FUZZY CLASSIFICATION OF SATELLITE IMAGERY
BY NEURAL NETWORKS

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ABSTRACT:
The conventional method for classification of satellite imagery is based on Bayes' theorem. The applied condition is to be "each pixel must belong to either of the classes". In other words, none of the pixels can belong to more than one class ('mixed' pixel), nor belong to none of the classes ('unknown' pixel). However, the 'mixed' pixel is necessarily existent in the case of satellite imagery. Also, the existence of 'unknown' pixel is inevitable as the number of class settings is restricted. This paper discusses the fuzzy classification of the satellite imagery. The classes are defined as fuzzy sets in spectral space. With this, the 'mixed' and 'unknown' pixels can be considered by the fuzzy set operations. It is difficult to directly give a membership function of the class fuzzy set in a multi-spectral space. Therefore, it is approximately estimated from the training data. By defining the membership function on the least squares criteria from the training data, I/O system equivalent to this function can be realized with back propagation algorithm of the neural network. The performance of the fuzzy classification is evaluated in comparison with the conventional supervised classification. The fuzzy classification method is able to provide a land cover classification superior to that derived from the conventional method. This paper also describes a method to effectively visualize the fuzzy classification result using ROB color composite.

KEY WORDS: Fuzzy classification, Neural network, Back Propagation Algorithm, Visualization of Fuzzy Event

1. INTRODUCTION

Let us consider the so-called image classification problem in which each pixel of satellite imagery is assigned to either of the classes C_m (m=1,2,...,M). The most commonly employed theorem is Bayes' theorem as shown below:

\[ P(C_m|x) = \frac{P(x|C_m)P(C_m)}{P(x)} \]  

(1)

With this we can calculate the probability that the pixel having measurement vector x is in the class C_m. Most of the statistical image classification procedures have been theoretically based on this Bayes' theorem. A variety of procedures are deduced depending on the kind of multi-variate distribution that is assumed (Mulder et al., 1990; Wang, 1990). The conditions under which Bayes' theorem is applicable are as follows:

\[ \sum_{m=1}^{M} P(C_m|x) = 1 \]  

(3)

The applied conditions mean that each pixel must belong to one and only one of the classes C_m (m=1,2,...,M). In other words, none of the pixels can belong to more than one class, i.e. C_m and C_{m'} ('mixed' pixels), nor belong to none of the classes ('unknown' pixels). However, the 'mixed' pixel is necessarily existent in the case of satellite imagery. Also, the existence of 'unknown' pixels is inevitable as the number of class settings is restricted due to the lack of detailed knowledge on the trend of land coverage in the area concerned.

In the set theory, assuming that the class C_m is a subset in a spectral space X, and that the membership grade of a measurement vector x to the set C_m has been defined, the conditions to apply the conventional procedures are equivalent to

\[ \mu_m(x) = 0 \text{ or } 1 \]  

(4)

and

\[ \sum_{m=1}^{M} \mu_m(x) = 1 \]  

(5)

Finally, by assuming random error for measurement vector of the training data which satisfy equation (4), P(x|C_m) can be obtained from the histogram, and further, the Bayes' theorem gets applicable form conditions of equation (5).

Now, let us regard the class C_m to be a fuzzy set in a spectral space X. This means that, the definition of C_m becomes uncertain and vague in relation to the spectral space, and hence, C_m cannot be explained as a simple error. If classes are defined in the practical classification as grass land, wasteland and urbanized area, the above assumption can be regarded to be rather reasonable.

In this case, conditions of equations (4) and (5) become

\[ \mu_m(x) = [0,1] \]  

(6)

causing the Bayes' theorem inapplicable. However, if a continuous membership function can be identified from some training data in the form of
equation (6), this allows us to treat 'mixed' and 'unknown' pixels. Considering the case of \( m = A, B \), the 'mixed' and 'unknown' pixels can be expressed respectively as

\[
\mu_{A \cap B} = \mu_A(X) \land \mu_B(X) \quad (7)
\]

and

\[
\mu_{A \cup B} = 1 - \mu_{A \cap B}(X)
\]

\[
= 1 - [\mu_A(X) \lor \mu_B(X)] . \quad (8)
\]

In this framework, a method is constructed to identify classes as fuzzy sets in multi-spectral space and to classify the satellite imagery into defined class fuzzy sets.

Besides, the probability measure of fuzzy event \( C_m \) can be defined by the Lebesque-Stieltjes integral:

\[
P(C_m) = \int \mu_m(x) \, dp \quad (9)
\]

where \( \Omega \) is the sample space and \( \mu_m(x) \) is the characteristic function of a fuzzy event \( C_m \), that is, the membership function (Zadeh, 1968). Wang (1990) defined the membership function based on maximum likelihood algorithm with fuzzy mean and fuzzy covariance matrix replacing the conventional mean and covariance matrix. This becomes possible, under the assumption that \( P(X/C_m) \) has a multi-variate normal distribution. In practice it is rare that the data have a normal distribution, as they may rather have a multi-modal or asymmetric distribution.

