HIGH ALTITUDE AERIAL TRIANGULATION WITHOUT GROUND CONTROL

I.Colomina
Institut Cartogràfic de Catalunya
Balmes, 209-211
E-08006 Barcelona
Catalonia - Spain
Comission III

Abstract

One of the already envisaged applications of the Global Positioning System is the in-flight determination of image exposure stations, thanks to its kinematic positioning capabilities. Many simulations have been performed in the last years which, together with the expected positioning accuracy when operating in differential mode (at the decimetre level), prove that accuracy requirements are met for most photogrammetric applications.

Nevertheless, many questions are still to be solved before operational systems for everyday photogrammetric practice are available. Some of the questions are on the GPS hardware and software side, such as cycle slips. Others are on the photogrammetric side: projection centre-to-antenna offset, unknown biases in the inner orientation elements, etc.

The possibility of recovering the inner orientation elements by using control points and auxiliary data (DTM) is discussed in connection with the determination of local datum transformations between GPS derived relative positions and the ground object space.

1 Introduction

Today, it is generally accepted that the navigational capabilities of the NAVSTAR GPS (NAVigation System with Time And Ranging Global Positioning System) will allow, in the future, for assisted or automatic guidance in photogrammetric flights following a predefined mission plan. It is accepted also that its kinematic positioning capabilities will allow for the elimination of all or a significant part of ground control in aerial triangulation [1, 7,11,15]. This former point is achieved through satellite aided coordinate observations of image projection centres. Moreover, if attitude orientation data were determined to a sufficient level of precision then, for certain mapping applications, even the step of aerial triangulation could be skipped. Attitude determination can be achieved by different means: using three antenna receivers properly placed on the aircraft's fuselage [10], laser gyros, or incorporating inertial navigation systems.

By means of computer simulations, the attainable block accuracy using GPS derived exterior orientation parametres has been thoroughly investigated [6] for several photo scales and block configurations. A remarkable result is the favourable propagation of projection centre coordinates precision to the ground point precision and that the attitude data have only a limited influence. Therefore, this paper concentrates on the –for the time being– simpler and more cost effective first set of data. That is, the case where the aircraft carries one single receiver on board, aimed at the determination of positions.

Concurrently with the photogrammetric investigations, theoretical and practical progress has been made on the GPS side. Worthy of mention here is the experience reported by Mader *et al.* [12] where they proved the feasibility of subdecimetre precision positioning (about 80 Km path length, 150 m flying altitude and a speed of 200 Km/h).

After the photogrammetric preliminary simulations studies, some experiments for practical concept validation and testing are being conducted. First results described by Frieß [8] for a large scale aerial triangulation block demonstrate that accuracies of better than .1 m are attainable in the determination of antenna centres at exposure moments, and indicate that the potential accuracy is at the centimetre level. Nevertheless, if the accuracy potential for ground points shown by the computer simulations is to be met, other factors will have to be considered: systematic errors in the image coordinates and biases in the inner orientation elements; separation between camera projection centre and antenna phase centre; and correlations in the set of derived GPS coordinates. As an additional difficulty, it is not to be forgotten that in many countries the knowledge of geoid undulations is uncertain, a problem that arises in any attempt to combine three dimensional coordinate differences from satellite observations with the old independent horizontal and vertical control networks. Finally, one should be aware that the expression *without ground control* means in fact *with minimal control* since, again, in many countries the transformation parametres from the GPS geocentric reference system (WGS 84) to the local system is either not known or not known to the required accuracy level. Even if the datum transformation is available, the expression *without ground control* has to be taken with care since, as Schwarz *et al.* [15] pointed out, biases in the inner orientation elements *can be readily corrected for so long as one control point is available.*

This paper adresses one of the former questions which might be of relevance in certain situations: the possibility of applying self calibration to the elimination of errors in the focal length and the principal point coordinates. Because of their correlation with other amounts (projection centre-to-antenna offset and datum transfer parametres) the discussion must be conducted globally. From this point of view, a combined adjustment program based on the operational bundle program of the Institute of Cartography of Catalonia (ICC) has been developed. It is an experimental version that performs the simultaneous adjustment of these kinds of data:

- the classical photogrammetric network (photo observations and object control points) with additional self-calibrating parametres (with the inclusion of focal length and principal point coordinates),
- the satellite network (antenna centre and object point coordinate observations),
- auxiliary object point information (elevation data from DTMs),
- the antenna offset,
- and the datum transformation parametres.

The introduction of auxiliary object information (DTM) has been considered after the simulated examples described by Ebner *et al.* [5] and after the experiments reported by Rosenholm *et al.* [13] where elevation data improved the results obtained with conventional control for absolute orientation of stereo models. Thus, through simulation the program allows for a general treatment of the topics mentioned before, and hopefully for the processing of the first sets of actual data.

2 System configuration and solution strategy

A first problem is that, in a real installation, the satellite derived coordinates refer to the antenna phase centre and not to the camera projection centre. Because the camera is maneuvered to compensate the aircraft's row, pitch and yaw, it is necessary to record these movements in order to determine the varying offset vector that relates both centres. As suggested by Lucas [11] a simpler installation will be adequate for the first experiments: the camera will be locked. The three components of the vector will be constant and, either they are known or ,for some specific network configurations, are estimated in the adjustment. For the simulation studies reported in this paper, it is sufficient to introduce a single camera-antenna vector whose coordinates are unknown parametres in the adjustment. Knowledge of the vector at each exposure moment can be simulated by proper weighting.

