition equations" or "observations" against "quasi-observations" started to be raised in the computational approaches.

Practically all of such developments are recorded in numerous publications around the world. Thus, before drawing our attention here to the specific developments, it would be appropriate to identify the significant publications or information sources in this regard.

#### 6.2.3.1 Publications

A. Journals. The following journals are special in their presentation of analytical concepts and methods in the English language:

1. Photogrammetria: Currently, ISPRS Journal of Photogrammetry and Remote Sensing; four issues per year.
2. Photogrammetric Engineering and Remote Sensing: Journal of the ASPRS; twelve issues per year.

3. Photogrammetric Record: Journal of the British Society of Photogrammetry; two issues per year.
4. CISM Journal: Journal of the Canadian Institute of Surveying and Mapping; four issues per year.
5. Australian Surveyor: Journal of the Australian Society of Surveyors; four issues per year.
6. ITC Journal: Journal of the International Institute of Aerospace Survey and Earth Sciences (ITC); four issues per year.

Certain other national journals in their respective languages deserve mentioning here, in particular, the Belgian, French, German, Russian and Swiss.

B. Conference Proceedings: The following conference proceedings are regularly published:

1. ISPRS Archives: During or following each ISPRS Congress (quadrennial) or each Inter-Congress ISPRS Commission Symposium.
2. ASPRS Proceedings: During or following each ASPRS Convention (two per year, Annual and Fall Conventions).

There are also many national and regional international conferences publishing their proceedings from time to time.

C. Books: Practically all text books and manuals in photogrammetry contain analytical concepts to a certain degree. However the following are so far the only books specifically devoted to analytical photogrammetry:

1. Merritt, Everett (1958): Analytical Photogrammetry; Pitman, N.Y.
2. Ackermann, F. (1973): Numerische Photogrammetrie (Herbert Wichmann Verlag, Karlsruhe, Germany)
3. Ghosh, Sanjib K. (1988): Analytical Photogrammetry (2<sup>nd</sup> Ed.); Pergamon Press.

Numerous books with significant contents in analytical approaches published in various world languages are appearing on the market.

#### 6.2.3.2 Related to Single Images

The theory and mathematical model for central perspective projection being well established through the pioneering prior works of men like Pulfrich, von Gruber, or Finsterwalder, the basis of Collinearity Condition was already there. This condition implies that the object point, the perspective center (or the exposure station) and the image point must lie on the same straight line. However, in its application through the computational procedures there were two problems. Firstly, the condition equations are non-linear and, secondly, in usual cases more observations are made than the minimum necessary for unique solutions. Therefore, to obtain practical and statistically acceptable solutions, it was found appropriate and convenient (1) to use linearized forms of the equations, (2) assuming iterative approaches, to consider only the first order terms, and (3) to use the least squares approach to account for the redundant data. It was almost universally found convenient to use the "Taylor" expansion for such linearization instead of using Newton's first order approximation. By mid 1950s the use of the collinearity equations was deep-rooted, its form being different according to the specific application case. For example, the standard form, linearized, was found convenient for

simple images consolidated into strip or block triangulation whereas its direction cosine form was found convenient for camera calibration (Brown 1956).

Mathematical models for interior orientation parameters have been established (Brandenberger 1948) as also those for camera calibration to include radial and tangential (decentering) lens distortions. The following general hypotheses of Conrady (1919) were accepted:

- (a) The objective (lens) axial ray passes undeviated through the lens;
- (b) The distortion can be represented by a continuous function; and
- (c) The sense of distortion should be positive for all image displacements in outward radial direction.

Tham (1946) established certain convincing ideas on lens distortion. Thereafter, through various research at numerous facilities the best accepted mathematical model to express a radial distortion is an odd order polynomial typified in the publication of Brown (1956) and Washer (1941, 1957).

With regard to the tangential distortion, Washer (1957) called it the Prism effect. Based on his concepts and the hypotheses of Conrady (1919), the mathematical model mostly accepted internationally was the one presented by Brown (1966).

It was already known prior to World War II that the emulsion carrier (film or glass) is subjected to dimensional distortions, which are functions of the material, environment (like temperature, humidity or pressure), aging and treatment (like chemical processing or drying). While the effect could be checked against camera calibration data, its compensation in the analytical approach was found easily through a two-dimensional similarity transformation of the photo-coordinates. Differential (systematic) distortion could be corrected by adapting affine (linear) transformation or by using projective equations. Simple equations were being innovated and programs were being developed to these effects in the 1950s. However, irregular distortions caused primarily by lack of film flatness or image motion continued to be causes of concern. The réseau (grid) photography developed in the UK, first described in 1951 by H.A.L. Shewell at the Commonwealth Survey Officers' Conference and published later (Shewell 1953), provided meaningful possibilities in this regard.

Mathematical modelling of atmospheric refraction has always followed the ideas obtained from Geodesy. However, most modern concepts easily adaptable to analytical procedures were established by Leyonhufvud (1953). Following further research the most accepted mathematical model is an odd order polynomial with regard to the radial distance of a point on a vertical photo. The concept is based on the acceptance of a Standard Atmosphere. There being several well known standard atmospheres [like US Standard, Air force Rome Development Center (ARDC) and International Civil Aviation Organization (ICAO) Standards] controversy persists, although all these are practically the same up to about 20 km flying height. Satisfactory concepts in this respect with regard to oblique photography and satellite imageries are yet to be developed.

The problem of Image Motion Compensation (IMC) remained unsolved until Kawachi (1965) derived the

formulas with regard to only the rotational movements and certain corrective (although partial) approaches were developed to provide film or camera movements during exposure by using one of several IMC devices. The complexities has been resolved by Ghosh (1985) through augmenting the collinearity condition equations. This approach is fully computational.

#### 6.2.3.3 Related to Stereo Images

It was around 1953 that the classic analog concept of relative orientation by way of elimination of y-parallax evolved into the condition of coplanarity through the efforts of Schut (1956-57) at the National Research Council of Canada. This condition implies that the two perspective centers (or exposure stations), any object point and the corresponding image points on the two conjugate (overlapping) photographs of the stereo-pair must all lie in a common plane. This condition is fundamental to relative orientation or space intersection. Like the collinearity equations, this condition equation is also non-linear and need to be linearized (for computer utilization) with iterative solutions in mind. The relative orientation formulation developed by E.H. Thompson (1959) showed complete elimination of trigonometric functions with a consequent ease and speed of computer utilization. Separately, Paul Herget in developing a system of analytical control extension, by using vector notation, minimized the perpendicular distances between pairs of corresponding rays in order to achieve a solution for relative orientation. He employed an ingenious trick whereby ground control equations took the same form as relative orientation equations (Herget and Mahoney 1957).