This study depends neither on the Bayes' theorem nor on the theory of the probability of fuzzy events. The proposed method in this study persistently identifies the membership function from fuzzy training data obtained purely under equation (6).

### 2. DEFINITION OF CLASS FUZZY SET

#### 2.1 Fuzzy Training Data

It is difficult to directly give a membership function of the class fuzzy set \( C_m \) in a multi-dimensional spectral space. Accordingly, the training data are obtained with an indirect method as follows:

i) Choose a region where its feature is considered to be homogeneous in the imagery

ii) Give the membership grades of that region to the class \( C_m(m=1,2,\ldots,M) \).

iii) Refer to measurement vectors of the pixels in the region. Accordingly, the same number of training data as the pixels in the region is obtained.

iv) Repeat processes i) through iii).

The above method is not so difficult in comparison with the method to obtain the training data employed in the conventional procedures. In addition, it allows more information to be extracted from the imagery as the condition in equation (4) does not exist.

### 2.2 Definition of Class Fuzzy Set

The training data given by the above method have the following features with respect to the conventional way of giving the membership function to the fuzzy set.

i) Training data can not be obtained for the entire region of the spectral space.

ii) Membership grade of the same measurement vector to the class \( C_m \) is sometimes given as a different value by plural training data.

Thus, the membership function for the spectral space \( X \) is approximately estimated from the training data. Assuming that the membership grades for a pixel measurement vector have random errors with equal and independent variances, and \( \mu_m(x) \) is a continuous function, then \( \mu_m(x) \) can be defined by the criteria of least squares as follows:

\[
\min. \sum_{i=1}^{n} [M_{im} - \mu_m(x_i)]^2 \quad (10)
\]

where \( x_i \) is a measurement vector of sample pixel \( i (i=1,2,\ldots,n) \), and \( M_{im} \) is a membership grade of \( x_i \) in the class fuzzy set \( C_m \).

### 3. CALIBRATION OF MEMBERSHIP FUNCTION OF CLASS FUZZY SET

As equation (10) presents the least squares method, statistical identification is possible if an appropriate form of the function can be given. However, the membership function is supposed to be a complex non-linear function, and it is very difficult to give an appropriate function form to a multi-dimensional space. In this study, accordingly, a method is proposed to approximately realize equation (10) by applying the neural network as reported by Rumelhart, Hinton and Williams (1986).

#### 3.1 Formulation of Neural Network

Consider a three-layer network with \( S \) input units, one hidden layer with \( T \) units, and a single output unit. The components of \( x_i = (x_{i1}, x_{i2}, \ldots, x_{is}) \) constitute the inputs to the \( S \) input units \( i_s \), \( s = 1,2,\ldots,S \), respectively. The output of the input unit, \( O_{is} \), is taken to be equal to the input. That is

\[
O_{is} = I_{is} \quad (11)
\]

The state of the hidden layer unit \( I_{it} \), \( t = 1,2,\ldots,T \), is determined as

\[
I_{it} = \sum_{s=1}^{S} w_{st}O_{is} + \theta_{t} \quad (12)
\]

where \( w_{st} \) is the weight assigned to the connection from input unit \( s \) to hidden unit \( t \) and the parameter \( \theta_{t} \) associated with the hidden unit \( t \) is bias. The output of the hidden layer is represented by a
sigmoid function as follows:

\[ f(l_i) = \frac{1}{1 + \exp(-l_i / \theta_0)} \]  

where \( \theta_0 \) is a parameter which modifies the shape of the sigmoid function, and is often called "temperature" parameter from the analogy with statistical mechanics. The state and the output of the output unit are determined similarly as:

\[ I_i = \sum_{t=1}^{T} w_t O_t + \theta \]

\[ O_i = f(I_i) \]

where \( w_t \) is the weight assigned to the connection from the hidden unit \( t \) to the output unit and \( \theta \) is a bias.