A second problem is the recording of the exposure moments in the receiver time reference system, which seems to be a pure hardware question. Any error introduced at this stage will be assumed to be added to the standard error of the antenna centre coordinates.

The actual raw observations consist of photo measured coordinates and carrier phase or pseudo range measurements. Although a general adjustment of both types of data is feasible, it is not practical [2]. If the satellite derived coordinates are used together with their covariance matrix, the sequential approach is correct and seems to be the general accepted solution strategy. Therefore, in the following the satellite network will in fact be a point field with a given covariance matrix.

3 Mathematical model

An aerial triangulation block with the incorporation of satellite aided coordinate observations can be regarded as a particular case of a combined terrestrial and satellite network. The adjustment of such combined networks requires hybrid functional and stochastic models. A problem in the functional model is that the observation equations must be formulated with respect to a common reference system and, either the transformation parametres (weighted according to the a priori knowledge of their precision) are solved for in the adjustment or the transformation is determined and applied independently of the hybrid adjustment. The second option is a common procedure when using GPS derived coordinate differences for geodetic network densification [3]. Nevertheless, the first approach is less restrictive and exhibits some advantages for photogrammetric networks (the possibility that the transformation partly absorbs eventual biases in the inner orientation elements). Adjustment of the block in the satellite reference system and further absolute orientation of the resulting point field can be viewed as a sequential version of the same approach.

The formulation of a correct stochastic model is a much more involved question [16,17] and, provided that systematic errors are appropriately modelled, requires estimation of the variance components. Because of its algorithmic implications and because this paper deals with simulated data the stochastic model has been kept rather simple.

3.1 Functional model

Taking into account the above considerations the functional model has been developed in a way that it is an extension of the classical bundle formulation. This allows for an easy transformation of the old bundle adjustment programs in order to incorporate the new observational data and the new unknown parametres. First, the traditional functional relationships for bundle photogrammetric networks are considered.

$$
\begin{aligned}
S_x(x_i^j, y_i^j) &= -f\,\frac{m_{11}^j(X_i - X^j) + m_{12}^j(Y_i - Y^j) + m_{13}^j(Z_i - Z^j)}{m_{31}^j(X_i - X^j) + m_{32}^j(Y_i - Y^j) + m_{33}^j(Z_i - Z^j)}, \\
S_y(x_i^j, y_i^j) &= -f\,\frac{m_{21}^j(X_i - X^j) + m_{22}^j(Y_i - Y^j) + m_{23}^j(Z_i - Z^j)}{m_{31}^j(X_i - X^j) + m_{32}^j(Y_i - Y^j) + m_{33}^j(Z_i - Z^j)},
\end{aligned} \tag{1}
$$

$$x_k = X_k,\ y_k = Y_k,\ z_k = Z_k, \tag{2}$$

where (1) are the well known collinearity equations for the (x_i^j, y_i^j) image coordinates of the i object point observed on the j image, and where equations (2) state that the k object point is a control point of known coordinates (x_k, y_k, z_k). In (1) S_x, S_y are the selfcalibrating functions defined by a number of additional parametres among which the coordinates of the principal point are included. It is convenient to take the attitude matrix in (1) as the transposed of the rotation matrix of the j image system

$$(m_{pq}^j)^T = \mathbf{R}^j. \tag{3}$$

The knowledge of the elevation data can be formulated, for the i object point as

$$z_e(X_i, Y_i) = Z_i. \tag{4}$$

The position and attitude parametres considered so far are referred to a local object system which is related to the GPS reference system in the usual way through a seven parametre transformation

$$\mathbf{X}_{GPS} = \mathbf{X}_0 + (1+\mu)\mathbf{R}\mathbf{X}_{local}, \tag{5}$$

where $\mathbf{R}$ is the rotation matrix of the local system with respect to the GPS system. From (3) and (5), new equations can be derived for the aircraft's antenna phase centre determined coordinates at the moment of exposure of the j image

$$\mathbf{x}_g^j = \mathbf{X}_0 + (1+\mu)\mathbf{R}(\mathbf{X}^j + \mathbf{R}^j\mathbf{X}_a), \tag{6}$$

where $\mathbf{X}^j = (X^j, Y^j, Z^j)^T$, $\mathbf{X}_a$ is the constant vector of the antenna offset in the image system, and $\mathbf{x}_g^j$ is the observed coordinate vector of the antenna in the GPS system. The former equation relates three observed values to sixteen unknown parametres; six orientation parametres of the j image and ten extra parametres common to the whole network. Some of these parametres and the parametres in equation (1) produce similar effects on the observed quantities and, therefore, are strongly correlated. For the sake of numerical stability and to model the situation where a number of these parametres are known –say, the antenna offset or the datum transformation– fictitious observation equations are to be introduced for the new ten parametres as well as for all the selfcalibrating additional parametres and the focal length. When the datum transformation is known, equation (6) results in the formulation proposed by Lucas [11].

If one or more antenna receivers are located on the ground a similar equation to (5)

$$\mathbf{x}_{g_i} = \mathbf{X}_0 + (1+\mu)\mathbf{R}\mathbf{X}_i \tag{7}$$

can be set up for object points which might be observed or not on the images. If not, they can still contribute to the network strength via equation (4) when elevation data is available.

3.2 Stochastic model

For the photogrammetric block, image observations have been considered uncorrelated, as well as the control points, to which covariance matrices are assigned. The term precision for the elevation data refers to the overall precision of the DTM. It is assumed, therefore, that a suitable node point distribution and precision allow the interpolation of the z_e function to a given degree of precision.