On the other hand, the superiority of the numerical relative orientation (over empirical and graphical methods) was definitively established. Also were established the processes of improving such relative orientations (Ghosh 1964). Notwithstanding the analytical conditions of collinearity and coplanarity, the on-line solutions at analytical plotters are all developed practically around such analogical-numerical concepts.

The process of absolute orientation (i.e. scaling, translating and levelling of a stereo model with respect to a ground reference coordinate system) is simply a problem of coordinate transformation. The equation must be linearized before it can be used. The method of least squares may also be used. This approach was standard already by the early 1950s.

It was readily found that during a sequential procedure of aerotriangulation the scale of a previous model needs to be transferred to the next model. This is similar to the requirements of the analog aeropolygon method. This process was mathematically modeled at the NRC Canada (Schut 1956-57) and is known as the scale restraint condition. This condition implies that with regard to a point in the triple overlap area (i.e. area of overlap between adjacent models) the two intersections in individual models must take place at the same spatial location. This condition is always used in conjunction with the coplanarity equations.

Theoretical concepts of bi-projective transformation (Das 1952) and the use of distances in the object space as control for stereo-models (Das 1973, Okamoto 1981) are purely computational

approaches that would prove extremely efficient in various applications of stereophotogrammetry.

#### 6.2.3.4 Related to Multiple Images

The application of analytical procedures on which most discussions and efforts have been made is that of phototriangulation. As early as the beginning of World War II, the need was typified in the following quote (Schermerhorn and Neumaier 1939): "The problem of control points was and is still, to a certain extent, the bottleneck in photogrammetric map production". Initial efforts were with regard to the adjustment of analog aerotriangulation. Later efforts concentrated on fully analytical procedures. Their classifications and historical developments would be apparent in Fig. 6.2

##### A. Adjustment of Analog Aerotriangulation

Historically, the development may be noted in terms of three stages:

**Stage 1: Adjustment of individual strips** along with associated data analyses and interpretations. The works of Thompson (1953), Roelofs (1949) and Gotthardt (1944) give typical indications of the initial studies. One would notice at this stage the prolonged discourses on the causes and propagation of random errors over those of systematic errors. One can refer to one single publication to typify the culmination of this stage in the OEEPE (Organisation Européenne d'Études Photogrammétriques Expérimentales) report for studies up to the end of 1959 (Solaini and Trombetti 1961). This study, initiated in 1956, concerned international efforts at twelve research centers and analyzed the results of some dozens of strip triangulations by using different adjustment procedures.

Several scientists got involved in such studies in the OEEPE group or separately and left their marks in numerous publications of each of them, such as, W.K. Bachman, A.J. Brandenberger, A. Bjerhammer, A. J. van der Weele, A. Verdin, P. A. Vermeir, J. Zarzycki and M. Zeller. The efforts of the ISP Commission III in this regard were very significant (see Cassinis and Cunietti 1964). The OEEPE (1973) publication indicates the termination of experimental researches of this stage, having the attention already passed from the treatment of isolated strips to that for an entire block. The highlights of this stage were: (1) Adjustment of aerial strip triangulation was approached by using condition equations; (2) The least squares principle was being applied to the condition equations; and (3) The polynomial corrections of point coordinates were affected by considering third order in X, and second order in Y and Z strip coordinates. It was also felt at the end of this stage that the measuring instruments and the operational procedures needed improvements more than the mathematical adjustment procedures.

**Stage 2: Adjustment of Blocks of Strips**. By the end of 1950s in view of the developments that electronics brought about in the computation processes new challenges concentrated on simultaneous adjustment of blocks of strips, the models of which have been formed by analogical procedures. In this regard, apart from numerous individual efforts in the world, the one most significant study which would indicate the progress is the report on the coordinated group study under ISP Commission III on "Massif Central" polygon

(Cassinis and Cunietti 1964). There were twenty tests on the whole performed in six countries. The following would give the highlights of the tests:

- In six out of eleven cases, the bridging of models was done by analytical methods.
- The strips were adjusted in the block, not only with analog or empirical procedures but also with analytical procedures using polynomials (second and third degree) and least squares method.
- Transverse (tie) strips were used in the adjustments.
- Most desirable disposition of control points were investigated with concluding ideas on precision, economy and time related efficiency considerations.
- Comparative studies were made between procedures using models formed with comparator observations against those established with analog plotting instruments.
- With the final objective of analyzing the intrinsic precisions, certain approaches were studied for the separation (filtration) of random errors from the systematic errors.

Two specific adjustment programs deserve special mention in this regard, one developed at the NRC, Canada (Schut 1966) and the other at the IGN, France (Masson d'Autume 1960). At this stage, however, one could note the closing of the era of aerotriangulation by strips (analog aeropolygon or areolevelling), the opening of aerotriangulation by blocks (or sub-blocks) and the appearance of a new trend with dismemberment of the strips into its constituent unit, the model.

**Stage 3: Adjustment by Independent Models.** Among the technological developments of the 1960s there is the computer with its exceptional possibilities of logic, memory and calculation capabilities which brought forth in photogrammetry very important changes not only in the data processing but also in the instruments themselves. The first step in this evolution was the development of the "semianalytic triangulation". The instrument bridging through coorientation and scale transfer was being replaced by computational procedures and was thus able to improve the precision by way of eliminating instrumental errors occurring in instrumental bridging. By so doing, only the formation of individual models was done at the instrument (analog or analytical) whereas the bridging, formation and adjustment of the block was being performed off-line at a computer. Numerous tests were performed world-wide. One can refer to the works of F. Ackermann, G.S. Schut, G. Inghilleri, E.H. Thompson, G. Togliatti, S.K. Ghosh, C.W. King, V.A. Williams and H.H. Brazier, to name a few. Yet, further block triangulation studies continued (Ackermann 1966, van den Hout 1966). One found, however, that the more a block is subdivided into the elements, the simpler the equation structures became. On the other hand, the problem of obtaining the adjusted values became more cumbersome. Thus, the various methods of adjustment procedures would not be basically different in the theoretical formulation of the fundamental equations, but they would differ in the computational procedures needed to handle a large amount of data and this in order to solve systems with unknowns of other kinds, and to elaborate procedures for evaluating the relative and absolute precisions of the adjusted coordinates. This also required the skill of the computer technologist rather than that of the photogram-

metrist. People were looking for "tricks" in the computer utilization rather than in the photogrammetric procedures. Thus, of necessity, people were yielding to the computer. In the program ITC-Jerie Anblock, the adjustment of planimetry is completely different from that for altimetry (van den Hout 1966). Obviously such approaches were inspired by previous works of recognized experts (Ackermann 1964, Jerie 1964).

