MODELLING ORIENTATION PARAMETERS OF SENSOR PLATFORMS

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1. Introduction

Recent advances in photogrammetry may be characterized by the introduction of computer technology into all photogrammetric procedures. The first link in such procedures, the primary data acquisition, plays an important role in this development, because the transformation of a classical photography into a computer readable format is a time-consuming and costly process. For this reason the substitution of the photo-chemical layer by photo-electrical sensors is the subject of current research. The three-line scanner and the airborne laser profiling system are two typical examples of the new sensor technologies which are mentioned here as the background to the studies presented in this paper.

Comparing modern scanners with conventional photogrammetry the main principles become obvious : the scanners provide less information per exposure, which must be compensated by a higher exposure rate. In consequence the exterior orientation parameters of the scanner platform must be determined with a higher rate. Closely connected to this point is the problem of a weak geometry of the scanner data. Single spot laser scanning represents an extreme. When only a single measurement is executed at every point of time, no geometric redundancy is obtained. Then the determination of the exterior orientation parameters is not possible in the conventional, indirect manner with the help of terrestrial control points. In this case the exterior orientation parameters must be directly measured by additional devices with a sufficient accuracy. These brief arguments explain why the direct measurement of the exterior orientation parameters of a modern sensor platform and their consideration in the whole evaluation process is indispensable.

This paper presents a mathematical model describing the dynamic and stochastical properties of any time-dependent parameter. The evaluation of measured data takes full advantage of this model for the solution of a series of problems such as : stochastical description of the measurement process, filtering and smoothing of observations, detection of gross measurement errors, stochastical description of the filtered data. To summarize, the dynamic modelling of measured exterior orientation parameters provides filtered data with their stochastic properties for input into further evaluations of sensor data.

The measured orientation parameters represent a trajectory of the sensor platform, which is disturbed by the observation errors. As the observation errors are unknown, we are unable to reconstruct the true track of the sensor platform. An approach to the true movement will be found by the introduction of a model, which is built up here by the application of autoregressive integrated (ARI-) processes. The time-characteristic of each orientation parameter will be modelled by a specific representation of an ARI-process.

In chapter 2 the theoretical background of the time-series model will be outlined. Chapter 3 will present some applications of the theory : 1. modelling the position parameters of an aircraft flight measured by the Global Positioning System GPS, and 2. modelling the attitude parameters of an aircraft and of a space shuttle measured by an Inertial Navigation System INS.

2. Theoretical background

2.1 Autoregressive integrated processes

Autoregressive integrated (ARI-) processes are a widely applied class of stochastic processes for various kinds of time-series (Haykin 1979, Kay and Marple 1981).
An autoregressive (AR-) process of order p describes a stationary time-series x_t by

$$x_t = - \sum_{i=1}^{p} a_i \cdot x_{t-i} + e_t \qquad V(e_t) = \sigma_e{}^2 \tag{1}$$

The time-series considered in our applications are generally not stationary. These time-series have to be transformed into stationary time-series by the elimination of trends. A usual way is to take derivatives of the time-series. The autoregressive <u>integrated</u> process of order (p,d) is achieved if the d-th derivative of the original time-series can be described by a stationary AR(p) process. Formally, the derivation can be expressed by an additional number d of process parameters a_i in eq.(1). Then the ARI(p,d) process fully describes the dynamic behaviour of any orientation parameter by a number of (p+d) process parameters a_i and by the variance $\sigma_e{}^2$ of the prediction errors.

2.2 The filtering algorithm

Two kinds of equations build up a Gauss-Markov model to estimate the filtered time-series x_t from the observed series y_t

$$E(y_t) = x_t \qquad V(y_t) = \sigma_n{}^2 \tag{2}$$

$$E(e_t) = 0 = x_t + \sum_{i=1}^{p} a_i \cdot x_{t-i} \qquad V(e_t) = \sigma_e{}^2 \tag{3}$$

Eq.(2) expresses the observation process with $\sigma_n{}^2$ being the variance of the observation noise. Eq.(3) presents the ARI model to which the unknown time-series x_t has to correspond.

The estimation of the filtered time-series x_t presumes the knowledge of the stochastic part of the Gauss-Markov model, i.e. the variances $\sigma_n{}^2$ and $\sigma_e{}^2$. As the variances in general are a priori unknown, they must be estimated by applying the variance component estimation (VCE) technique. The formulation in the frequency domain published by Förstner (1984) is recommended for our application.

Together with the unknown ARI process paramters a_i we obtain a highly nonlinear estimation problem which can be solved in an iterative computation scheme (Lindenberger 1987). The conditions to be fulfilled by the data for successfull solution of the GM model and the VCE are mentioned in the same paper (e.g. the variances $\sigma_n{}^2$ and $\sigma_e{}^2$ have to be of same order of magnitude).

Gross errors in the observations would disturb the validity of the GM model in eq.(2). These gross errors are automatically detected and are taken into consideration by individual weights in eq(2). The weights are calculated following the robust estimation theory (Danish method, Krarup et al. 1980). On the other hand, a robust treatment of eq. (3) reduces the influences of edges and discontinuities in the data, which disturb the ARI model.

