GENERAL FREE NET THEORY IN PHOTOGRAMMETRY

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ABSTRACT

The conventional free net theory in photogrammetry has been constructed under the assumption that the three-dimensional similarity transformation is valid between the model and object spaces. However, unlike geodesy, a model formed from overlapped pictures can, in general, be transformed into the object by a three-dimensional projective transformation having fifteen independent coefficients. In this study, a new free net theory is derived based on this fact and is further applied to some simulated examples in order to clarify its practical characteristics.

INTRODUCTION

The free network theory in geodesy was first derived by Meissl in 1962 and soon applied to photogrammetry very effectively (Ebner(1974), Gruen(1976), Granshaw(1980), and others). This free network theory in photogrammetry can readily be extended to the general case where a picture has eleven independent orientation parameters. In this paper, the general orientation problem of overlapped photographs is briefly described, the general free network theory is derived based on the general orientation theory of photographs, and its practical characteristics are discussed by applying it to simulated photographs.

GENERAL ORIENTATION PROBLEM OF PHOTOGRAPHS

We will begin with the discussion on the general orientation problem of a stereopair of photographs, because it is very important for deriving the general free network theory (The detailed discussion on this problem is seen in Okamoto(1981a, 1981b)). The general collinearity equations relating an object point P(X,Y,Z) and its measured image point $p_c(x_c,y_c)$ are described as

$$x_{c1} = \frac{{}_1A_1X + {}_1A_2Y + {}_1A_3Z + {}_1A_4}{{}_1A_9X + {}_1A_{10}Y + {}_1A_{11}Z + 1} \tag{1}$$

$$y_{c1} = \frac{{}_1A_5X + {}_1A_6Y + {}_1A_7Z + {}_1A_8}{{}_1A_9X + {}_1A_{10}Y + {}_1A_{11}Z + 1}$$

for the left picture, and in the form

$$x_{c2} = \frac{{}_2A_1X + {}_2A_2Y + {}_2A_3Z + {}_2A_4}{{}_2A_9X + {}_2A_{10}Y + {}_2A_{11}Z + 1}$$

$$y_{c2} = \frac{{}_2A_5X + {}_2A_6Y + {}_2A_7Z + {}_2A_8}{{}_2A_9X + {}_2A_{10}Y + {}_2A_{11}Z + 1} \qquad (2)$$

for the right photograph, respectively. The condition that Equations 1 and 2 are valid for all object points photographed in common on the left and right pictures can be formulated as

$$\begin{vmatrix} x_{c1}\,{}_1A_9 - {}_1A_1 & x_{c1}\,{}_1A_{10} - {}_1A_2 & x_{c1}\,{}_1A_{11} - {}_1A_3 & x_{c1} - {}_1A_4 \\ y_{c1}\,{}_1A_9 - {}_1A_5 & y_{c1}\,{}_1A_{10} - {}_1A_6 & y_{c1}\,{}_1A_{11} - {}_1A_7 & y_{c1} - {}_1A_8 \\ x_{c2}\,{}_2A_9 - {}_2A_1 & x_{c2}\,{}_2A_{10} - {}_2A_2 & x_{c2}\,{}_2A_{11} - {}_2A_3 & x_{c2} - {}_2A_4 \\ y_{c2}\,{}_2A_9 - {}_2A_5 & y_{c2}\,{}_2A_{10} - {}_2A_6 & y_{c2}\,{}_2A_{11} - {}_2A_7 & y_{c2} - {}_2A_8 \end{vmatrix} = 0 \qquad (3)$$

which is equivalent to the coplanarity condition of corresponding rays. Under the condition of Equation 3, we can define one space (X_M, Y_M, Z_M) which can be transformed into the object space (X,Y,Z) by the three-dimensional projective transformation having 15 independent elements, i.e.,

$$X_M = \frac{B_1X + B_2Y + B_3Z + B_4}{B_{13}X + B_{14}Y + B_{15}Z + 1}$$

$$Y_M = \frac{B_5X + B_6Y + B_7Z + B_8}{B_{13}X + B_{14}Y + B_{15}Z + 1} \qquad (4)$$

$$Z_M = \frac{B_9X + B_{10}Y + B_{11}Z + B_{12}}{B_{13}X + B_{14}Y + B_{15}Z + 1}$$

