

**A POSITIONING STUDY  
OF LONG RANGE, GREAT OBLIQUE ANGLE  
AND SMALL FRAME PHOTOGRAPH**

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**ABSTRACT**

It is difficult to determine the ground position of photo points on such a special aerial picture as the long range, great oblique angle and small frame photograph. This paper covers: (a) Special method of ground point positioning performed on Zeiss Planicomp C130 Analytical Plotter, and the experimental accuracy of photo point positioning on the special aerial photograph shot at an oblique angle of 72.5 degrees and an altitude difference of 200m; (b) The digital rectification method used in digital image processing technology and the unique digital control differential rectification method by Z2 Orthocomp Analytical Orthoprojector, in the light of the DTM data given by the plotter, and the experimental accuracy of the ground point position on an orthographic photograph obtained by using these two methods. (c) The theoretical accuracy of positioning of such a special aerial photograph.

**INTRODUCTION**

The scale of a special aerial photograph taken at an extremely long range, an extremely great oblique angle and a small frame varies greatly. Approximately, the variation of the lateral and longitudinal scales is 1:3 to 1:4. Thus, it is difficult for

such a special aerial photograph to establish a correct stereoscopic model on Zeiss Planicomp C130 Analytical Plotter using the frequently used orientation method. Therefore, it is in no position to discuss the positional accuracy of the ground points. As the oblique angle of the photograph is great and its scale varies greatly, it is impossible to get an orthographic photograph on Zeiss Z2 Orthocomp Analytical Orthoprojector by the commonly-used exterior orientation and scanning direction. With the unique methods described in this paper, the ground point positional accuracy obtained through direct measurement by Zeiss Planicomp C130 Analytical Plotter and through measurement on the orthographic photograph can meet the requirement of a map on the scale of 1:50,000, under condition that the control points can not be actually measured but found from a map of 1:10,000.

#### 1. The Special Method of Ground Point Positioning and the Experimental Accuracy

The base altitude ratio of such a special aerial stereo pair is but only 1:50 (the space lab photos is 1:10) and the crossing angle corresponding to this is very small, approximately  $1.6^{\circ}$ , that because the focus distance is very long and the base line of the photograph very short. Therefore, the accuracy of forward intersection is low and it is hard to establish an accurate stereoscopic model on Zeiss Planicomp C130 Analytical Plotter by the commonly used orientation method. For this reason, it is advisable to find out first the optimized original value of the element of exterior orientation of the photograph in the light of the data provided by the navigational system. However, the accuracy of the absolute orientation obtained after that is still far from meeting the requirements. For instance, after introducing the original value of the element of exterior orientation of photograph into the experimental stereo pair 1159/1163, the result of the absolute orientation obtained in accordance with the commonly used method is as listed below:

POINT	DXG	DYG	DZG
101	-8.523	-16.805	-1.684
103	6.414	30.656	-0.971
112	8.316	-39.160	1.184
104	-6.207	25.246	1.469
MEAN	7.441	29.125	1.354

To this end, model orientation is done with double-point aft intersection in space. Since beam angle is rather small ( the maximum is 5.5 degrees ) and is much affected by the control point accuracy and the measurement accuracy, the result of bundle orientation is still not satisfactory. The result of the bundle orientation obtained on the basis of the absolute orientation mentioned above is as follows:

POINT	DXG	DYG	DZG
101	1.117	-1.981	-97.721
103	1.292	-11.802	-135.186
104	-0.968	-8.620	-97.622
112	-2.355	20.225	-138.565