Finally, the satellite aided antenna centre coordinate observations are taken as uncorrelated, since there was no information available to the author concerning this question.

4 Algorithmic aspects

The adjustment of the satellite-photogrammetric network is done in the usual way: linearization of the model given in section 3, least squares estimation of the corrections to the unknowns; and iteration to convergence. In a classical bundle adjustment, in each iteration step, the normal equations are formed and solved by direct methods (e.g. Cholesky factorization). If the subscript 1 stands for the object point corrections set (Δ_1), 2 for the image orientation corrections (Δ_2), and 3 for the additional parametres (Δ_3) then the normal equations can be written as follows

$$\begin{pmatrix} N_{11} & N_{21}^T & N_{31}^T \\ N_{21} & N_{22} & N_{32}^T \\ N_{31} & N_{32} & N_{33} \end{pmatrix} \begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{pmatrix} = \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix}. \tag{8}$$

The well known sparsity structure (see figure 1) of N_{11}, N_{21} and N_{22} is exploited by eliminating the group unknowns Δ_1. This leads to the reduced normal equations

$$\begin{pmatrix} N_{22} - N_{21}N_{11}^{-1}N_{21}^T & N_{32}^T - N_{31}N_{11}^{-1}N_{21}^T \\ N_{32} - N_{31}N_{11}^{-1}N_{21}^T & N_{33} - N_{31}N_{11}^{-1}N_{31}^T \end{pmatrix} \begin{pmatrix} \Delta_2 \\ \Delta_3 \end{pmatrix} = \begin{pmatrix} R_2 - N_{21}N_{11}^{-1}R_1 \\ R_3 - N_{31}N_{11}^{-1}R_1 \end{pmatrix}, \tag{9}$$

which are of the *banded-bordered* type. The heavy task is solving the linear system (9) which can optionally be loaded without going through the creation of (8).

In [16] Strunz classifies the non-photogrammetric data sources that provide observations for combined adjustments into three groups: classical geodetic measurements; navigation systems; and object information. In [16] it is also mentioned that the last two data groups do not change the sparse structure in (8) and (9). Advantage can be taken of this fact in order to extend the classical bundle programs to more general combined programs without much effort. This is just the case for the new equations (4), (6) and (7) introduced in section 3. The ten new unknowns (which will be referred to as Δ_4) can be arranged in the border behind the selfcalibrating parametres, a strategy already proposed by Ebner [4].

Figure 1 outlines the situation described above. There, N_{41}, N_{42} and N_{44} are the new partial normal equations which appear by considering the contributions of observation equations after linearization of (6) and (7). Equation (4) contributes only to the 3×3 diagonal submatrices in N_{11}, whereas N_{43} is a null submatrix. N_{41} is sparse but considering it as a full submatrix does not add a significant computational load. The old basic program of the ICC forms directly the reduced normal equations (9) so that N_{41} is technically introduced simultaneous to N_{31}, and N_{42} is simply loaded after the reduced normals have been formed.

Another feature of the old basic program is the partial inversion of the normals –for the non-zero submatrices– following the method reported by Sarjakoski in [14] which, together with an appropriate symbolic organization of the transposed normal matrix, works very efficiently. Because of this computational facility inherited by the new program, the figures given for the simulations in the next section are rigorous theoretical precision measures. Although not discussed in this paper, correlation coefficients can be analysed as well.

5 Simulation studies

A synthetic block of 49 images at 1:75000 photo scale was generated. Most of the project parametres aim at resembling practical block configurations at the ICC where orthophotomaps are being digitally produced from aerial photography at 1:70000 scale. Because of the consideration of elevation data, two block versions have been investigated. One version –block P– was generated taking altitudes from the Catalan Pyrinees with large slopes and height differences. The second version –block N– corresponds to a normal flat terrain. A summary of the block characteristics is given in table 1. Ground control