Equation 4 is equivalent to the projective one-to-one correspondence between the model and object spaces. Using the results above, we can further find important characteristics of the coplanarity condition that seven independent orientation parameters must mathematically be provided for the construction of the stereo model, because the 22 independent orientation parameters must become known for the unique determination of all photographed object points from their measured image coordinates (x_{c1}, y_{c1}) and (x_{c2}, y_{c2}).

GENERAL FREE NETWORK THEORY IN PHOTOGRAMMETRY

In the simultaneous determination of both orientation parameters of overlapped pictures and coordinates of photographed object points, the basic equations are the collinearity equations. The general collinearity equations, i.e.,

$$
\begin{aligned}
x_c &= \frac{A_1 X + A_2 Y + A_3 Z + A_4}{A_9 X + A_{10} Y + A_{11} Z + 1} = C/B \\
y_c &= \frac{A_5 X + A_6 Y + A_7 Z + A_8}{A_9 X + A_{10} Y + A_{11} Z + 1} = D/B
\end{aligned}
\tag{5}
$$

can be linearized in the form

$$
\begin{aligned}
x_c = F_{x0} &+ (X/B)_0 \Delta A_1 + (Y/B)_0 \Delta A_2 + (Z/B)_0 \Delta A_3 + (1/B)_0 \Delta A_4 \\
&- (CX/B^2)_0 \Delta A_9 - (CY/B^2)_0 \Delta A_{10} - (CZ/B^2)_0 \Delta A_{11} \\
&+ ((A_1 B - A_9 C)/B^2)_0 \Delta X + ((A_2 B - A_{10} C)/B^2)_0 \Delta Y + ((A_3 B - A_{11} C)/B^2)_0 \Delta Z \\
y_c = F_{y0} &+ (X/B)_0 \Delta A_5 + (Y/B)_0 \Delta A_6 + (Z/B)_0 \Delta A_7 + (1/B)_0 \Delta A_8 \\
&- (DX/B^2)_0 \Delta A_9 - (DY/B^2)_0 \Delta A_{10} - (DZ/B^2)_0 \Delta A_{11} \\
&+ ((A_5 B - A_9 D)/B^2)_0 \Delta X + ((A_6 B - A_{10} D)/B^2)_0 \Delta Y + ((A_7 B - A_{11} D)/B^2)_0 \Delta Z
\end{aligned}
\tag{6}
$$

Setting up Equation 6 for all photographs under consideration, we have a system of linear equations in a matrix form as

$$\mathbf{A} \Delta \mathbf{x} = \mathbf{c} \tag{7}$$

in which

A: a coefficient matrix of the system of linear equations

Δx: a vector of corrections to unknowns

c: a vector of constants.

Without object space controls the matrix **A** is singular and its rank deficiency is fifteen. In the general free network theory we have therefore fifteen linearly independent vectors satisfying the following relationship (Mittermayer(1972), and Ebner(1974)):

$$\mathbf{A}\,\mathbf{G} = \mathbf{0} \tag{8}$$

where **G** is a matrix constructed from the 15 linearly independent vectors, i.e.,

$$\mathbf{G} = (g_1, g_2, \ldots, g_{15}) \tag{9}$$

These 15 vectors can easily be found by linearizing the three-dimensional projective transformation (Equation 4) and expressed in the form of Equation (10) or Equation (11). In Equation (11) the first seven vectors are related to a linearized three-dimensional similarity transformation and the last eight vectors are associated with a model deformation.