By the end of 1960s one finds that the use of analytical photogrammetry was no longer limited to research institutes (academic or national mapping related organizations). It began to be used (due to the operational ease, obtainable precision and rapidity of production) in the private sector together with the commercialization of programs developed at the institutes. For example, the Stuttgart University program developed under the direction of Ackermann reached world-wide diffusion, as well as subsequent programs like RABATS developed by J.F. Kenefick associates and SPACE-M (or PAT-M) developed by the Canada Department of Energy, Mines and Resources.

#### B. Analytical Aerotriangulation

The major thrust of completely analytical aerotriangulation has been in the U.S.A. Inspired and initiated by people like Schmid (1959), the basic approach involves the observation of image coordinates only. The elemental unit is not the model any more but the photogram and the implied condition is that of collinearity of the optical ray containing the perspective center (camera station), an image point and the corresponding object point. During the development, however, there have been several digressions.

Much of the work during and immediately before World War II in the USA was done at the Tennessee Valley Authority. One of them, Ralph O. Anderson (1947) proposed a scheme in which orientation of photographs would be done semi-graphically while the main scheme of control extension would be done analytically. This, however, could not compete with pure analogical procedures primarily because of economic reasons.

During the war and the following years, the US Naval Photographic Interpretation Center developed a series of analytical solutions for camera calibration, space resection, interior and exterior orientation as well as relative and absolute orientation of stereo-pairs (Merritt 1951).

At the US Ballistic Research Laboratories, Aberdeen, MD. as a consequence of research directed towards ballistic camera operations in which several cameras may observe an event simultaneously, the application of these procedures into strip and block triangulation followed immediately. These were primarily the efforts of Hellmut Schmid (1951, 1959) who later joined the US Coast and Geodetic Survey. The principal features of Schmid's work are a rigorous least squares solution, the simultaneous solution of multiple photographs and a complete study of error propagation. Schmid (1974) was successful in extending his ideas in performing a three-dimensional geodetic triangulation by using passive (reflecting surface) earth satellites observed with ballistic cameras from 45 stations around the earth. He was probably the first photogrammetrist to look for solutions in anticipation of the use of high speed computers (off-line). His early reports

were written in vector notation. Later on he introduced matrix notations.

The first operational system of analytical aerotriangulation was developed at the British Ordnance Survey in 1947 with analytical radial triangulation in order to provide control for the large scale (1:2500) resurvey of Britain. This approach was abandoned in favour of using spatial triangulation with réseau photography measured at the Cambridge Stereoscomparator (Shewell 1953). The complete system modified in view of computer implementation was described by Arthur (1959).

Paul Herget (1957) in his method of analytical control extension proposed the simultaneous solution of an entire strip but the eventual implementation of his system developed ultimately as a cantilever strip, photo by photo. The Herget method, under contract from the US Engineer Research and Development Laboratories, and taken up by Cornell University, was next developed into a method capable of simultaneous solution of a block by way of utilizing either ground point or exposure station control (McNair et al 1958). This method was adopted by the US Geological Survey and developed, what is known, as the "Direct Geodetic Restraint Method" (Dodge 1959).

There have been interesting developments in Japan (Ryokichi 1960) and in the USSR (Lobanov 1960) also apart from those in the European and North American countries.

At the National Research Council of Canada, Schut (1957), among others, recognized the theoretical superiority of a simultaneous block solution but discarded it in favour of a cantilever strip formation because of computer limitations. The strip triangulation was originally programmed in 1953 for the IBM 650 which contained a 2000-word drum. Later the program was used with a simulator on the IBM 1620. Increasing use of analytical triangulation shown by the demand for copies of the program required further revision and its large scale production. A FORTRAN version with a complete description is given in Schut (1973).

Duane Brown, earlier an associate of H. Schmid, has made major contributions to the analytical treatment of aerotriangulation as also in various engineering problems. Brown's principal contributions are the following: (a) Treatment of all orientation parameters as either known or unknown; (b) Solution of normal equations achieved by partitioning their matrix to separate the orientation elements and ground points; (c) The method of introducing ground control points and the air-station parameters with appropriate weights, thus making it possible to include auxiliary data without, however, disturbing the basic mathematical model; and (d) Development of a new mono-comparator which works on the principle of self-calibration (Brown 1969).

Certain important contributions were made at Cornell University under the guidance of Arthur McNair by Anderson (1964) and Mikhail (1963) in developing the Triplet method (Anderson and McNair 1966). In this method the rigid elemental unit for the strip or sub-block formation is obtained by using three consecutive photographs (hence the name Triplet). Each triplet is overlapped with the adjacent one. The method was adapted for official use and yielding good results by the US Coast and Geodetic Survey (currently NOS, NOAA) (Keller and Tewinkel 1966).

Through the group studies on Analytical Block Triangulation sponsored by ISP Commission III during 1964-68 it has been shown that (1) Excessive sidelap (e.g., 60% as against 20%) does not yield much improved accuracy in block adjustment; (2) Control in the periphery of a block (at least in the corners) would greatly improve the accuracy of block triangulation; (3) Additional auxiliary data as additional control in the central area of the block would also improve the accuracy; (4) The precision is practically independent of the block size but is directly related to the available control, its quality and distribution.

By the end of the 1960s, we reach a stage when simultaneous analytical block triangulation reached a level of maturity. Comparators (both mono- and stereo-) of various manufactures and designs came on the market, powerful computers were available and usable economically as well as complex and refined programs for computations and adjustments were developed. The simultaneous procedure known also as "Bundles" method was improved and adopted by many organizations (Matos 1971, Wong and Elphinstone 1972, Schenk 1972) and at numerous centers in Germany, Finland, Italy, Canada and the USA. In spite of these developments, however, the method of independent models remains very popular and is generally found to be more cost-effective. In this, the solution of the normal equation system has been found to be critical as far as the preparation of the computer programs is concerned. A direct method by using submatrices as units and a Cholesky solution was adopted finally. This method has been called Hyper Cholesky (abbr. Hychol) and it has proved to be suitable particularly for banded or banded-bordered matrices (Ackermann et al 1973).

Brown et al (1964), in taking advantage of the characteristics of sparseness of the solution matrix, utilized the indirect method of Block Successive Over-Relaxation (BSOR). The unique characteristic of this approach lies in the use of an indexing scheme whereby (a) only non-zero submatrices of the Normal equations would be formed, and (b) these would be stored and operated on in a collapsed form. This procedure, however, had two disadvantages, viz., (1) a very slow convergence in case of few control points and (2) the problem caused by the impossibility of computing the inverse matrix. The shortcomings of the BSOR reduction were later avoided by utilizing an algorithm called Recursive Partitioning developed by Gyer (1967).