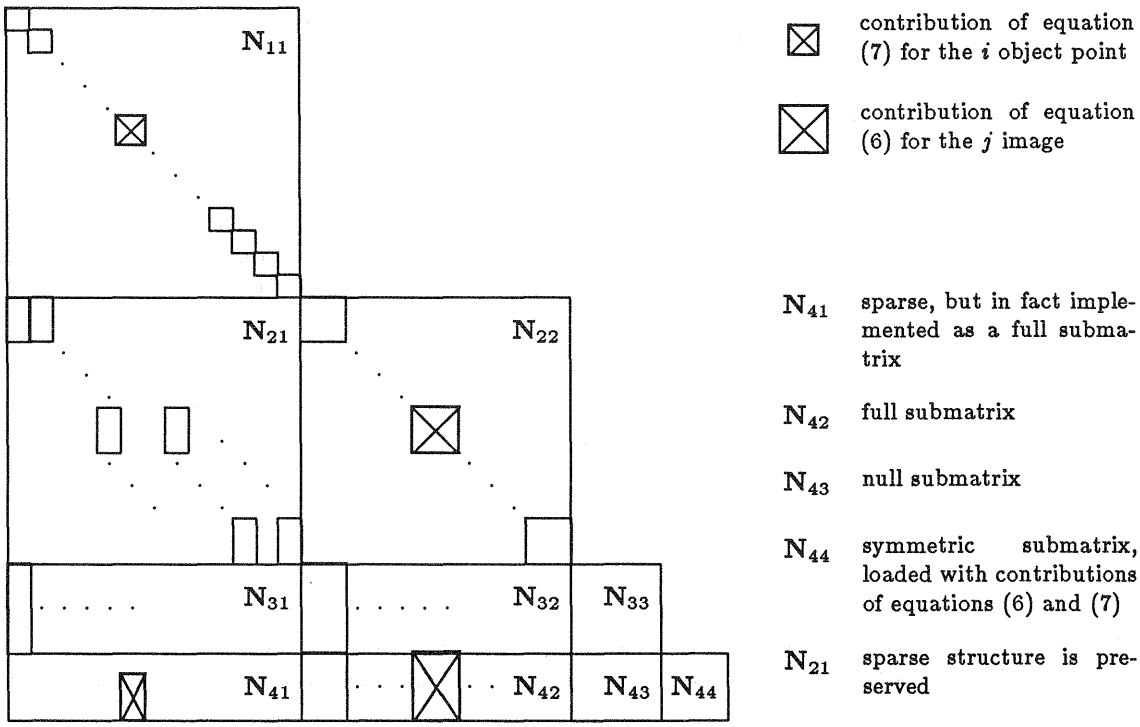

Figure 1: Structure of the extended normal equations

consist of four full control points located at the corners of the block and one vertical control point at the centre of the block. For the less favourable case concerning elevation data, that is block N, two more synthetic blocks were generated, blocks D and G. Block D is just block N but using pairs of tie points, and block G is a larger block (13 strips, 13 images per strip). Both blocks D and G have identical terrain characteristics to those of block N. For all blocks, strictly vertical images following the same regular grid pattern as the ground object points were assumed.

The availability of the DTM as well as its precision has been assumed because an elevation data base is being compiled at the ICC. Its intended accuracy ranges between $1m$ and $2m$. A point which is not discussed although of the utmost importance, is the numerical evaluation of terrain slopes, since it has to do with the convergency properties of the adjustment. Studies are being conducted in order to approximate terrain derivatives in a suitable way to accelerate or, at least, not to impair convergency speed of the bundle adjustment.

Tolerance values for the precision of aerial triangulation results have been established as a function of the intended map scale (1:25000) and contour interval (10 m). In a mapping project, the precision budget left for aerial triangulation depends not only on the map scale and contour interval but also on the equipment used in the compilation phase. Thus, the given precision tolerance ranges are only indicative. They are $\sigma_h < 1.25 - 1.7m$ for horizontal circular precision and $\sigma_v < 1 - 1.5m$ for vertical precision.

Key input values for the simulation results are those concerning the precision of both photogrammetric and coordinate satellite observations. The simulations so far described in the literature cover a wide range of values; from $3\mu m$ and $.1m$ in [11] to $15\mu m$ and $0 - 10m$ in [6]. Because the goal of the simulations reported in this paper is to prove the feasibility of some hybrid network types, the input standard errors have been kept fixed ($10\mu m$ and $.5m$) whereas the network configuration is the variable parametre. In this respect one must be aware of the relative validity of the results of any simulation.

Datum transfer parametres are three translations, one scale factor and three rotations about the Z, X and Z axes, i.e. longitude, latitude and azimuth. The scale factor in the datum transformation has been regarded as known since it is the easiest and most invariant amount to be determined in a separate survey and since it is highly correlated with the focal length.

Figure 2 gives the description of the tables with simulation results.

5.1 Simulation results for block P

Reference simulations, i.e. classical aerial triangulation results, are given in table 2. As already known selfcalibration cannot be applied to the inner orientation parametres in this case. On the other hand, the

Common		Block-dependent		
			Block P	Block N
f	152 mm			
h_g	11400 m			
s	1:75000	h_{min}	500 m	100 m
n_s	7	h_{max}	2800 m	560 m
n_i	7	h_{mean}	1591 m	318 m
n_p	77	d_{min}	0	0
n_f	5.7	d_{max}	1.25	0.25
σ_i	10 μm	d_{mean}	0.28	0.06
p	60%	σ_e	2 m	1 m
q	60%			
σ_g	.15 m			
σ_s	.50 m			

f :	focal length
h_g :	flying height above ground
s :	photo scale
n_s :	number of strips
n_i :	id. of images per strip
n_p :	number of object points
n_f :	average point folding
σ_i :	standard error of photo obs.
p :	forward overlap
q :	side overlap
σ_g :	standard error of ground control
σ_s :	standard error of satellite aided coordinate observations
h :	ground elevation
d :	ground slope
σ_e :	DTM standard error

Table 1: Simulated block parametres

elevation auxiliary data give results –for ground points– within tolerances. Standard deviations of the X, Y coordinates of projection centres –a basic precision parametre for digital image rectification– fall only within tolerances when using ground control and DTM data simultaneously, an interesting result in itself.

The results after introduction of GPS data are given in three groups (tables 3, 4 and 5). The first two groups correspond to the determination of datum transfer parametres (table 3) and to the selfcalibration of interior orientation parametres (table 4) independently. In the third group (table 5), the results for the more complex situation of datum transfer determination and selfcalibration are given. For all three groups, simulations have been run twice: with and without one GPS receiver on the ground, located in the block centre.

The results of table 3 show that any network configuration is acceptable and that having a receiver on the ground is not necessary although it improves the general block precision. The largest improvement factors when using a receiver on the ground are obtained if a DTM is the only ground control, and the smallest when using both DTM and control points. The angular parametres exhibit a regular behaviour. The analysis of the figures in table 4 shows, again, that any network configuration is acceptable and that a receiver on the ground is not required –the only significant improvement being obtained when using just DTM data.