If measured image coordinates (x_c, y_c) have random errors, the system of linearized observation equations can be given by

$$\mathbf{v} = \mathbf{A}\Delta\hat{\mathbf{x}} - \mathbf{L} \tag{12}$$

where **v** is a vector of residuals to the observations. The free network adjustment is then carried out as follows:

$$\mathbf{v}^T\mathbf{P}\mathbf{v} \longrightarrow \min$$
$$\text{under the condition } \mathbf{G}^T\Delta\hat{\mathbf{x}} = \mathbf{0}$$

where **P** is a weight matrix of the observations.

TEST WITH SIMULATION MODELS

The method presented in the previous sections was tested with simulated photographs. In the construction of the simulation models, three convergent photographs were considered to be employed and the image coordinates of 25 object points were calculated by means of the conventional collinearity equations under the following conditions(See Figure-1):

flying height:	H = 1500m
focal length of the camera:	c = 15cm
picture format:	23 x 23cm^2
convergent angles:	ca. ± 20deg.
maximum height difference among the 25 object points:	ca. 100m

$$
\begin{array}{lllllllllllllllll}
\mathbf{g}_1{}^T = (& A_9A_1, & A_9A_2, & A_9A_3, & A_9A_4-A_1, & A_9A_5, & A_9A_6, & A_9A_7, & A_9A_8-A_5, & A_9A_9, & A_9A_{10}, & A_9A_{11}, & \ldots\ldots, & 1, & 0, & 0, & \ldots\ldots) \\
\mathbf{g}_2{}^T = (& A_{10}A_1, & A_{10}A_2, & A_{10}A_3, & A_{10}A_4-A_2, & A_{10}A_5, & A_{10}A_6, & A_{10}A_7, & A_{10}A_8-A_6, & A_{10}A_9, & A_{10}A_{10}, & A_{10}A_{11}, & \ldots\ldots, & 0, & 1, & 0, & \ldots\ldots) \\
\mathbf{g}_3{}^T = (& A_{11}A_1, & A_{11}A_2, & A_{11}A_3, & A_{11}A_4-A_3, & A_1 1A_5, & A_{11}A_6, & A_{11}A_7, & A_{11}A_8-A_7, & A_{11}A_9, & A_{11}A_{10}, & A_{11}A_{11}, & \ldots\ldots, & 0, & 0, & 1, & \ldots\ldots) \\
\mathbf{g}_4{}^T = (& A_1, & 0, & 0, & 0, & A_5, & 0, & 0, & 0, & A_9, & 0, & 0, & \ldots\ldots, & -X, & 0, & 0, & \ldots\ldots) \\
\mathbf{g}_5{}^T = (& A_2, & 0, & 0, & 0, & A_6, & 0, & 0, & 0, & A_{10}, & 0, & 0, & \ldots\ldots, & 0, & -X, & 0, & \ldots\ldots) \\
\mathbf{g}_6{}^T = (& A_3, & 0, & 0, & 0, & A_7, & 0, & 0, & 0, & A_{11}, & 0, & 0, & \ldots\ldots, & 0, & 0, & -X, & \ldots\ldots) \\
\mathbf{g}_7{}^T = (& 0, & A_1, & 0, & 0, & 0, & A_5, & 0, & 0, & 0, & A_9, & 0, & \ldots\ldots, & -Y, & 0, & 0, & \ldots\ldots) \\
\mathbf{g}_8{}^T = (& 0, & A_2, & 0, & 0, & 0, & A_6, & 0, & 0, & 0, & A_{10}, & 0, & \ldots\ldots, & 0, & -Y, & 0, & \ldots\ldots) \\
\mathbf{g}_9{}^T = (& 0, & A_3 & 0, & 0, & 0, & A_7, & 0, & 0, & 0, & A_{11}, & 0, & \ldots\ldots, & 0, & 0, & -Y, & \ldots\ldots) \\
\mathbf{g}_{10}{}^T = (& 0, & 0, & A_1, & 0, & 0, & 0, & A_5, & 0, & 0, & 0, & A_9, & \ldots\ldots, & -Z, & 0, & 0, & \ldots\ldots) \\
\mathbf{g}_{11}{}^T = (& 0, & 0, & A_2, & 0, & 0, & 0, & A_6, & 0, & 0, & 0, & A_{10}, & \ldots\ldots, & 0, & -Z, & 0, & \ldots\ldots) \\
\mathbf{g}_{12}{}^T = (& 0, & 0, & A_3, & 0, & 0, & 0, & A_7, & 0, & 0, & 0, & A_{11}, & \ldots\ldots, & 0, & 0, & -Z, & \ldots\ldots) \\
\mathbf{g}_{13}{}^T = (& A_4, & 0, & 0, & 0, & A_8, & 0, & 0, & 0, & 1, & 0, & 0, & \ldots\ldots, & XX, & XY, & XZ, & \ldots\ldots) \\
\mathbf{g}_{14}{}^T = (& 0, & A_4, & 0, & 0, & 0, & A_8, & 0, & 0, & 0, & 1, & 0, & \ldots\ldots, & YX, & YY, & YZ, & \ldots\ldots) \\
\mathbf{g}_{15}{}^T = (& 0, & 0, & A_4, & 0, & 0, & 0, & A_8, & 0, & 0, & 0, & 1, & \ldots\ldots, & ZX, & ZY, & ZZ, & \ldots\ldots)
\end{array}
\tag{10}
$$