The history of analytical aerotriangulation would not be complete if we do not mention the conceptual contributions of self-calibration applied to aerotriangulation (Ebner 1976, Kilpela" 1980), the inclusion of auxiliary data (Inghilleri 1961; Blachut 1957, Brandenberger 1967; Zarzycki 1964) and geodetic measurements (Brandenberger 1959, Wong and Elphinstone 1972). Any of these would help enhance the quality of block triangulation.

An important contribution was made by Case (1961) when he showed how a substitution of parameters may greatly reduce the rank of the normal equation matrix, while at the same time decreasing the error accumulation by constraining the parameters in terms of weight or of function or both. The concept was developed further by Schmid and Schmid (1965) in a general procedure for the method of least squares where all elements of the mathematical model are considered as observation data, the burden of classification being on the weight (constraint) matrix. Thus, for example, by simply

assigning infinite weight to the control as against the orientation parameters, one can use the same computer program for space resection as against space intersection. Further contributions have been subsequently made (e.g., Mikhail 1970) with regard to constraints.

Notwithstanding the advancements in analytical aerial triangulation, it has been observed that no result is free from gross errors, search and elimination of which are seldom cost-effective. Baarda (1967) developed for Geodesy a procedure called "Data snooping" which initiated similar studies in aerial triangulation. Baarda's concept of Internal reliability for all bound values of a data snooped system has been used to define also External reliability of the upper bounds of the influence of non identifiable gross errors (Förstner 1985, Gruen 1979). In any case, it is also established that a clear subdivision between very small gross errors and the random errors may not be possible and, therefore, certain errors would remain in the adjusted block. A trend in research toward perfection in this regard is being noticed.

An interesting extension of the self-calibration technique as applicable to Bundle Block Adjustment has been tried by Ebner (1976) in which additional parameters to consider systematic image deformation (two-dimensional) and model deformation (three-dimensional) are included. Computationally, these could be handled by stochastic mathematical models (by treating the additional parameters as free unknowns) or by treating them as problems of Collocation (Moritz 1978) in terms of prediction and filtering of noise and signals.

With the advent of the analytical plotters in the early 1960s came the concept of on-line solutions (Cunietti et al 1964). However, with regard to on-line aerotriangulation people found that sequential procedures are comparatively more cost-effective than the simultaneous solution at an analytical plotter (Strahle 1971, Hobbie 1978). The block formation with least squares adjustments can be also achieved with success in such approaches (Dorrer 1978).

#### 6.2.4 Unconventional Technologies and Advancements

Man's involvement in space and international conflicts like World War II, Korean and Vietnam wars brought about certain technological developments in imaging systems and data processing procedures. These posed tremendous challenges in photogrammetry. Initially some of the pertinent studies were necessarily classified. However, apart from unconventional applications of analog and conventional photogrammetry, innovations and procedural developments in analytical photogrammetry occurred which deserve mention here. Such developments have been noticed during the 1960s and later.

Case (1967) demonstrated how the standard collinearity equations can be modified to handle some panoramic and strip photographic systems. Recent advancements indicate subsequent supportive researches related to various unconventional imaging systems (Masry 1969, Derenyi 1973, Clerici 1977, Kratky 1983, Ghosh 1975, Takamoto 1976 are some typical examples). However, apart from such adaptations of analytical photogrammetry to unconventional technologies and systems one can notice two distinct areas of new thinking and development viz., (a) Digital Terrain Models (DTM) and (b)

Real-Time Photogrammetry, which deserve particular attention here.

##### 6.2.4.1 Digital Terrain Models (DTM)

The DTM concept had its origin in the work performed by Charles L. Miller (1957) and his associates at the Massachusetts Institute of Technology in the USA. The objective was first to expedite highway design by digital computation based on photogrammetrically obtained terrain data in three-dimensions.

Eventually, the concept has been developed by considering ordered arrays of numbers that represent the spatial distribution of terrain characteristics. In the most usual case, the spatial distribution is represented by X and Y (planimetric) coordinates and the terrain characteristic is recorded as the terrain elevation, Z. Recent literature (ASP 1978) has referred to these and other distributions (like latitude, longitude against elevation) as Digital Elevation Models (DEM) to distinguish them from other models which describe different terrain characteristics. The data are also organized as equations of surface defined by polynomials or Fourier series. One may note also that characteristics other than mere elevation, such as terrain slope, land value, ownership, land use or soil type, may also be included in the DTM (Doyle 1978). The distribution of points and their coordinates required in the process of digitizing are not automatically fit as final output for numeric mapping. This necessitates interpolations of various kinds (Schut 1976). Numerous researches and program developments in this regard have already been undertaken (Ebner et al 1980).

Some of the direct applications of DTM are: Generation of profiles, Generation of contour lines, Generation of perspective views, Earthwork calculations, Terrain simulation, Terrain object models (in plaster) and Computer-controlled cartography providing digital cartographic data base. Several national mapping organizations, notably in the USA, Canada, FR Germany and Australia are producing digital cartographic data bases. One can reasonably expect that in the near future most of the time-critical manual cartographic operations would be superseded by completely automated systems with only the critical data-interpretation, judgement and decision making operations requiring human intervention.

Digital mapping based on the DTM concepts and related developments is flexible and offers advantages to allow very efficient and cost-effective mapping particularly by using automated correlation plotters like the Gestalt Photo Mapper by Hobrough (Allam 1978) and this together with the constantly improving computer technology. Such approaches are expected to be of great help in the near future.

##### 6.2.4.2 Real-time Photogrammetry

The revolutionary technological developments that started in the 1970s in fields like microelectronics, semiconductor crafts and photonics have influenced fundamental new thoughts and researches aimed at obtaining better efficiencies in the recording, data processing, data storage and administration phases of photogrammetric operations. In computer science, a "real-time" system is understood as "the



processing of information or data in a sufficiently rapid manner so that the results of the processing are available in time to influence the process being monitored or controlled" (Sipple and Sipple 1972).

In view of improving efficiency and with regard to certain time-critical operations, in photogrammetry certain "on-line" operations have been previously meant to be in connection with the "real-time loops" in analytical plotters or the "real-time data transfers" from spaceborne sensors. Real-time, however, does not mean zero time. For example, real-time in the context of data acquisition and data processing is generally considered such that the response time of a process must be within one video cycle which is between 0.03 and 0.04 second, depending on the video standard. On the other hand, real-time performance depends not only on the amount of data but also on the complexity of data, type of required results, algorithms and hardware used.

At a recent symposium organized under the auspices of the ISPRS Commission V (Archives 1986, 26/5) one would note that a real-time photogrammetric system is meant to process digital images, where the heart of the system consists of an image-processing unit. Image enhancement could very much be a part of the process. Detection, recognition and information extraction are pertinent phases of such systems.