### 3.2 Calibration of Membership Function

The learning is accomplished so that the square of the error is minimized as follows:

\[ E_i = \frac{1}{2} (M_{im} - O_i)^2 \]  

where the factor of one-half is inserted for mathematical convenience in later discussion. The input-output system which is almost equivalent to the membership function \( \mu_{im}(x) \) is regarded as:

\[ \mu_{im}(x) = O(x) \]

where \( O(x) \) shows the output of the network after sufficient learning is finished. A gradient descent using the back propagation training algorithm is employed. For example, the convergence of the weight \( w_{st} \) is achieved by taking incremental change \( \Delta w_{st} \) as follows;

\[ \Delta w_{st} = -\eta \frac{\partial E_i}{\partial w_{st}} \]  

where \( \eta \) is the learning rate. By utilizing \( f'(x) = f(x)[1-f(x)] \), \( \Delta w_{st} \) is written

\[ \Delta w_{st} = -\eta \frac{\partial E_i}{\partial O_t} \frac{\partial O_t}{\partial O_{it}} \frac{\partial O_{it}}{\partial O_{ist}} \frac{\partial O_{ist}}{\partial w_{st}} \]

\[ = -\eta\left( \frac{\partial E_i}{\partial O_t} \frac{\partial O_t}{\partial O_{it}} \frac{\partial O_{it}}{\partial O_{ist}} \frac{\partial O_{ist}}{\partial w_{st}} \right) \]

\[ = \eta(M_{im} - O_i)(1-O_i)w_tO_i(1-O_t)O_{is} \]

Now \( \delta = (M_{im} - O_i)(1-O_t) \) is defined and written as;

\[ \Delta w_{st} = \eta \delta w_t O_i (1 - O_t) O_{is} \]  

The weight \( w_{st} \), biases \( \theta_t \) and \( \theta \) are trained in the same manner as follows;

\[ \Delta w_t = \eta \delta_i O_{it} \]  

\[ \Delta \theta_t = \eta \delta_i w_t O_{it} (1 - O_{it}) \]  

\[ \Delta \theta = \eta \delta_i \]

If the neural network has more than three layers, and input-output function \( f \) is the sigmoid function, it is proved that equation (17) converges to zero (Funahashi, 1989). Equation (17) does not completely converge to zero since the training data in this study are assumed to include the same measurement vector but different membership grades. It is possible, however, to get results equivalent to those obtained by the least squares method without assuming the function form.

### 3.3 Experiment

Let us try a simple experiment. The membership grades given for one-dimensional measurement \( x \) were estimated by the back propagation algorithm. A single unit was prepared in the input layer for a measurement \( x \). The hidden layer with three units was defined. The 'temperature' parameter \( \theta_0 \) and the learning rate \( \eta \) were assumed to be 1.9 and 0.3 respectively. The neural network was trained for 3,000 iterations. The membership grades given as training data and the estimated values by the trained network are shown in Fig. 1. A correlation coefficient between them is 0.986. As is apparent from this result, a three layer neural network can estimate two-modal and asymmetric distribution of membership grades.

![Fig. 1 Estimation of Membership Grades by Neural Network](image-url)

### 4. VISUALIZATION OF FUZZY CLASSIFICATION RESULT

So far the easiest method to represent the classified result of satellite imagery is the fabrication of land cover/land use map. In this method, the classified results are displayed as colored imagery by assuming a single color to each class. The color is not defined limitedly and discontinuously in its nature, but rather in a continuous 3-D space like...
RGB or HSI (Hue, Saturation and Intensity) space in general (Harris et al., 1990). By using this method, the result of fuzzy classification can be effectively expressed with colors.

Let us take the most conventional RGB space as an example. Color expression similar to the false color composite can be applied by selecting 3 classes among those settled and assigning either of R, G and B to the membership grade of each class. In the RGB space in general, the color (hue and saturation) is expressed as the difference in direction of the vector, and the brightness of the color as the difference in the amplitude of the vector. From the above, the manner of class mixture is expressed as the color, and possibility of not the 'unknown' pixel as the brightness of the color. In this case, the correspondence of the class mixture and the color is implied with chromaticity diagram (RGB composite image in the trigonometric coordinates under that the brightness is set invariant). This can be utilized as a general explanation to interpret results of image classification.

In general, the color is defined as a point in a regular triangular pyramidal space determined by orthogonal RGB axes (Harris et al., 1990). In other words, M-dimensional data can be expressed with colors if M-angular pyramidal space is defined (M>3; each axis is made correspond to arbitrary hue on the Munsell hue circle) and M-dimensional data are expanded in this space. This allows direct expression of the fuzzy classification with colors. With this method, however, attention must be brought to the fact that the result of classification and the displayed color do not correspond 1:1 to each other, because displayed colors are eventually defined through projection into the orthogonal RGB space. However, there is an advantage that the manner of class mixture is roughly expressed as the color (mainly hue and saturation) and its possibility as the brightness.

5. APPLICATION AND EVALUATION

In order to evaluate the performance of the fuzzy classification by neural networks, an experimental study has been carried out on Landsat TM data.