$$
\begin{aligned}
\mathbf{g}_1^{T} &= (A_9A_1,\ A_9A_2,\ A_9A_3,\ A_9A_4-A_1,\ A_9A_5,\ A_9A_6,\ A_9A_7,\ A_9A_8-A_5,\ A_9A_9,\ A_9A_{10},\ A_9A_{11},\ldots\ldots,\ 1,\ 0,\ 0,\ldots\ldots) \\
\mathbf{g}_2^{T} &= (A_{10}A_1,A_{10}A_2,A_{10}A_3,A_{10}A_4-A_2,A_{10}A_5,A_{10}A_6,A_{10}A_7,A_{10}A_8-A_6,A_{10}A_9,A_{10}A_{10},A_{10}A_{11},\ldots\ldots,\ 0,\ 1,\ 0,\ldots\ldots) \\
\mathbf{g}_3^{T} &= (A_{11}A_1,A_{11}A_2,A_{11}A_3,A_{11}A_4-A_3,A_{11}A_5,A_{11}A_6,A_{11}A_7,A_{11}A_8-A_7,A_{11}A_9,A_{11}A_{10},A_{11}A_{11},\ldots\ldots,\ 0,\ 0,\ 1,\ldots\ldots) \\
\mathbf{g}_4^{T} &= (-A_2,\ A_1,\ 0,\ 0,\ -A_6,\ A_5,\ 0,\ 0,\ -A_{10},\ A_9,\ 0,\ldots\ldots,-Y,\ X,\ 0,\ldots\ldots) \\
\mathbf{g}_5^{T} &= (A_3,\ 0,\ -A_1,\ 0,\ A_7,\ 0,\ -A_5,\ 0,\ A_{11},\ 0,\ -A_9,\ldots\ldots,\ Z,\ 0,-X,\ldots\ldots) \\
\mathbf{g}_6^{T} &= (0,\ -A_3,\ A_2,\ 0,\ 0,\ -A_7,\ A_6,\ 0,\ 0,\ -A_{11},\ A_{10},\ldots\ldots,\ 0,-Z,\ Y,\ldots\ldots) \\
\mathbf{g}_7^{T} &= (A_1,\ A_2,\ A_3,\ 0,\ A_5,\ A_6,\ A_7,\ 0,\ A_9,\ A_{10},\ A_{11},\ldots\ldots,-X,-Y,-Z,\ldots\ldots) \\
\mathbf{g}_8^{T} &= (A_4,\ 0,\ 0,\ 0,\ A_8,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0,\ldots\ldots,XX,XY,XZ,\ldots\ldots) \\
\mathbf{g}_9^{T} &= (0,\ A_4,\ 0,\ 0,\ 0,\ A_8,\ 0,\ 0,\ 0,\ 1,\ 0,\ldots\ldots,YX,YY,YZ,\ldots\ldots) \\
\mathbf{g}_{10}^{T} &= (0,\ 0,\ A_4,\ 0,\ 0,\ 0,\ A_8,\ 0,\ 0,\ 0,\ 1,\ldots\ldots,ZX,ZY,ZZ,\ldots\ldots) \\
\mathbf{g}_{11}^{T} &= (A_1,\ 0,\ 0,\ 0,\ A_5,\ 0,\ 0,\ 0,\ A_9,\ 0,\ 0,\ldots\ldots,-X,\ 0,\ 0,\ldots\ldots) \\
\mathbf{g}_{12}^{T} &= (A_3,\ 0,\ 0,\ 0,\ A_7,\ 0,\ 0,\ 0,\ A_{11},\ 0,\ 0,\ldots\ldots,\ 0,\ 0,-X,\ldots\ldots) \\
\mathbf{g}_{13}^{T} &= (0,\ A_1,\ 0,\ 0,\ 0,\ A_5,\ 0,\ 0,\ 0,\ A_9,\ 0,\ldots\ldots,-Y,\ 0,\ 0,\ldots\ldots) \\
\mathbf{g}_{14}^{T} &= (0,\ A_2,\ 0,\ 0,\ 0,\ A_6,\ 0,\ 0,\ 0,\ A_{10},\ 0,\ldots\ldots,\ 0,-Y,\ 0,\ldots\ldots) \\
\mathbf{g}_{15}^{T} &= (0,\ 0,\ A_2,\ 0,\ 0,\ 0,\ A_6,\ 0,\ 0,\ 0,\ A_{10},\ldots\ldots,\ 0,-Z,\ 0,\ldots\ldots)
\end{aligned}
\qquad (11)
$$