The hardware components of a real-time photogrammetric imaging system may contain (1) Charge Coupled or Charge Injection Devices (CCD or CID) or even Photodiodes, (2) Image Data Transfer facilities and (3) Frame-grabbing facilities (to hook onto non-standard analog signals like Analog-Digital converters).

In these regards, biomechanic applications were among the earliest real-time semi-photogrammetric systems found in literature (e.g., Woltring 1974). These applications were followed by other unconventional applications like industrial measurements (Pinkney 1978), and machine vision (Haggren 1986). It is apparent that real-time photogrammetric systems offer, due to their fast operational abilities, further possibilities with regard to system calibration, network design, control feedback and interactive processing (Gruen 1988).

### 6.3 INSTRUMENTS

The historical development of instruments in analytical photogrammetry can be viewed with regard to the following groupings (functional component related):

- (1) Instruments for data acquisition (comprising comparators, point marking and transferring devices).
- (2) Instruments for data processing (comprising image space plotters, analytical plotters and hybrid or converted plotters).
- (3) Instruments for data display and presentation (comprising plotting tables, software systems and orthophotoprinters).

One would notice, however, considerable interactions amongst such components in view of diverse photogrammetric activities, conventional (in aerial and terrestrial mapping) and unconventional (technologies or applications related). Such interactions would be mentioned as deemed appropriate. With regard to analytical

solutions and use of computers, these can be viewed as being off-line (like a comparator) or on-line (like an analytical plotter).

#### 6.3.1 Data Acquisition Instruments

##### 6.3.1.1 Comparators

A. Monocomparators. The first monocomparator was uniaxial. Designed, under the guidance of Abbé, by the Zeiss factory at Jena at the turn of the century, the Abbé-Comparator was in strict adherence of Abbé comparator principles. These principles, formulated in 1890 but initially published in 1906 state that a comparator design be based on two requirements: (i) To exclusively base the measurement in all cases on a longitudinal graduation with which the distance to be measured is directly compared; and (ii) To always design the measuring apparatus in such a way that the distance to be measured will be a rectilinear extension of the graduation used as scale.

The Abbé principles were subsequently adopted in most mono- and stereocomparators. However, to this date, there are comparators where the principles are violated. Komess No. 2 instrument introduced by VEB Carl Zeiss in 1964 (ISPRS 1964) was one with full implementation of the principles of Abbé. Uniaxial or single-coordinate comparators are not favoured in photogrammetry. However, to this day, such instruments (e.g., Mann Type 841) find their use with success in numerous fields of science and engineering like readings on spectrograms, interferograms and X-ray diffraction patterns.

It was understood always that a stereocomparator comprises, in principle, two monocomparators, of which one photo-carriage (specifically the left-side one in most stereocomparators) could always be used as a monocomparator. However, numerous single carriage comparators were designed before and after World War II in view of manufacturing economy and certain conveniences (like portability and specialized applications).

The most common measuring device is the leadscrew or spindle in combination with a nut. Some have single leadscrews with which only one coordinate can be read at a time. After all points on the photo are read in one coordinate (say, x), the photo is rotated through 90° and the other coordinate (y) for all points are read next. This requires double pointing, and consequently, more time and some complications in data refinement. Accordingly, double leadscrew comparators were developed.

A drum attached to the end of the screw, graduated in decimal divisions with regard to the pitch of the screw typified in Mann 422 is very common in such equipment. During the 1950s, however, by using glass-scales and verniers (or micrometers) the need for temperature control in the laboratory was reduced. One earlier developed example in this regard was the Zeiss Jena Koordinatenmessgerät 3030 where a spiral micrometer has been used. This permits easy reading to 0.1 μm.

Grid (réseau) plate and micrometers were used next in the 1960s for example in the Canadian NRC monocomparator. Further developments involved the use of optical diffraction gratings having a precisely known number of lines per unit length (2160 lines per mm) and a moiré fringe pattern (Ferranti fringes) derived therefrom. Such measuring systems are entirely free from friction

and wear. Minor error sources like dust or scratches have no appreciable errors. This was also the period when other systems of measurements were developed, like interference (Chitayat 1960), DIG (a non-incremental system capable of determining absolute positions over an unlimited range, developed by Bausch & Lomb, Inc.) or computer-controlled measurement with TV cameras, operator consoles, point locators and computer systems with card, tape or hard copy output possibilities (ASPRS 1980).

During the 1960s the readout systems evolved from manual (optical-mechanical) to hard-copy printout to automated systems permitting point identification and recording of data on punched cards, punched tapes (paper or magnetic) in such formats as are directly usable at a digital computer or an automatic plotter. A notable example is the OMI Nistri TAL/P introduced around 1964. Thus possibilities of human blunders were minimized, observational speed was increased and the working systems approached near real-time efficiencies.

In spite of the fact that the ISPRS 1976 Helsinki Congress witnessed an influx of Analytical Plotters (see section 6.3.2), one sophisticated monocomparator was also introduced at the same Congress, viz., Zeiss Oberkochen PK-1 having a two-dimensional linear measuring system with index grating, binocular viewing at various magnifications and linear pulse measurements with guide-error compensation (ISPRS Archives 1976).

The historical development of monocomparators would be incomplete if we fail to mention the superb mathematical concept of self-calibration implemented in the Multilaterative monocomparator introduced by Brown (1969). It is a single-image, portable comparator operating on the principles of trilateration. A self-calibrating solution provides all x, y photo coordinates with no further need of refining the data with regard to comparator generated errors. The only operational weakness of this instrument is that it requires four settings of the same photo for reading scale distances along the same glass scale (pivoted).

B. Stereocomparators. Following the invention of the floating mark in 1892 (see Section 6.2.1), the first ever instrument for three-dimensional analytical procedures was the stereocomparator developed separately, independently and almost simultaneously by C. Pulfrich of Zeiss in Germany and H.G. Fourcade in South Africa, both in 1901. In the Zeiss stereocomparator, the optical system shifted in y and the plates in x while the differential movements  $p_x$  and  $p_y$  (x and y parallaxes) were applied to the left- and right-side plates, respectively.

Similar parallax adding design continued in most stereocomparators manufactured way into the 1960s (e.g., Carl Zeiss 1818, Wild STK-1 and Stereometer SM-4 by Drobishev of Stankoimport, USSR).

Note: In this concept, if the point coordinates on the left-side photo are  $x'$  and  $y'$ , the x and y coordinates of the conjugate point on the right side photo are given by  $x'' = x' + p_x$  and  $y'' = y' + p_y$ , obtained by adding the differential coordinates (parallaxes) needed in restoring stereoscopic coincidence at that point.

Zeiss introduced in 1926 (originally made by van Heyde of Dresden) their new stereocomparator in which the plates were shifted in both x and y

directions (Hugershoff 1930). This design concept was adopted in several comparators with success (e.g., Nistri RIC-1, SOM-Sopolem stéréocomparateur, Mann - USA 1740 A and Cambridge stereocomparator).