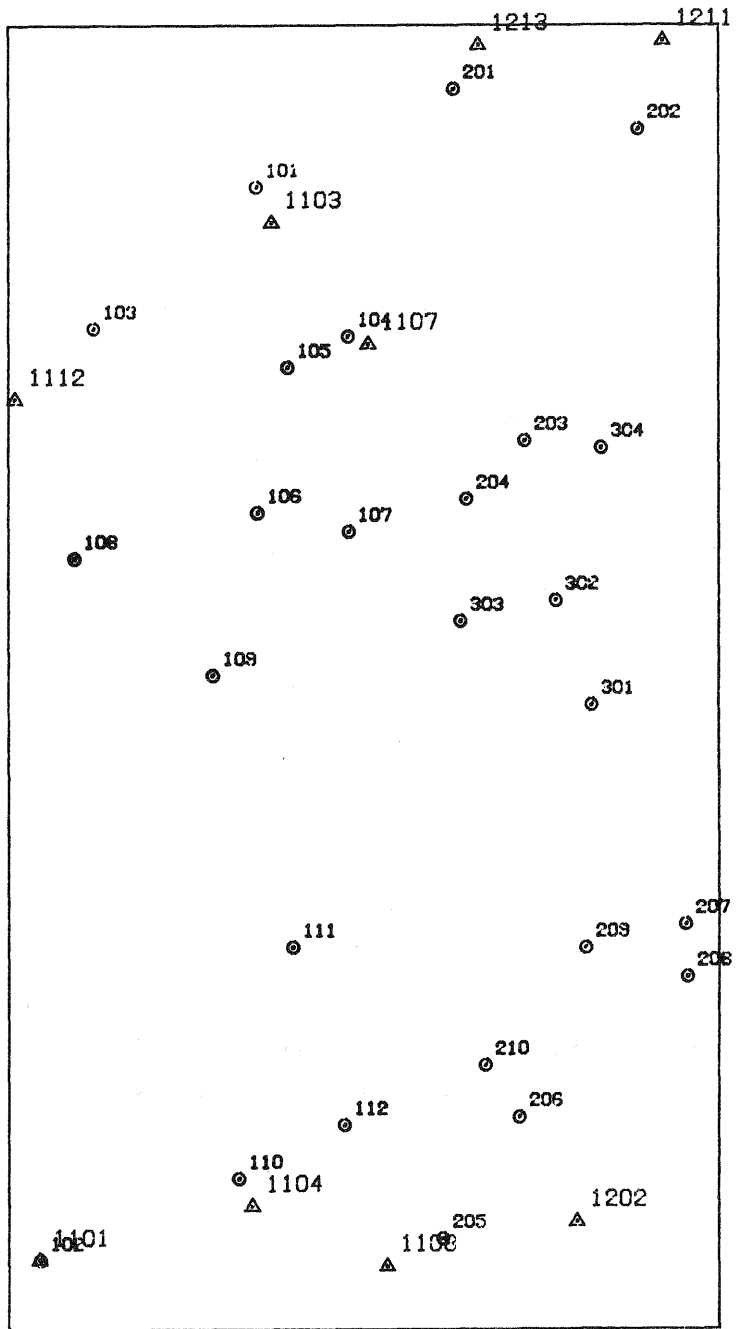
Seen from the orientation result above, the horizontal accuracy is somewhat improved, but the vertical one is much degraded. Based on this, now have the absolute orientation done once again by calling the absolute orientation software, and the orientation result is as follows:

POINT	DXG	DYG	DZG
101	-1.216	-5.539	0.469
112	4.168	-3.637	-0.395
103	-1.336	3.430	0.355
104	-1.617	5.797	-0.447
MEAN	2.411	4.742	0.419

From the result above, it is clear that the orientation accuracy has much been improved and thus meet the orientation accuracy required by mapping on the scale of 1:25,000.