The question arises in the assessment of focal length and principal point precision. According to Hakkarainen [9], the standard error of a calibration procedure is less than 5 μm for the focal length. In view of this value, the results of the simulations are excellent in all cases. The standard deviations of the principal point are also excellent. Even in the *worst* case –only DTM as control, no receiver on the ground– standard errors are less than that of photo observations ($\sigma_i = 10\mu m$). If, for instance, orthophotomaps are to be produced, a joint analysis of interior and exterior orientation elements precision is necessary which in this case leads, again, to acceptable results.

Table 5 shows the results of simultaneous datum transfer and inner orientation parametres determination. Although the precision in the ground object space falls always within tolerances, it is obvious that a receiver on the ground is required. Then, any network configuration is acceptable. An interesting result is that a DTM can be used as an alternative to ground control. Some precision is lost for the horizontal components (worsening factors of 1.48 for points, and 1.19 for projection centres) and some precision is gained for the vertical components (improvement factor of 1.20 for points). The joint consideration of ground control and elevation data gives excellent results, even for the inner orientation parametres. Nevertheless, at this point, one should be not too optimistic about DTM contributions before going through the results for flat terrain blocks.

5.2 Simulation results for blocks N and D

In table 6 the reference simulations are presented. Contrary to the reference table (2) in the former section no selfcalibration attempt is described. Instead, the results of conventional aerial triangulation for block D –couples of tie points– are given. The figures deserve two comments. First, the introduction of elevation data –block N– makes the horizontal standard errors of projection centres to be within tolerances. Second, the increase in the number of tie points leads to big precision improvements when only elevation data is used. Tolerance values are not yet reached, but this favourable behaviour will be exploited when using satellite data.

The analysis steps of section 5.1 are reproduced. Table 7 deals with the case where the datum parametres are unknown. If there is a receiver on the ground, all network configurations are feasible. If not, DTM data alone do not allow for datum transfer determination. Thus, the configuration I, S_o, E (block N) fails to meet the tolerances. Alternatives to this critical configuration are discussed in the next section.

Table 8 shows the results when the unknown parametres are those of inner orientation. Again, out of the configuration D, S_o, E (block N), the remaining cases give acceptable standard errors and the utilization of a receiver on the ground is not required. An interesting result with respect to elevation data is that of configuration D, S_o, S_g, E. The critical case D, S_o, E is discussed later.

Finally, the most difficult situation of simultaneous datum determination and selfcalibration corresponds to table 9. After the results of table 5 the configuration with no receiver on the ground has been disregarded. Contrary to the situation for block P and because of the small slopes the DTM elevation data is not sufficient although a receiver is on the ground. Thus, a new network type S_o, S_g, E adds to the list of critical configurations.

5.3 Simulation results for blocks D and G

The former analyses show weaknesses of elevation data when used as the only control data. The overall influence of the DTM in the adjustment depends on the terrain slope features and on the number of elevation points. Therefore, some improvement in block precision should be expected either by increasing the number of tie points or, because of the block strength when satellite *aerial control* is used, by increasing the number of images. These hypotheses are confirmed by the simulation results in tables 10 –pairs of tie points– and 11 –a larger block– which cover the three critical configurations in section 5.2. In all six simulated cases the mapping reference specifications are fulfilled. Thus, the three critical configurations found above are no longer a problem so long as one is aware of the behaviour of DTM data in these kind of hybrid adjustments.

6 Conclusions

It is possible to selfcalibrate the inner orientation parametres and to estimate the datum transfer parametres. If both parametre sets are to be determined simultaneously, a satellite receiver on the ground is required. Some control data is also required which can be a number of control points and/or a DTM. Depending on the terrain slope characteristics, excessively large standard errors of horizontal related components might occur in small blocks if only elevation data is used as control. This problem can be solved by observing more points; for instance, using pairs of tie points. It is not to be forgotten that if simultaneous selfcalibration and datum transfer determination is not possible, eventual errors in the interior parametres can be partly absorbed by the datum transformation.

If elevation data is available, it should be used as auxiliary information in bundle adjustments even for conventional aerial triangulation. This indeed applies to high altitude flights. Nevertheless, the utilization of a DTM in conjunction with satellite derived orientation elements works more efficiently according to the simulation results (compare tables 2 and 3, and tables 6 and 7). The DTM observations have been regarded as uncorrelated, a stochastic model which might deviate from the actual situation [13, p. 248]. On the other hand, the assumed standard errors for satellite derived and photogrammetric observations are rather pessimistic. One can expect, therefore, that the main conclusions will remain valid.

A last point related to the use of DTM data is the convergency behaviour of the iterative adjustment, something not studied in these experiments and which deserves further investigations with actual data.

Although not discussed here, knowledge of local geoid undulations at least is becoming increasingly important. Another question related to geodesy is that a 7 parametre transformation is not usual for datum transformations and that a 5 parametre one could be more adequate. Again, this will not change the main conclussions.

The current software for bundle adjustment can be easily adapted without much effort, since the new observation and normal equations considered in the extended model fit nicely into the structure of classical bundle programs. On the other hand, until GPS derived orientation data is not deeply analysed the required processing software will be an open question. It might be necessary, for example, to correct for drifts in each strip. Then, the general datum transformation used in the reported simulations would be useless.

Acknowledgement

The basic bundle program was developed at the ICC and at the *Institut für Geodäsie und Photogrammetrie* (ETH-Zürich) under the advice of Armin Grün. For this reason, for time devoted to discussions, and for his invaluable support for more than two years, the author wishes to thank Prof.A.Grün.

References

[1] Ackermann,F., 1984. Utilization of navigation data for aerial triangulation. *International Archives of Photogrammetry*, Vol. 25, Comm. III, pp. 1-9.