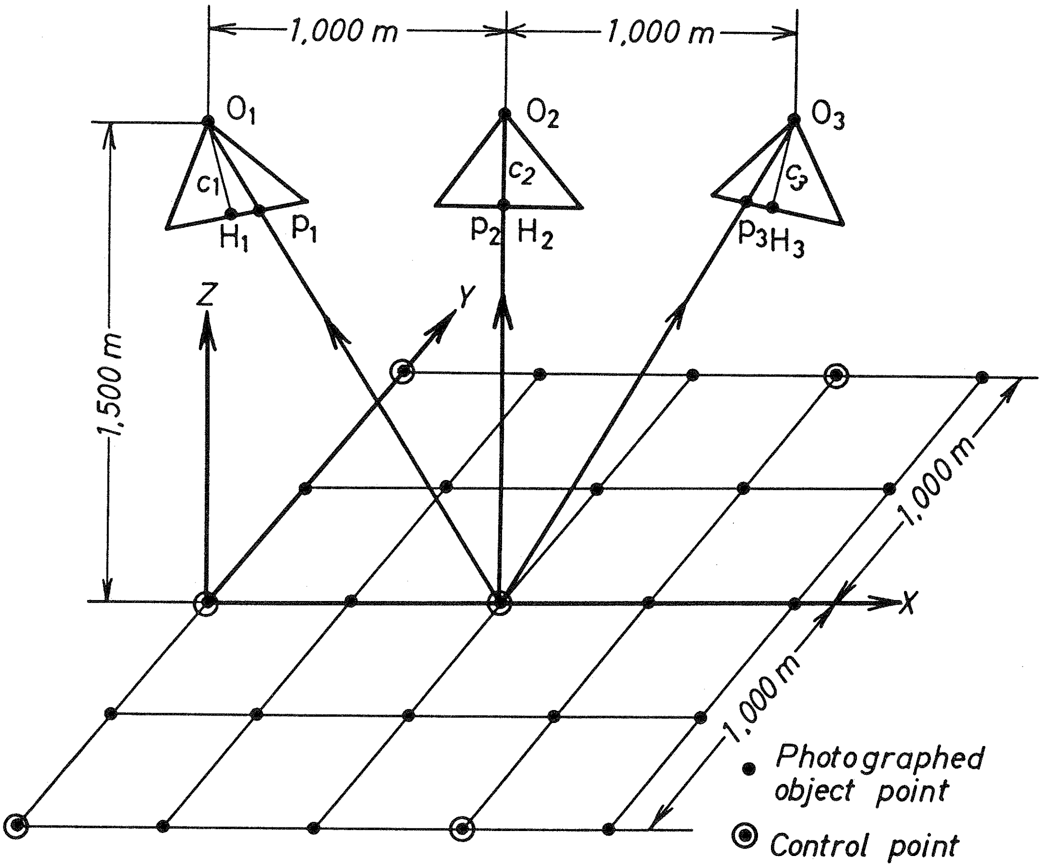

Figure-1: Three convergent photographs

Then, the image coordinates are affinitively transformed as

$$\begin{aligned} x_{ci} &= a_{1i}x_i + a_{2i}y_i + a_{3i} \\ y_{ci} &= a_{4i}x_i + a_{5i}y_i + a_{6i} \end{aligned} \tag{13}$$

in which the six coefficients $a_{ji}(j=1,\ldots 6;\ i=1,2,3)$ are given as shown in Table-1.

Table-1: Coefficients of the affine transformation

Photo.Nr.	a_1	a_2	a_3(mm)	a_4	a_5	a_6(mm)
1	1.03	0.04	110.10	0.02	0.98	-150.37
2	0.99	0.03	110.94	0.02	1.02	100.25
3	1.01	-0.03	-100.92	0.03	0.98	-180.31

Finally, the perturbed photo coordinates were provided in which the perturbation consisted of random normal deviates having a standard deviation of 5 microns.

The free network theory is essentially a linear theory. Thus, we must have fairly good approximation values for unknowns because mathematical models in photogrammetry are usually non-linear. Further, in order to obtain a high external accuracy of calculated object points we need an iterative calculation, because the solution depends upon the given approximation values of the unknowns. In this investigation, a conventional orientation method(the DLT method(Abdel-Aziz and Karara(1971)) was first applied for obtaining the particular solutions with fixed control points. Then,the approximation values of the unknowns in the free network adjustment were calculated by contaminating the control point coordinates with random errors having a standard deviation of 10cm. The iterative calculation in this free network adjustment was performed by replacing the approximation values for the control points by the true values in each iteration step. Also, only the general case employing the 15 linearly independent vectors was analyzed. The calculation was carried out for following two kinds of control point arrangement(See Figure-2);