5.1 Fuzzy Training Data

The study area is Yokohama, Japan. The TM imagery was taken in October, 1988. The bands 1,2,3,4,5 and 7 were used for the classification. Four land cover classes (urban, grass, forest, water) were defined. A training site, 156 by 256 pixels in size, was specified. About 10,000 pixels were obtained as the fuzzy training data from the training site by the procedure shown in section 2.1. Further, in this work, one hundred and twenty sample pixels were selected by random sampling, that is, 120 by 4 membership grades were used for learning of the neural networks. One hundred and twenty pixels were not sufficient for land cover classification. This is due to the performance of the personal computer, NEC PC-9801 RA. The program for training the neural network is now being installed into the EWS, Sun SPARC Station IPC. The number of sample pixels will be increased.

5.2 Fuzzy Classification

The neural network was calibrated for each land cover class. The input layer had six units for six TM bands. It was very difficult, in general, to choose the number of hidden layers and the number of units in each hidden layer, as it depends on the type of application (Hepner, 1990). A single hidden layer with six units was defined in this work.

The neural network for each class was trained for 30,000 iterations. The learning rate, \( \eta \), was constantly defined as 0.3. Table 1 shows the overall accuracy of the learning given by the correlation coefficient between the network output and the membership grade. The accuracy of learning is considered to be an appropriate level.

Table 1. The Accuracy of Learning of Neural Networks

<table>
<thead>
<tr>
<th>Class</th>
<th>( \theta_0 )</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>urban</td>
<td>3.50</td>
<td>0.957</td>
</tr>
<tr>
<td>grass</td>
<td>3.90</td>
<td>0.933</td>
</tr>
<tr>
<td>forest</td>
<td>3.15</td>
<td>0.925</td>
</tr>
<tr>
<td>water</td>
<td>2.45</td>
<td>0.991</td>
</tr>
</tbody>
</table>

5.3 Comparison with Convention Supervised Classification

For comparison, maximum likelihood classification (M.L.C.) was applied. The sample pixels were "hardened" so that the individual sample pixel was assigned into the land cover class to which a maximum membership grade was given. These were regarded as ground truth data for M.L.C. The fuzzy classification was carried out on the training sample by using the calibrated neural networks. The result was "hardened" in the similar way.

Table 2 gives the number and percentage of correctly classified pixels. An 82.5 percent overall accuracy was achieved by the fuzzy classification, and a 61.7 percent overall accuracy was achieved by M.L.C. This result shows that the fuzzy classification by neural networks is also applicable to a one-pixel-one-class classification method.

Table 2. Number of Correctly Classified Pixels

<table>
<thead>
<tr>
<th>Class</th>
<th>Total Number of Pixels</th>
<th>Correctly Classified Pixels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>F.C.*</td>
</tr>
<tr>
<td>urban</td>
<td>60</td>
<td>53 (88.3%)</td>
</tr>
<tr>
<td>grass</td>
<td>27</td>
<td>22 (81.5%)</td>
</tr>
<tr>
<td>forest</td>
<td>22</td>
<td>14 (63.6%)</td>
</tr>
<tr>
<td>water</td>
<td>11</td>
<td>10 (90.9%)</td>
</tr>
<tr>
<td>Total</td>
<td>120</td>
<td>99 (82.5%)</td>
</tr>
</tbody>
</table>

* F.C.: Fuzzy Classification
5.4 Visualization of Fuzzy Classification Result

The calibrated neural network was applied to the entire of the training site. The membership grades to four fuzzy sets 'urban', 'grass', 'forest' and 'water' were given to each pixel. By the union operation of fuzzy sets 'grass' and 'forest', the membership grades to a new fuzzy set 'green' were calculated and assigned to each pixel.

Let us visualize the fuzzy classification result. The R, G and B color sequences were linearly assigned to the membership grades to fuzzy sets 'urban', 'green' and 'water' respectively. Fig.2 shows the possibility of each land cover class by the brightness of corresponding color. Fig.3 shows the RGB color composite. The possibility of class mixture is represented by the composite of corresponding color and the possibility of 'unknown' class is represented by the brightness of composed color. (Unfortunately Fig.2 and Fig.3 are black-and-white prints. Interesting color outputs will be presented at Washington D.C. Congress.)

6. CONCLUSION

Although the 'mixed' and 'unknown' pixels are inevitable in the classification of the satellite imagery, it is difficult to consider their existence by means of the conventional methods based on the probabilistic Bayes' theorem which are often employed. Defining the class in the classification of satellite imagery as fuzzy sets is more reasonable since this allows the existence of the above phenomenon by usual fuzzy set operations.

By defining the membership function of class fuzzy set on the least squares criteria from the training data, I/O system equivalent to this function can be realized with back propagation algorithm of the neural network. Thus, the class fuzzy set can be defined without assuming the concrete function form of its membership function.

The result of fuzzy classification, that is, the possibility of 'mixed' or 'unknown' pixel, can be expressed as color or brightness respectively by expanding membership grade to each class in the RGB space or enlarged color space.

REFERENCES