2.3 Capability of the algorithm

What are the main results from the algorithm ? Under the assumption that the true trajectory of the sensor platform can be modelled by an ARI-process, we obtain a filtered data set x_t which is the most probable representation of the true track. Together with the estimated ARI process parameters, several demands of further evaluation of the sensor data will be satisfied.

In addition, the ARI-model yields important results, relevant for the analysis of the stochastic model. The estimated variance of the observation noise $\sigma_n{}^2$ describes the observation process. The variance of the prediction errors $\sigma_e{}^2$ gives a fidelity measure how well the the ARI-model is suited for the real physical process. The inversion of the normal equation system out of equations (2) and (3) provides accuracy criteria of the filtered data set, especially the variance of the filtered data $\sigma_x{}^2$ and the autocorrelation coefficients r(h). It is emphasised here that all stochastic results are obtained without any a priori information.

Any systematic effects in the time-series, such as drifts of the orientation parameters with time, cannot be taken into consideration by the algorithm. For this reason the estimated accuracies must be understood not as absolute but as relative accuracies.

3. Applications

3.1 Position coordinates from NAVSTAR-GPS

The NAVSTAR Global Positioning System GPS enables the determination of the x,y,z position coordinates of one or more GPS receivers. The application of GPS is of particular interest in photogrammetry for the inflight positioning of the aerial camera. This reduces drastically the ground control requirements for aerial triangulation (Frieß 1986).

In the case of a stationary GPS receiver the accuracy estimation of GPS measurements is relatively simple due to the redundancy of the observations. In contrast to this, the application of a GPS receiver in a moving vehicle, such as an aircraft, renders the accuracy estimation more difficult or rather impossible if a non redundant satellite constellation (i.e. 4 satellites or less) is available or if an unsuitable receiver is used. Then the suggested algorithm with ARI-processes can be utilized for stochastic investigations.

As an example some results of an accuracy study on real GPS data are presented in the following. The data for the analysis originate from a photogrammetric test flight with a Sercel GPS receiver TR5SB carried out by the Survey Departement of Rijkswaterstaat in the Netherlands on June 10th and 12th 1987 (D. Boswinkel, R.Witmer, J.W.v.d. Vegt 1988). The position coordinates in the earth-fixed WGS system derived from the primary phase measurements are subject of this analysis. Systematic effects which influence the determination of the coordinates are eliminated to a great extent in advance by simultaneous measurements with a second GPS receiver at a stationary reference point. Both receivers recorded with a registration rate of 1 measurement every 0.6 seconds. The data of the whole flight were divided according to the 6 photo-strips with about 130 registrations each. Only the data within the strips are of interest and the aircraft then shows a very uniform kind of movement.

Each position coordinate is considered separatly by it's own ARI-process. The mean ARI process order was found to be (5,2); so 7 ARI process parameters fully describe the model. The parameters of the stochastic model were estimated without any a priori information. In table 1 the estimated standard deviations of the noise-component in the GPS coordinates are listed. The main result is that the internal accuracy of the positions determined from a moving GPS receiver is about 0.02 m.

Table 1: Estimated noise σ_n [m] of GPS coordinates

Strip	Date	X	Y	Z
1	12.06.87	0.027	0.011	0.038
3	10.06.87	0.020	0.009	0.026
4	10.06.87	0.024	0.009	0.027
5	10.06.87	0.020	0.013	0.024
6	12.06.87	0.014	0.011	0.014
7	12.06.87	0.012	0.007	0.010
rms of strips 3,4,5		0.021	0.010	0.026
rms of strips 1,6,7		0.018	0.010	0.021

These results can be compared with an estimation of the internal accuracy out of the least squares adjustment of the GPS positions. The LS adjustment determines the weight coefficient matrix Q and gives in general an estimate of the weight unit sigma naught, i.e. the GPS range measurement accuracy. In the case of a moving GPS receiver observing 4 satellites the sigma naught value is not determinable because of the missing redundancy. Out of different considerations the range measurement precision of the Sercel receiver in dynamic application can be assumed reasonably to be $\sigma_0 = 0.006$ m. This value is twice as large as the precision obtained by a stationary receiver and depends mainly on the velocity of the aircraft. With this assumption the accuracies of the coordinates given in table 2 are calculated (Frieß 1988).

Table 2: Estimated internal accuracy σ_n [m] of GPS coordinates from least squares adjustment with assumed range measurement accuracy of $\sigma_0 = 0.006$ m

	X	Y	Z
rms of strips 3,4,5	0.022	0.009	0.017
rms of strips 1,6,7	0.020	0.009	0.020

Comparing the estimated internal accuracies from the ARI model in table 1 without any a priori stochastic information and from the least squares adjustment with apriori knowledge of the sigma naught (table 2), we note a very good agreement. It demonstrates that the ARI model is suited to be applied for modelling the dynamic characteristics of GPS coordinates during a photogrammetric flight and that the results are realistic.