[2] Anderson,J.M., 1985. A combined photogrammetric and Doppler adjustment. *Photogrammetric Engineering and Remote Sensing*, Vol. 51, pp. 655-666.

[3] Beckmann, A., Larisch, H.-J. and Schuster,O., 1987. Bestimmung eines herkömmlichen TP-Netzes mit satellitennutzender Vermessung. *Vermessung, Photogrammetrie, Kulturtechnik*, Vol. 12-87, pp. 471-479.

[4] Ebner,H., 1984. Combined adjustment of photogrammetric and non-photogrammetric information. *International Archives of Photogrammetry*, Vol. 25, Comm. III, pp. 130-138.

[5] Ebner,H. and Müller,F., 1986. Processing of digital three line imagery using a generalized model for combined point determination. *International Archives of Photogrammetry*, Vol. 26, Comm. III, pp. 212-222.

[6] Frieß,P., 1986. A simulation study on the improvement of aerial triangulation by navigation data. *International Archives of Photogrammetry*, Vol. 26, Comm. III, pp. 269-283.

[7] Frieß,P., 1987. The NAVSTAR Global Positioning System for aerial triangulation. *Schriftenreihe des Instituts für Photogrammetrie der Universität Stuttgart*, Vol. 12, pp. 33-45.

[8] Frieß,P., 1988. Empirical accuracy of positions computed from airborne GPS data. Presented paper to the 16th ISPRS Congress, Comm. III, Kyoto.

[9] Hakkarainen,J., 1984. Geometrical stability of aerial cameras. *International Archives of Photogrammetry*, Vol. 25, Comm. I, pp. 93-103.

[10] Hartl,P. and Wehr,A., 1986. Chancen der GPS-Satellitennavigation für die Luftphotogrammetrie. *Bildmessung und Luftbildwesen*, Vol. 54, pp. 221-240.

[11] Lucas,J.R., 1987. Aerotriangulation without ground control. *Photogrammetric Engineering and Remote Sensing*, Vol. 53, pp. 311-314.

[12] Mader,G.L., Carter,W.E., Douglas,B.C. and Krabill, W.B., 1986. Decimeter precision aircraft positioning using GPS carrier phase measurements. Presented paper to the Joint Workshop on Combined Adjustment of Heterogeneous Geodetic and Photogrammetric Data, Munich.

[13] Rosenholm,D. and Torlegård,K., 1987. Absolute orientation of stereo models using digital elevation models. *Technical papers of the ASPRS-ACSM Convention*, Vol. 4, pp. 241-250.

[14] Sarjakoski,T., 1984. On sparse matrix techniques for computing weight coefficient matrices. *International Archives of Photogrammetry*, Vol. 25, Comm. III, pp. 945-953.

[15] Schwarz, K.P., Fraser, C.S. and Gustafson, P.C., 1984. Aerotriangulation without ground control. *International Archives of Photogrammetry*, Vol. 25, Comm. I, pp. 237-250.

[16] Strunz,G., 1986. Processing of non-photogrammetric data in combined block adjustment. *International Archives of Photogrammetry*, Vol. 26, Comm. III, pp. 231-240.

[17] Welsch, W.M., 1986. Problems of accuracies in combined terrestrial and satellite control networks. *Bulletin Géodésique*, Vol. 60, pp. 193-203.

TABLES OF SIMULATION RESULTS

D	σ_{X_0}	σ_{x_0}	σ_{X_p}	$\sigma_{\bar{X}}$
I	σ_{Y_0}	σ_{y_0}	σ_{Y_p}	$\sigma_{\bar{Y}}$
S_o	σ_{Z_0}	σ_f	σ_{Z_p}	$\sigma_{\bar{Z}}$
S_g	σ_λ	σ_{X_a}	σ_ω	σ_X
G	σ_ψ	σ_{Y_a}	σ_ϕ	σ_Y
E	σ_α	σ_{Z_a}	σ_κ	σ_Z

$(X_0, Y_0, Z_0, \lambda, \psi, \alpha)$:	datum transformation parametres	$m, mgon$
(x_0, y_0, f) :	inner orientation parametres	μm
(X_A, Y_A, Z_A) :	camera-to-antenna offset	m
$(X_p, Y_p, Z_p, \omega, \phi, \kappa)$:	exterior orientation parametres (average)	$m, mgon$
$(\bar{X}, \bar{Y}, \bar{Z})$:	ground object points (essential average)	m
(X, Y, Z) :	ground object points (9-fold average)	m

Symbols on the left column indicate which data were used in the adjustment, as follows. D datum transformation parametres, I interior orientation parametres, S_o satellite data for aircraft's antenna, S_g satellite data for one ground point, G conventional ground control and E elevation auxiliary information.