Residuals of Ground Coordinates

No	X(M)	Y(M)	Z(M)	DX(M)	DY(M)	DZ(M)
101	24537.7 24541.0	25138.2 25134.0	109.6 108.6	-3.3	4.2	0.46
102	23104.5 23107.0	18850.4 18853.0	92.28 92.7	-2.5	-2.6	-0.42
103	23472.9 23468.0	24198.7 24198.0	102.48 102.1	4.9	0.7	0.38
104	25147.7 25149.0	24145.7 24145.0	101.92 102.0	-1.3	0.7	-0.08
105	24740.1 25745.0	23927.2 23940.0	99.47 101.0	-4.9	-12.8	-1.53
106	24552.2 24548.0	22981.3 22983.0	94.62 95.5	4.2	-1.7	-0.88
107	25143.0 25149.0	22858.2 22858.0	94.60 94.6	-6.0	0.2	0.00
108	23341.4 23336.0	22681.1 22682.0	95.29 96.7	5.4	-0.9	-1.41
109	24258.7 24255.0	21910.1 21910.0	93.68 93.8	3.7	0.1	-0.12
110	24428.4 24422.0	18588.6 18589.0	84.14 84.7	6.4	-0.4	-0.56
111	24781.2 24779.0	20107.2 20115.0	85.05 86.8	2.2	-7.8	-1.75
112	25109.1 25116.0	18941.5 18941.0	83.78 83.1	-6.9	0.5	0.68
201	25850.3 25848.0	25781.4 25778.0	134.21 133.0	-2.3	3.4	1.12
202	27073.3 27068.0	25508.3 25513.0	172.81 175.0	5.3	-4.7	-2.19
203	26312.8 26315.0	23458.7 23456.0	110.39 110.0	-2.2	2.7	0.39
204	25927.7 25931.0	23072.2 23071.0	98.52 98.9	-3.3	1.2	-0.48
205	25767.6 25768.0	18198.9 18196.0	80.13 79.9	-0.4	2.9	0.23
206	26278.5 26275.0	19001.1 18997.0	86.16 85.5	3.5	4.1	0.66
207	27384.9 27380.0	20276.8 20267.0	81.75 83.0	4.9	9.8	-1.25
208	27397.4 27392.0	19923.2 19921.0	82.69 83.0	5.4	2.2	-0.31
209	26726.7 26719.0	20118.6 20113.0	83.46 83.0	7.7	5.6	0.46
210	26052.2 26052.0	19343.3 19338.0	88.01 87.9	0.2	5.3	0.11
301	26752.6 26758.0	21725.2 21715.0	215.77 214.0	-5.4	10.2	1.77
302	26522.4 26519.0	22397.0 22402.0	151.03 152.7	3.4	-5.0	-1.67
303	25897.5 25889.0	22275.4 22265.0	214.50 215.5	8.5	-7.6	-1.00
304	26825.7 26821.0	23404.1 23408.0	211.78 213.5	4.7	-3.9	-1.72
Mean Square of Error				±4.67	±5.12	±1.05



Check Points and Their Distribution

According to the above mentioned method, a ground point positioning experiment for four models has been carried out with the oblique angle of the photo being 72.5 degrees and the ground height difference being 200m. For the orientation accuracy of the four models, the maximum horizontal error is 8m, while the maximum vertical error is 0.7m. Within this range, 26 check points have been selected and their distribution is as shown in the diagram above. Among them, 22 points are on even surface with their numbers ranging from 101 to 210, and 4 points are on the top of the mountain and their numbers are 301 through 304. Points with 4 digits are the control points. All of which are identifiable ground points found from a map of 1:10,000. If the three-dimension co-ordinate measured from the map of 1:10,000 is considered as the true value, and the three-dimension co-ordinate measured by Zeiss Planicomp C130 Analytical Plotter as the observation value, the co-ordinate error obtained by comparison between the two is as shown in the table (beside the diagram). The experimental accuracy of photo point positioning finally obtained is expressed in terms of mean square error, i.e.:

$$m_x = \pm 4.67m$$

$$m_y = \pm 5.12m$$

$$m_z = \pm 1.05m$$

## 2. The Unique Digital Control Differential Rectification and the Experimental Accuracy

Such special aerial photograph is rectified by digital control differentiation on Zeiss Z2 Orthocomp Analytical Orthoprojector, whereas it can't be done with other method normally used. Thus, it is suggested that the digital control differentiation rectification can not be realized on Z2 so as to obtain an orthographical photograph unless special methods of off-line exterior orientation, scanning along course direction of the airplane, etc., are adopted. Also, we have done the experi-

ments on digital rectification and digital mosaic using digital image processing technology. The positional accuracy of the ground points has been tested for the orthographic photograph obtained by using these two methods. As many as 22 points have been selected from the two orthographical photographs 1159 and 1147 as check points with numbers in the range 101 to 211 and their distribution is as shown in the diagram. The result of the test is expressed in the terms of mean square error, which is referred to as the very orthographical rectification accuracy of such special aerial photographs.