All subsequent evaluation of the sensor data, for example the aerotriangulation, is based on the filtered GPS coordinates. These filtered data with their stochastic informations are achieved from the ARI-model. Table 3 summarizes the estimated accuracies of the filtered coordinates and their autocorrelation coefficients. It is an important result that in this example only the correlations from one to the next point (within 0.6 sec.) remain significant.

Table 3: Accuracy criteria of filtered GPS data

Estimated accuracies σ_x of filtered coordinates [m]

	X	Y	Z
rms of strips 3,4,5	0.017	0.007	0.021
rms of strips 1,6,7	0.013	0.006	0.016

Correlation coefficients of filtered data d=0.6 sec

	X	Y	Z
r(1d)	0.46	0.69	0.52
r(2d)	-0.06	0.11	-0.01
r(3d)	0.06	-0.15	-0.10

3.2 Attitude data from INS

Attitude data, i.e. the roll-, pitch- and yaw- angles of the sensor body axis to the terrestrial fixed coordinate system can be directly measured by an inertial navigation system (INS) belonging to the equipment of every larger aircraft. The evaluation of INS measurements is in most cases very difficult and depends essentially on the physical model which describes the systematic influences of a large number of physical effects.

The analysis of INS data presented in this chapter examine only the registered attitude angles. Any systematic drift effects caused by remaining errors in the gyros of the INS are not taken into consideration. For this reason the filtered attitudes cannot be estimated as absolute values, but must be rectified for the systematic drifts. Then the estimated accuracies of the measured angles are understood as the precision of one reading of the gyro.

The data for the analysis come from two different dynamic applications of INS. In the first example the attitude data of an INS from a NASA Space Shuttle mission are presented. The analysed data are registered during the metric camera experiment of the ESA/NASA's D1 Spacelab mission on 2nd December 1983. The registrations took place every 2 seconds, the angles are measured in degrees. The attitudes refer to the earth-fixed Greenwich True of Date system. The registrations were subdivided according to 21 photo-strips consisting between 120 and 800 points. As the estimated accuracies are very similar between the strips, only the root mean square values from all the strips are listed in table 4.

Table 4: INS attitude data from Space Shuttle

Estimated standard deviations in [deg]

	YAW	PITCH	ROLL
observation noise σ_n	0.0060	0.0048	0.0060
ARI-model errors σ_e	0.0019	0.0012	0.0014
filtered data σ_x	0.0027	0.0027	0.0031

Correlation coefficients of filtered data d=2 sec

	YAW	PITCH	ROLL
r(1d)	0.71	0.62	0.67
r(2d)	0.46	0.39	0.47
r(3d)	0.18	0.10	0.15
r(4d)	0.13	0.13	0.13

The mean ARI process order in this case is (6,1). The estimated accuracies are not very different between the three angles, so that a common measurement precision can be assumed. The estimated autocorrelation coefficients show a decrease within the first five seconds.

The attitude data of the second example are registered during an aircraft flight with a Litton LTN-72 Inertial Navigation System of a Falcon jet. The registration rate was 10 Hz. The characteristic of the aircraft flight is very different to the space shuttle flight, which is much more smooth. This is expressed by the ARI process parameters; here the ARI process order was only (2,2). The analysis refers to the central part of the flight, not disturbed by take-off and landing maneuvers. The results are presented here with reservation. Some discrepancies in the results caused by unsteadinesses in the aircraft trajectory are subject of further research. During undisturbed parts of the flight the standard deviation of the ARI-prediction errors σ_e decrease below 0.0001 deg.

Table 5: INS attitude data from aircraft

Estimated standard deviations in [deg]

	YAW	PITCH	ROLL
observation noise σ_n	0.0023	0.0035	0.0030
ARI-model errors σ_e	0.0005	0.0013	0.0026
filtered data σ_x	0.0011	0.0013	0.0017

Correlation coefficients of filtered data d=0.1 sec

	YAW	PITCH	ROLL
r(1d)	0.86	0.79	0.80
r(2d)	0.57	0.40	0.41
r(3d)	0.29	0.09	0.08
r(4d)	0.09	-0.07	-0.09

The estimated observation noise σ_n is in full accordance with other investigations from a stationary INS of same type. In the stationary mode the precision of the attitude measurement is about four times higher than in the dynamic mode, which was expected in advance.

4. Conclusions

This paper introduced autoregressive integrated stochastic processes for modelling the dynamic characteristics of the exterior orientation parameters of a sensor platform. The ARI-Model in combination with a variance component estimation enables the entire functional and stochastical description of the orientation parameters. The main advantages of this model are the easy handling, the low number of necessary ARI process parameters and the dispensation of any a priori information concerning the stochastical model.

The successful application of the ARI model to very different kinds of sensor orientation parameters improves the power of the concept. Comparisons to other methods for accuracy estimation demonstrate that the obtained results are realistic.

5. References

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