By late 1950s, the above mentioned concept was improved in using glass plates with measuring réseau grids. Negatives or diapositives are placed with emulsion surface directly in contact with these grid plates, one for each photo (as in Zeiss PSK and PEK). Around the same time, measuring procedures based on grids printed on the negative at the time of exposure or in the stereocomparator were also developed (as with Hilger-Watts Stereocomparator, see Lawrence 1949). Side by side, different types of measuring (floating) marks other than black circular shaped dots were developed (e.g., luminous marks at Nistri RIC-1 or half-cross graticules at the Hilger-Watts stereocomparator). Other innovations involved three-plate (two-model) instruments as in Nistri TA3-A as well as the addition of a recording camera, a typewriter, electronic data registering devices (like Wild EK-6), special mechanical reading systems (like Ferranti system by using moiré fringe pattern), and photo carriage movements supported and guided by frictionless air bearings.

By the 1980s, practically all manufacturers of comparators have ceased to offer such instruments. The only exception to this trend are the Zeiss (at Oberkochen with its PSK series and at Jena with its Stecometer). One notable development has been the Dicometer by Zeiss Jena to replace the Stecometer (Starosczyk et al. 1986). Instead of the classical stereocomparator movements of x, y and  $p_x$ ,  $p_y$ , the Dicometer features two quite independent cross-slide movements, one for each plate ( $x'$ ,  $y'$  and  $x''$ ,  $y''$ ). This has been done possibly with a view to using the Dicometer in the future as the analogical (opto-mechanical) portion of an analytical plotter.

With regard to stereoscopic viewing and mensuration, the most recent invention concerns the "Floating line" where the specific distance and direction of a line in a stereo model can be viewed and established with digital data. The idea was first presented in Ghosh and Boulianne (1984) with subsequent world patents and numerous related publications. It must be mentioned, however, that a similar analogical concept with very limited scope was introduced in the 1940s in the Topographic Stereometer of Drobishev (USSR).

#### 6.3.1.2 Point marking and transferring devices

With the development of aerotriangulation (see Section 6.2.3.4) as early as the 1940s it was found desirable (ISPRS Archives 1948) to mark and transfer unmistakable points on the photos with a view to having uniform point marks at ideal locations and avoidance of ambiguity in identification, selection and even presignalization. The first success in the strives for mechanizing the process was realized by P. Dongelmans of the International Training Center, Delft during the mid 1950s. His Snap Marker comprises a metal frame containing a plexiglass disk to be placed on the photo. The setting mark, a steel ball of 0.2 mm diameter, protrudes from the lower side of the plexiglass disk which in turn exercises a resilient pressure on the photo. For exact setting, the marker can be displaced in the metal frame in orthogonal directions with the aid of two knurled screws. A spring-loaded small hammer strikes the steel ball to produce a circular hole of 0.1 mm diameter in the emulsion of the

photo (on film or glass plate), surrounded by a grey bulge. Two snap markers would be necessary to transfer a point under a stereoscope.

Zeiss Oberkochen marketed in 1960 (Brucklacher 1961) their point marking and transfer device (MK). This device consists of a rectangular metal frame in which a mechanical marking unit on one side and a glass plate with point setting on the other side are accommodated in such a way that they can be rotated. A circle of 1 mm diameter can be engraved around the point in the emulsion of photo with the aid of a marking cylinder. A point mark in the center of the identification circle can be produced with a minute needle prick being activated by a knurled knob intended for producing such artificial points. Two such units are necessary for marking and transferring points under a stereoscope.

Further developments in these instruments were made during the 1970s. All of these can accommodate film and glass plate photos. Some allow for permanently marking an annular ring around images of interest or around a small drilled or heat-marked hole in the emulsion to permit rapid location (like, Kern PMG 2, Wild PUG 4, Zeiss Oberkochen PMI and Zeiss Jena Transmark B laser point equipment). Most of these are equipped with zooming optics for continuous magnification in viewing.

Further modernization is noted in the Wild PUG 5 developed during the 1980s. It uses two ultrasonic marking heads. An electric vibration is transformed into a mechanical vibration to generate heat for the conical marking head to inscribe the mark as it penetrates the emulsion. Various marks of diameters between 40 and 150  $\mu\text{m}$  are provided along with viewing magnification variable continuously between 5.5x and 30x.

In off-line procedures, two different paths have been followed, each with a certain degree of automation. In the first, point selection, marking and transferring are separated from mensuration, such as with a Wild PUG, to be followed by reading at a monocomparator. In the second procedure, point selection (without marking), selection and mensuration are combined in a single instrument, the stereocomparator. Little has been done to automate the point selection, marking and transfer at a comparator. In this regard, the only instrument developed in the 1970s is the Kern CPM 1 which serves as point marking and transferring device as well as a monocomparator and a stereodigitizer.

One development is worth mentioning here - this being the latest in the automation of monocomparator operation - the Automatic Réseau Measuring Equipment (ARME, see Roos 1975). This instrument, when given calibration (or approximate) values of coordinates of réseau (grid) intersections on a photograph, under computer control drives to the vicinity of the point and next automatically centers on and measures the point coordinates.

Attempts were made in the late 1960s to automate the stereocomparator process, as exemplified in the OMI-Bendix TA3/PA. However, such instruments, equipped with correlators, never arrived at sufficient accuracy. Finally, these functions have been incorporated in the analytical plotters, in the more recent versions of which the instrument is able to find the originally selected point solely from the previously measured coordinates, thereby reducing the necessity to mark the points to be

transferred and measured. Some highly promising experimental versions (see De Meter 1962) of automated total (marking, transferring and measuring) systems did not, however, eventually materialize as they failed to be cost-effective.

### 6.3.2 Data Processing Instruments

It was in two companion papers presented at Ottawa, Canada in 1957 that Helava (1957-8) and Moore (1957-8) first introduced the concept of the analytical plotter, which was realized first in the AP1 and AP2 instruments of the early 1960s. Hobrough (1959) described the Stereomat with its tremendous potentials for automation in photogrammetry. The technology, however, remained almost exclusively in the partnership domain of the US Army (the user) and the OMI-Bendix (the manufacturer). Only few non-military users obtained instruments of the "commercial" version of the OMI AP-C series until other manufacturers came up with instruments like the Galileo (Italian) Digital Stereocartograph in 1972. The situation came to a surge of growth in 1976 when, at the ISPRS Helsinki Congress some eight new analytical plotters were introduced. The principal reason of this dramatic change was the development of "desktop" mini computers such as the DEC PDP-11, Data General NOVA and Hewlett Packard HP-1000 providing fast and reliable photogrammetric operations. People were still searching for less expensive yet adequate in accuracy instruments as would be seen in the following.