Digital rectification: The maximum error for the co-ordinate of the ground points is 31.39m, while the positional mean square error of the ground points  $m_s = \pm 21.44m$ .

Digital control differentiation rectification: The maximum error for the co-ordinate of the ground points is 60.96m, while the positional mean square error of the ground points  $m_s = \pm 25.20m$ .

From the experimental accuracy described above, it is concluded that the mosaic of the orthographical photographs obtained with such methods may be used as a map of 1:50,000.

### 3. Analysis of Ground Point Positioning Accuracy

The factors affecting the ground point positioning accuracy are mainly the photograph orientation error and ground point measurement error.

#### (1) Ground point positioning error caused by photograph orientation

According to the principle of double-point aft intersection in space for the great oblique angle photograph, the error of photo element of exterior orientation caused by the control point error and measurement error is obtained. On this basis,

the positional error with the photo points on the ground is then discussed.

(a) Accuracy of double-point aft intersection in space

The following equations are used to evaluate the accuracy of photo element of exterior orientation obtained with the method of double-point aft intersection in space:

$$\begin{aligned}
 m_{\varphi} &= \sqrt{\theta_{11}} m'_0 ; & m_{\omega} &= \sqrt{\theta_{22}} m'_0 ; & m_{\kappa} &= \sqrt{\theta_{33}} m'_0 \\
 m_{X_S} &= \sqrt{\theta_{44}} m'_0 ; & m_{Y_S} &= \sqrt{\theta_{55}} m'_0 ; & m_{Z_S} &= \sqrt{\theta_{66}} m'_0
 \end{aligned}
 \tag{1}$$

Where,  $m'_0 = \frac{m_0}{\sqrt{2}}$ ,  $m_0$  is the mean error of unit weight measurement in single point measurement. As the control points based by aft intersection are taken from a map of 1:10,000, the mean error of point position is  $\pm 0.4\text{mm}$ , and the mean error of point position transferred into the experimental photo with a mean scale of 1:50,000 is  $\pm 0.08\text{mm}$ . Having the measurement error taken into account, it is therefore supposed that  $m'_0 = \pm 0.1\text{mm}$ . The direction cosine of the experimental photo is :

$$\begin{pmatrix}
 0.9992395 & 0.0352670 & 0.01663397 \\
 0.0264619 & 0.3000007 & -0.9535720 \\
 0.0286394 & 0.9532868 & 0.3007059
 \end{pmatrix}$$

Substitute the photo co-ordinate  $|x|=|y|=b$  and other data into the equation coefficient matrix of aft intersection method of half-photo four points, the weight coefficient is obtained as follows:

$$\begin{aligned}
 \sqrt{\theta_{11}} &= 0.001 ; & \sqrt{\theta_{22}} &= 0.0003 ; & \sqrt{\theta_{33}} &= 0.0089 \\
 \sqrt{\theta_{44}} &= 88/M ; & \sqrt{\theta_{55}} &= 104/M ; & \sqrt{\theta_{66}} &= 178/M
 \end{aligned}$$

After substituting  $M=50,000$  and other data into equations (1), the accuracy of double-point aft intersection in space for the experimental stereo pair is the following:

$$m_{\varphi} = \pm 0.34' ; m_{\omega} = \pm 0.10' ; m_{\epsilon} = \pm 3.06'$$

$$m_{X_S} = \pm 8.8m ; m_{Y_S} = \pm 10.4m ; m_{Z_S} = \pm 17.8m$$

(b) Mean error of the ground point position caused by photo orientation error

The mean error of the ground point co-ordinate caused by photo orientation error is given by the following equations:

$$\begin{aligned} m_{X1} &= \sqrt{\theta_{XX}} m_o \cdot M \\ m_{Y1} &= \sqrt{\theta_{YY}} m_o \cdot M \\ m_{Z1} &= \sqrt{\theta_{ZZ}} m_o \cdot M \end{aligned} \quad (2)$$