Figure 2: Description of the result tables

Classical bundle adjustment	D	-	-	1.97	.67	D	-	-	2.05	1.30	D	-	-	1.61	.59
	I	-	-	1.98	.67	I	-	-	2.01	1.25	I	-	-	1.62	.60
		-	-	1.14	1.32		-	-	1.02	.90		-	-	.73	.77
		-	-	9.1	.63		-	-	7.4	1.03		-	-	7.1	.53
	G	-	-	9.1	.63		-	-	7.4	.97	G	-	-	7.1	.53
		-	-	3.5	1.10	E	-	-	4.8	.76	E	-	-	3.2	.70
Classical bundle adjustment and selfcalibrat. of interior orientation parametres	D	-	175	12.93	1.01	D	-	205	15.40	1.90	D	-	119	8.83	.97
		-	191	14.03	1.26		-	215	16.05	1.79		-	134	9.92	.98
		-	331	25.31	2.36		-	92	6.98	1.19		-	76	5.76	1.07
		-	-	11.3	1.02		-	-	7.5	1.50		-	-	7.2	.76
	G	-	-	10.0	1.07		-	-	7.5	1.42	G	-	-	7.2	.76
		-	-	4.1	2.00	E	-	-	4.9	.90	E	-	-	3.4	.81

Table 2: Block P: reference simulations

Combined bundle-satellite adjustment		.50	-	.76	.60		.60	-	1.09	.96		.36	-	.64	.48
	I	.55	-	.67	.54	I	.82	-	.95	.88	I	.39	-	.60	.48
	S_o	.40	-	.57	.80	S_o	.74	-	.61	.67	S_o	.32	-	.43	.62
		1.8	-	3.0	.51		1.8	-	3.1	.87		1.3	-	2.8	.44
	G	1.3	-	3.2	.48		1.0	-	3.1	.80	G	.8	-	2.9	.43
		1.7	-	3.1	.67	E	2.9	-	3.1	.57	E	1.4	-	2.8	.53
Combined bundle-satellite adjustment with one ground receiver		.35	-	.64	.49		.34	-	.77	.66		.28	-	.60	.45
	I	.35	-	.59	.48	I	.37	-	.70	.64	I	.30	-	.56	.45
	S_o	.30	-	.51	.75	S_o	.40	-	.53	.65	S_o	.26	-	.42	.61
	S_g	1.5	-	3.0	.43	S_g	1.7	-	3.0	.60	S_g	1.2	-	2.8	.40
	G	1.0	-	3.1	.42		1.0	-	3.1	.55	G	.8	-	2.9	.40
		1.7	-	3.1	.63	E	2.4	-	3.1	.55	E	1.4	-	2.8	.52

Table 3: Block P: unknown datum transformation parametres

Combined bundle-	D	-	4.9	.47	.52	D	-	9.6	.47	.82	D	-	4.2	.47	.47
satellite		-	4.9	.47	.52		-	9.3	.47	.80		-	4.3	.47	.47
adjustment	S_o	-	4.2	.35	.72	S_o	-	3.7	.35	.65	S_o	-	2.7	.34	.63
		-	-	2.9	.47		-	-	2.9	.78		-	-	2.7	.42
	G	-	-	2.9	.47		-	-	2.9	.75	G	-	-	2.8	.52
		-	-	3.1	.62	E	-	-	3.1	.56	E	-	-	2.8	.51

Combined bundle-	D	-	4.3	.47	.49	D	-	6.3	.47	.61	D	-	3.9	.47	.45
satellite		-	4.3	.47	.49		-	6.1	.47	.61		-	3.9	.34	.45
adjustment	S_o	-	4.2	.35	.72	S_o	-	3.5	.35	.64	S_o	-	2.7	.34	.60
with one	S_g	-	-	2.8	.43	S_g	-	-	2.9	.55	S_g	-	-	2.7	.40
ground	G	-	-	2.8	.43		-	-	2.9	.54	G	-	-	2.7	.40
receiver		-	-	3.1	.62	E	-	-	3.1	.54	E	-	-	2.8	.50

Table 4: Block P: selfcalibration of inner orientation parametres

Combined bundle-		7.14	91	6.71	.68		7.01	103	7.72	1.08		5.65	78	5.70	.58
satellite		6.70	117	8.52	.72		7.61	121	9.00	.99		5.67	94	6.90	.60
adjustment	S_o	7.51	79	6.07	.88	S_o	7.16	62	4.68	.71	S_o	5.60	54	4.10	.65
		1.8	-	3.3	.61		1.3	-	3.2	.98		1.3	-	2.9	.52
	G	1.3	-	3.3	.66		1.1	-	3.2	.89	G	.8	-	3.0	.55
		1.8	-	3.3	.75	E	3.2	-	3.2	.61	E	1.5	-	2.9	.55

Combined bundle-		.50	9.1	.77	.52		.50	9.8	.94	.77		.50	8.2	.74	.47
satellite		.50	9.3	.77	.50		.50	10.1	.90	.71		.50	8.4	.74	.46
adjustment	S_o	.50	8.1	.71	.79	S_o	.50	9.1	.69	.66	S_o	.50	7.3	.65	.62
with one	S_g	1.7	-	3.0	.46	S_g	1.8	-	3.1	.66	S_g	1.3	-	2.8	.42
ground	G	1.2	-	3.2	.44		1.1	-	3.1	.61	G	.8	-	2.9	.41
receiver		1.7	-	3.1	.66	E	3.0	-	3.1	.56	E	1.5	-	2.8	.52

Table 5: Block P: datum determination and selfcalibration

Classical bundle	D	-	-	2.02	.70	D	-	-	3.60	3.10	D	-	-	1.56	.61
adjustment	I	-	-	2.03	.70	I	-	-	3.49	2.98	I	-	-	1.56	.61
		-	-	1.17	1.34		-	-	1.64	.60		-	-	.65	.57
		-	-	9.3	.64		-	-	7.0	2.74		-	-	6.6	.55
	G	-	-	9.3	.64		-	-	7.0	2.60	G	-	-	6.6	.55
		-	-	3.5	1.11	E	-	-	8.5	.55	E	-	-	3.1	.52