#### 6.3.2.1 Image Space Plotters

These involve on-line computational processing based on the concepts of analog third order plotters where other than matching the two photographs for stereoscopic viewing, there is no "orientation" process. The measurement of a stereo-model is on a point-by-point basis. The computer generates X, Y, Z (model or terrain) coordinates. For continuous plotting, manual tracing of the detail is performed wherein the floating (or measuring) mark simply follows the details on one photo (usually the left-hand side). Contouring would require interpolation and plotted as a subsequent off-line operation. These are obviously less expensive instruments and were available during the late 1970s. Examples are Zeiss Oberkochen Stereocord, APPS (developed at the US Army Engineer Topographical Laboratories), APPS-IV of Autometric, Inc., Galileo Stereobit and indigenously (see Ladouceur et al. 1982) developed ones.

Because of their purchasing and operational low costs, such instruments have been enjoying a strong growth of interest in their areas of application where accuracy standards are often less demanding in view of their use for jobs like thematic mapping or map revision in forestry, agriculture, regional planning, etc.

Their uses have been extended during the mid 1980s with devices like CCD video cameras to capture the image of an existing map which is converted to digital form and displayed on the screen of a personal computer. Some of these instruments have their capabilities extended by using other gadgets (like scanning mirror stereoscopes) whereby they can be considered to be fully analytical plotters (although with limited accuracies) with feedback mechanisms and featuring ensured parallax-free

oriented stereo-models such as Carto AP-190 (Carson 1987) or Topcon PA-1000.

### 6.3.2.2 Analytical Plotters

In terms of their optical-mechanical design and algorithmic approaches, the development of analytical plotters has been following two different approaches, viz.,

(a) The image coordinates approach, where the inputs to the computational solution are the image coordinates (typically  $x'$ ,  $y'$  and  $x''$ ,  $y''$ ) and the outputs are X, Y, Z model or terrain coordinates supplemented by  $px''$  and  $py''$  values which are imparted to the right side photo to ensure an oriented model. This is typified in instruments like Galileo Digital Stereocartograph and Digicart 20 instruments, Autometrics APPS-IV or HDF-Maco 35/70 plotter.

(b) The object coordinates approach, where the inputs to the computational solution are the model or terrain X, Y, Z coordinates and the outputs are the image coordinate values imparted to the photos (in real-time) as small computer generated displacements ( $px'$ ,  $py'$ ,  $px''$  and  $py''$ ) required to ensure an oriented model and positioning the measuring mark. This approach is nothing but the classical Helava solution and is used in most well known instruments manufactured by OMI-Nistri, Zeiss Oberkochen, Wild, Matra, etc.

A wide range of development in analytical plotters took place since the 1984 ISPRS Congress at Rio-de-Janeiro. PC microcomputer based low-cost technology has been behind several new instruments during this period. These are typified by the development made by smaller firms like Adam Technology (MPS instruments, see Chamard 1987) and Yzerman (APY instruments, see Yzerman 1987) or by larger manufacturers like Wild BC-3 (with Data General Dasher 386), Kern DSR-15 (with a DEC MicroVAX II computer) or OMI AP-5 (See Klein 1987).

With regard to analytical plotters, one can also note the availability of new features of enlarged photo-carriers (such as are offered in Zeiss Oberkochen Planicomp and Intergraph Intermap Analytical Instruments as special options to the size of 24 x 33 cm or the OMI AS-11PA to the size of 23 x 46 cm).

The superb capabilities of the analytical plotters have been enhanced in the 1980s through certain significant developments. Although these would raise the production cost of such instruments, they also indicate the technological trends with regard to these instruments. Most significant ones are the following:

1. Integration of graphics work stations (e.g., Kern DEC PDP-11 based MAPS 300 system workable on-line with either analog stereo-plotters like PG2 or analytical plotters like DSR-15 or Zeiss Oberkochen Planimap system workable with either analog plotters like Planicart or analytical plotters like Planicomp) to have an auxiliary screen displaying the plotted manuscript map or chart.

2. Superimposition and stereo-superimposition of graphics (e.g., Wild S9-AP system or Zeiss P1 Planicomp system) whereby, for example, a monoscopic image of the plotted map can be compared with the 3-D image of the stereo-model, or stereo-superimposition of the plotted map on the stereo-model would be possible.

3. Image matching with correlators (e.g., Kern Digital Orthophoto Software System KDOSS or the Gestalt Photo Mapper GPM developed in 1970s) conceptually followed the Bunker-Ramo UNAMACE system (Bertram 1965) of the mid 1960s.

Digital image acquisition (with CCD sensors) and processing and digital workstations non-real-time applications are in existence already. It is expected that these would develop into cost-effective real-time operations, all around analytical plotters of tomorrow.

### 6.3.2.3 Hybrid or Converted Plotters

In view of available modern technologies and economic modernization of existing equipment, the conversion of several analog plotters into analytical plotters has been successfully accomplished during the 1980s. Examples are provided in Quasco Analytical (QA) conversion of Kern PG2 and Wild B8 and E. Coyote's (Zeiss Jena agent in the USA) conversion of several Topocart instruments (known as Anacarts) by using computers (DEC PDP-11) and software provided by Helava Associates. It would be of interest to see whether or not such innovative low-cost PC-based hybrid analytical plotters will become popular world-wide. Some thousands of such analog instruments are already installed in the world. They are suitable and wait for such conversion.

### 6.3.3 Data Display and Presentation Instruments

Apart from improvements in the classical plotting tables, the last two decades provided numerous novelties in the photogrammetric instruments in their tasks of display and presentation of results.

#### 6.3.3.1 Plotting Tables

Starting from the 1960s right through the 1980s, we have witnessed gradual and steady improvements in the plotting tables. We have witnessed direct on-line or off-line plotting capabilities including data storage possibilities. Plotting possibilities are available with a wide range of tools, for example, tangential-control scribing devices, dual or quadruple plotting heads and software-controlled plotting such as would provide automatic corrections based on prior calibration. Tilttable table surface and variable plotting speeds are now regular features at such plotting tables.

#### 6.3.3.2 Software Systems

It was by the late 1970s that the importance of clearly arranged tabulations supplemented by typewriter headings in clear text was understood in general. Such outputs are essential for checking, analyzing and editing of intermediate or final results. The demands in the world did help develop software systems for acquisition, processing, management and presentation (in graphical or digital forms) of photogrammetric - cartographic data. We have also witnessed object-oriented and structured data management systems with graphic editing capabilities. Such operating systems (like Zeiss Oberkochen PHOCUS or Wild Leitz MS-DOS and UNIX) constitute newer generations of analytical plotters. Associated Computer Workstations are now commonplace. These also are generally usable with numerous peripheral hardware equipment, generally designed after the customer's requirements.

