ADJUSTMENT OF AEROTRIANGULATION BY A MULTIPLEX METHOD

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Abstract

A method is proposed for the final setting an approximately oriented strip to existing control. Calculation of necessary corrections to the points coordinates is based an a combination of polinomial and bundle adjustment. From a viewpoint of geometry the method is similar to a procedure of absolute orientation of a relatively oriented strip in a multiplex, where the projectors supporting frame is slightly tilted, twisted and bended, in order to level stereoscopic model to ground control and to keep intersections of all the corresponding projecting rays simultaneously. Mathematicaly the method conforms to the selfcalibration rigorous adjustment in which the additional parameters circumscribe a strip deformation. In this method the adjustment procedure is obvions, the erroneous control points can be revealed easier than in selfcalibration with parameters defining an image distortion.

Mathematic principles

Methods of rigorous adjustment of analytic phototriangulation developed about 10 years ago found a general use. However, they couldn't force out completely the polynomial methods that have practical application as before. They will remain evidently applicable in the nearest future as well, at least for pre-adjustment purposes aiming at the checking of data errors and determination of an initial value of unknows.

Polynomial and rigorous methods of aerotriangulation adjustment seem to be the two poles apart with a wide gap between them. But any one of them has its own advantages and it is desirable to join these advantages.

As follows from its name, the rigorous adjustment with selfcalibration is more preferable since it yields good results. But it requires setting up and solving of large sets of normal equations which makes hard demands on the capacity of computers. Polynomial methods can be easily realized and require relatively short machine times. However, they frequently give results as good as those of rigorous methods.

Both regorous and polynomial methods don't give a single-valued solution since it is unknown the real character of image distortion and deformation of aerotriangulation network as a whole. The mathematical models of image distortion or propagation of errors in a strip are usually found empiricaly. In analytical form these models can be represented for rigorous(1) and polynomial (2) methods as follows:

$$\left.\begin{aligned} \sigma_x &= \varphi_x(x,y,Q) \\ \sigma_y &= \varphi_y(x,y,Q) \end{aligned}\right\}, \qquad (1)$$

where Q are selfcalibration parameters,

$$\left.\begin{aligned}\delta X &= \varphi_x(X,Y,Z,A)\\ \delta Y &= \varphi_y(X,Y,Z,B)\\ \delta Z &= \varphi_z(X,Y,Z,C)\end{aligned}\right\}, \quad (2)$$

where A,B,C are polynome coefficients. The reliability of a model is not the same for any particular case and depends upon many factors.

In this connection the errors of coordinate of ground control points are of great significance. Though various robust and statistic technigues for checking of ground control correctness have been developed, the problem of undefective points selection is not completely cleared up and solved. In polynomial methods, especially when the strip deformation is eliminated separately for each coordinate axis, the errors of control points cause a loss of an inner rigidity achieved at the stage of constraction of a relatevely oriented network. In regorous methods an error of a control point causes a local distortion in a certain zone around it and empairs mutual position of bundles.

It is possible to verify the inner rigidity of a network and the mutual agreement of its particular elements by the procedure of exterior orientation of each photograph. This simulates the process of rectification of photographs and estimates the accuracy of such rectification. As a rule in relatevely oriented strips the residuals for collineaity conditions are very small. After absolute orientation by means of polynomes the residuals increase, on the average, two or more times. A joint adjustment of several overlapping strips in a block may enhance the residuals to a certain extent due to an insufficiently reliable pass point transfer from one photo to another.

An idea follows from the above discussion that for the improvement accuracy of areotriangulation adjusted by polinomes it is important to keep the inner rigidity of network during geodetic orientation. To make it feasible the determination of polynome coefficients necessary for elimination of deformation must be accomplished with monitoring under conditions of collinearity.

Suppose we have a set of observation equations for geodetic orientation which has been written up for control points:

$$\left.\begin{aligned}\delta X + L_X &= V_X\\ \delta Y + L_Y &= V_Y\\ \delta Z + L_Z &= V_Z\end{aligned}\right\}, \quad (3)$$

where $\delta X, \delta Y, \delta Z$ are expressed by polynomes from (2). Let it be required that after correction of aerotriangulation network with the aid of coefficients A,B,C of polynomes the following conditions of collinearity have been observed for all terrain points:

$$\left.\begin{aligned}x &= -f\,\frac{(X_m+\delta X_m-X_s)a_1+(Y_m+\delta Y_m-Y_s)b_1+(Z_m+\delta Z_m-Z_s)c_1}{(X_m+\delta X_m-X_s)a_3+(Y_m+\delta Y_m-Y_s)b_3+(Z_m+\delta Z_m-Z_s)c_3}\\ y &= -f\,\frac{(X_m+\delta X_m-X_s)a_2+(Y_m+\delta Y_m-Y_s)b_2+(Z_m+\delta Z_m-Z_s)c_2}{(X_m+\delta X_m-X_s)a_3+(Y_m+\delta Y_m-Y_s)b_3+(Z_m+\delta Z_m-Z_s)c_3}\end{aligned}\right\}, \quad (4)$$

where X_m, Y_m, Z_m are coordinates of terrain points from a relatively oriented network.