Where,  $m_o$  is the mean error of unit weight measurement, and let it be supposed that  $m_o = \pm 0.002\text{mm}$ .

$$\begin{aligned} \theta_{XX} &= \frac{x^2(x-b)^2}{b^2 f^2} \theta_{66} + \frac{x^2(x-b)^2}{b^2 f^2} \theta_{11} + \frac{x^2(x-b)^2 y^2}{b^2 f^2} \theta_{22} \\ &+ \left(\frac{x}{b} - \frac{1}{2}\right)^2 y^2 \theta_{33} + \frac{2x^2(x-b)^3}{b^2 f^2} \theta_{16} \\ \theta_{YY} &= \left(\frac{(2x-b)y}{2bf}\right)^2 \theta_{66} + \frac{y^2(x-b)^2(2x-b)^2}{4f^2 b^2} \theta_{11} + \left(\frac{x-b}{b} + \frac{1}{2}\right)^2 \frac{y^4}{f^2} \theta_{22} \\ &+ \frac{y^4}{b^2} \theta_{33} + \frac{(x-b)(2x-b)^2 y^2}{2f^2 b^2} \theta_{16} \\ \theta_{ZZ} &= \frac{x^2(x-b)^2}{b^2} \theta_{11} + \frac{x^2 y^2}{b^2} \theta_{22} + \frac{2x^2(x-b)y}{b^2} \theta_{12} \end{aligned}$$

Where,  $x, y$  are the photo co-ordinates of the photo points, and  $b$  is the photo baseline. Now suppose that photo's  $|x|=|y|=d=b$ ,



while  $x=-b$ . Substitute these data into the above equations and the following is given:

$$\theta_{XX} = 0.0791463$$

$$\theta_{YY} = 0.01868106$$

$$\theta_{ZZ} = 8.8487358$$

Have these substituted into equation (2), the mean error of the ground point co-ordinate of the experimental photo caused by the photo orientation error is obtained as follows:

$$m_{X1} = \pm 0.028m$$

$$m_{Y1} = \pm 0.043m$$

$$m_{Z1} = \pm 0.885m$$

(2) Positioning error caused by measurement error

After the model is established, the photo point positioning error caused by measurement error is obtained by the following equations:

$$m_{X2} = \frac{H}{b} \operatorname{tg}(\alpha + \beta) \operatorname{Sint}(2m_o)$$

$$m_{Y2} = \frac{H}{b} \operatorname{tg}(\alpha + \beta) \operatorname{Cost}(2m_o) \quad (3)$$

$$m_{Z2} = \frac{H}{b} (2m_o)$$

From the analysis of the above equations, it gives that co-ordinate error of X and Y will both be affected by height measurement error. As  $0^\circ \leq t < 45^\circ$ , this influence on co-ordinate Y is more prominent than that on co-ordinate X. As  $45^\circ \leq t \leq 90^\circ$ , this affection to co-ordinate X is more severe than that to co-ordinate Y. In the equations,  $\beta$  is the included angle between the line connecting the projection centre to the photo point and the primary optical axis, and

t is the angle depending on the course line direction and is obtained according to course angle T provided by the navigation system. After substituting the experimental photo data into the above equations, the following is given:

$$m_{X2} = \pm 0.067m$$

$$m_{Y2} = \pm 3.817m$$

$$m_{Z2} = \pm 1.059m$$

(3) The theoretical accuracy of the ground point positioning

The positioning accuracy of such special aerial photo with an oblique angle of 72.5 degrees on the single model ground points is:

$$m_X = \sqrt{m_{X1}^2 + m_{X2}^2} = \pm 0.073m$$

$$m_Y = \sqrt{m_{Y1}^2 + m_{Y2}^2} = \pm 3.817m$$

$$m_Z = \sqrt{m_{Z1}^2 + m_{Z2}^2} = \pm 1.380m$$

The theoretical accuracy mentioned above does basically meet the case of the experiment.