### 6.3.3.3 Orthophotoprinters

The first commercially available one was the OMI OP/C-2 introduced in the early 1970s. Later on, Wild OR-1 Avioplan from 1976 and the Zeiss Oberkochen Orthocomp Z-2 from 1980 remain the only analytically controlled orthoprinters available on the market. Their development and use have been surprisingly limited as compared with other equipment systems.

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Note: The following abbreviations are used for oft used words and phrases.

ASPRS (formerly ASP) = American Society for Photogrammetry and Remote Sensing  
BU = Bildmessung und Luftbildwesen  
CISM = Canadian Institute of Surveying and Mapping  
CGS = Coast and Geodetic Survey  
ISPRS (formerly ISP) = International Society for Photogrammetry and Remote Sensing  
OEEPE = Organisation européenne d'études photogrammétriques expérimentales  
Phta = Photogrammetria (currently ISPRS Journal of Photogrammetry and Remote Sensing)  
PERS (formerly PE) = Photogrammetric Engineering and Remote Sensing  
PR = Photogrammetric Record.

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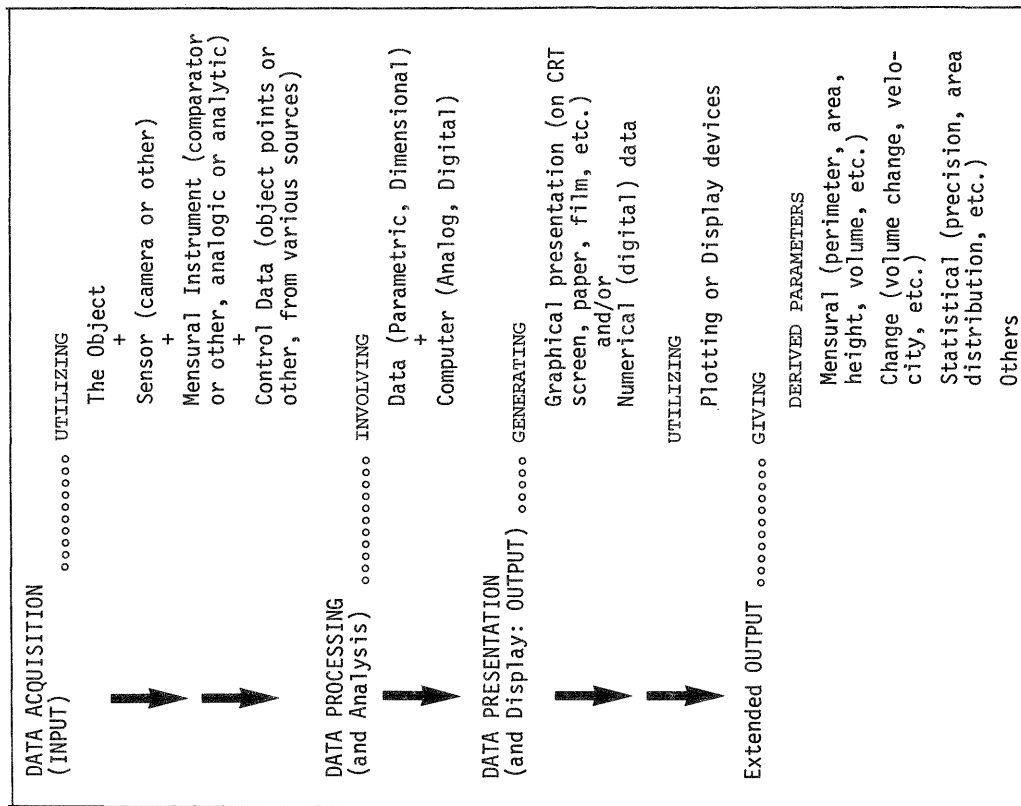
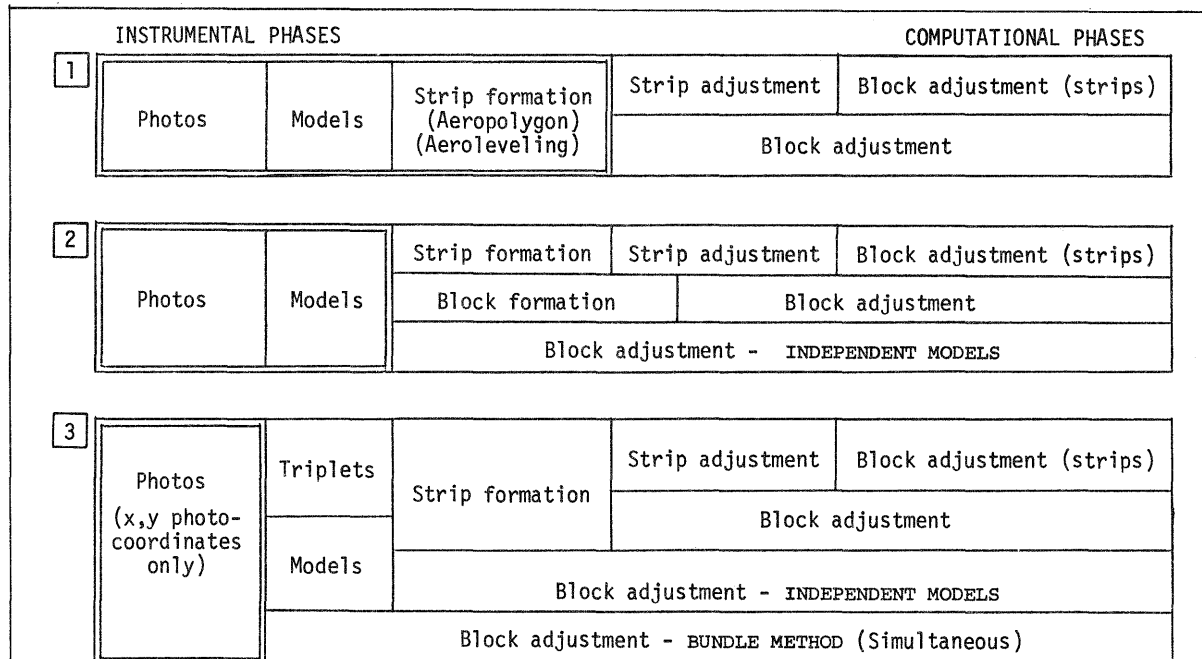


Fig. 6.1 : General working system in analytical photogrammetry



Note: Adapted from Ghosh (1988)

Fig. 6.2 : Outline of phototriangulation procedures. 1 : Analogical procedures; 2 Semi-analytical procedures; and 3 Analytical procedures.