Classical bundle	D	-	-	1.48	.58	D	-	-	2.23	1.89	D	-	-	1.12	.52
adjustment		-	-	1.48	.58		-	-	2.15	1.80		-	-	1.12	.52
using		-	-	.90	1.06		-	-	1.00	.50		-	-	.48	.49
pairs of		-	-	6.7	.53		-	-	4.9	1.58		-	-	4.7	.46
tie points	G	-	-	6.7	.53		-	-	4.9	1.58	G	-	-	4.7	.46
		-	-	2.6	.88	E	-	-	5.2	.45	E	-	-	2.3	.44

Table 6: Reference simulations: blocks N and D

Combined bundle-satellite adjustment		.50	-	.77	.56		1.31	-	2.35	2.23		.30	-	.62	.47
	I	.56	-	.68	.54	I	2.02	-	2.09	2.06	I	.35	-	.58	.47
	S_o	.40	-	.57	.79	S_o	1.68	-	.89	.52	S_o	.29	-	.39	.50
		1.8	-	3.1	.51		1.6	-	2.9	2.12		1.1	-	2.7	.44
	G	1.3	-	3.3	.49		.9	-	3.0	1.97	G	.6	-	2.8	.43
		1.8	-	3.1	.67	E	5.3	-	2.9	.46	E	1.3	-	2.7	.44
Combined bundle-satellite adjustment with one ground receiver		.34	-	.65	.49		.35	-	.85	.75		.24	-	.58	.45
	I	.35	-	.60	.49	I	.40	-	.77	.73	I	.27	-	.55	.44
	S_o	.29	-	.51	.75	S_o	.45	-	.56	.51	S_o	.24	-	.39	.50
	S_g	1.5	-	3.0	.43	S_g	1.5	-	2.9	.64	S_g	1.1	-	2.7	.40
	G	1.1	-	3.1	.43		.8	-	2.9	.63	G	.6	-	2.7	.40
		1.7	-	3.1	.64	E	2.7	-	2.9	.45	E	1.3	-	2.7	.44

Table 7: Block N: unknown datum transformation parametres

Combined bundle-satellite adjustment	D	-	4.9	.47	.52	D	-	26.0	.47	1.98	D	-	4.1	.47	.46
		-	4.9	.47	.52		-	24.7	.47	1.89		-	4.1	.47	.46
	S_o	-	4.2	.35	.72	S_o	-	2.5	.35	.51	S_o	-	2.0	.34	.49
		-	-	2.9	.47		-	-	2.8	1.96		-	-	2.7	.43
	G	-	-	2.9	.47		-	-	2.8	1.87	G	-	-	2.7	.42
		-	-	3.1	.62	E	-	-	2.8	.45	E	-	-	2.7	.43
Combined bundle-satellite adjustment with one ground receiver	D	-	4.4	.47	.49	D	-	7.8	.47	.69	D	-	3.8	.47	.44
		-	4.4	.47	.49		-	7.8	.47	.69		-	3.9	.47	.44
	S_o	-	4.2	.35	.72	S_o	-	2.5	.34	.51	S_o	-	2.0	.34	.49
	S_g	-	-	2.8	.43	S_g	-	-	2.8	.64	S_g	-	-	2.7	.40
	G	-	-	2.8	.43		-	-	2.8	.64	G	-	-	2.7	.43
		-	-	3.1	.62	E	-	-	2.8	.45	E	-	-	2.7	.43

Table 8: Block N: selfcalibration of inner orientation parametres

Combined bundle-satellite adjustment with one ground receiver		.50	9.3	.77	.52		.50	10.6	1.34	1.18		.50	8.0	.73	.46
		.50	9.2	.77	.50		.50	13.7	1.27	1.06		.50	8.2	.72	.46
	S_o	.50	8.2	.71	.79	S_o	.50	12.7	.67	.51	S_o	.50	7.1	.63	.50
	S_g	1.8	-	3.0	.46	S_g	1.6	-	2.9	.97	S_g	1.1	-	2.7	.41
	G	1.3	-	3.2	.45		.9	-	3.0	.89	G	.6	-	2.8	.41
		1.7	-	3.1	.67	E	5.3	-	3.0	.45	E	1.3	-	2.7	.44

Table 9: Block N: datum determination and selfcalibration

Combined														
bundle-	I	.87	-	1.58	1.49		-	16.6	.45	1.29		.50	9.9	1.07	.95
satellite		1.28	-	1.44	1.41	D	-	16.5	.45	1.29		.50	11.8	1.02	.86
adjustment	S_o	1.18	-	.67	.46	S_o	-	1.9	.29	.46	S_o	.50	10.4	.61	.46
with one		1.3	-	2.4	1.40		-	-	2.4	1.27	S_g	1.3	-	2.4	.81
ground		.6	-	2.5	1.34		-	-	2.4	1.27		.7	-	2.5	.74
receiver	E	3.9	-	2.1	.40	E	-	-	2.0	.40	E	3.9	-	2.1	.40

Table 10: Block D: Critical configurations

Combined														
bundle-	I	.67	-	1.25	1.20		-	12.0	.47	.98		.50	9.4	.99	.93
satellite		.91	-	1.19	1.16	D	-	13.1	.47	1.05		.50	10.3	.87	.80
adjustment	S_o	.82	-	.44	.48	S_o	-	1.2	.33	.47	S_o	.50	7.6	.61	.48
with one		.4	-	2.6	1.14		-	-	2.6	.96	S_g	.4	-	2.6	.84
ground		.2	-	2.6	1.12		-	-	2.6	1.03		.2	-	2.5	.74
receiver	E	1.8	-	2.5	.43	E	-	-	2.5	.43	E	1.8	-	2.5	.43

Table 11: Block G: Critical configurations