That is evident that the condition equations for expressions (4) will assume the following form:

$$aT + b \begin{vmatrix} \varphi_x(X,Y,Z,A) \\ \varphi_y(X,Y,Z,B) \\ \varphi_z(X,Y,Z,C) \end{vmatrix} + l = V, \qquad (5)$$

where T is vector of corrections for elements of exterior orientation of photographs.

Let us write the equations (5) for all points of all photographs, join them with the system (3), solve together and find the deformation coefficients A, B, C and by them the corrections for terrain point coordinates. By changing the weights of photogrammetric and geodetic measurements when setting up normal equations it is possible to achieve an optimum agreement of geodetic and photogrammetric coordinates on a ground control with a required degree of conservation of inner photogrammetric bonds in a network.

Such an adjustment, taken from a geometrical viewpoint reminds an absolute orientation of a relatively oriented strip at an aeroprojector multiplex in which the projectors supporting frame is slightly tilted, twisted and bended in order to level a stereoscopic model to ground control and to keep intersections of all the corresponding projecting rays simultaneously. That is why the method is called to be multiplex one. In mathematical sense, it can be considered as a rigorous adjustment with selfcalibration whose additional parameters characterize a strip deformation. In this method the adjustment procedure is easy-to-interpret, the erroneous control points can be revealed easier than in selfcalibration with parameters defining an image distortion.

It is known that random errors in photogrammetric series, when accumulated along the strip, have a pseudo-systematic behavior. That is why in polynomial adjustment the effect of random errors is partly approximated simultaneously with systematic errors. If a single vector of selfcalibration parameters common for all strips of a block is used, the pseudo-systematic character of random errors is not revealed nor eliminated when adjusting the bundles. It is one more important reason in favour of a multiplex method.

The multiplex method permits to include in processing readings of airborne instruments and other data of independent photogrammetric control. When adjusting a multi-strip aerotriangulation it is necessary to add to above mentioned conditions the requirement of equality for coordinates of pass points chosen in the overlapping zones of adjacent photos, i.e:

$$\left.\begin{aligned} X_i+\delta X_i-X_j-\delta X_j &= \delta X_i-\delta X_j+L_{\Delta x}=V_{\Delta x} \\ Y_i+\delta Y_i-Y_j-\delta Y_j &= \delta Y_i-\delta Y_j+L_{\Delta y}=V_{\Delta y} \\ Z_i+\delta Z_i-Z_j-\delta Z_j &= \delta Z_i-\delta Z_j+L_{\Delta z}=V_{\Delta z} \end{aligned}\right\}, \qquad (6)$$

where i and j are strips numbers.

In a common case when the axis direction of a strip is arbitrary with respect to geodetic coordinate system it is necessary to take into account this non-coincidence.

Since the equations for collinearity conditions are independent for each photograph, in a matrix of the normal equations for the multiplex method a sub-matrix can be isolated which represents a single-diagonal series of cells for individual photographs having a size of 6x6. Therefore, even in the course of normal equation formation it is possible to exclude unknown corrections for elements of exterior orientation of a photograph immediately after composing of equations for collinearity conditions of this photograph. The remaining equivalent system will have exactly the same structure as in a case of a conventional polinomial adjustment of one or a number of overlapping strips. Hence, the techniques for solution of normal equations used in polynomial adjustment can be completely borrowed for the multiplex method. Quite feasible also is the procedure of accuracy evaluation for point coordinates by inverse matrix, weight functions and weighted RMSE.

Conclusion

Therefore, the propounded multiplex method of aerotriangulation adjustment is based on well developed polynomial and rigorous techniques. It joints advantages of both: a sufficiently simple structure of normal equations without big demands for computer capability as well as ease of interpretation of a mathematical deformation model and of checking a ground control which is proper to a polynomial method; keeping inner photogrammetric unity and rigid bonds of adjacent elements of a network (points, photographs, bundles) which is proper to a rigorous method.

All this makes suppose that the multiplex method is quite worth of paying attention to it. It is realized in the USSR for strip aerotriangulation and is in practical use. Work on block aerotriangulation and its programming is now in progress and will be completed towards the end of 1988.