(1) case A where we have an appropriate arrangement of five control points mathematically required.
(2) case B in which the configuration of six control points is somewhat inadequate.

case A case B

Figure-2: Arrangement of control points

In addition, fictitious observations with loose variances were introduced for both the orientation elements and object point coordinates.

The obtained results regarding the standard error of unit weight, the average internal error of check points, and the average external error are shown in Table-2. We can find in Table-1 the following characteristics;

(1) The geometry of the used three convergent photographs was rather strong. Consequently, the conventional orientation method had a fairly good accuracy.

Table-2: The obtained orientation results

case		$\hat{\sigma}_0$ (μm)	average internal error at the ground scale(cm)	average external error at the ground scale(cm)
A	particular solution	2.9	4.6	8.2
	free net solution	2.7	3.0	6.8
B	particular solution	3.1	5.3	6.8
	free net solution	2.8	3.1	6.5

(2) Applying the free network adjustment, great improvements were recognized in the internal precision.
(3) On the other hand, the improvements in the external precision were not very great.
(4) The solutions of the free network adjustment converged very slowly.
(5) The solution sometimes diverged when very high weight were given on the free network constraints ($\mathbf{G}^T\Delta\hat{\mathbf{x}} = \mathbf{0}$)

DISCUSSIONS AND CONCLUSIONS

In this paper the conventional free network theory in photogrammetry has been extended to the general case where a photograph has eleven independent orientation parameters. This general theory may be characterized by the fact that it is related to a linearized three-dimensional projective transformation, while the conventional one is associated with a linearized three-dimensional similarity transformation. Thus, model deformation can be potentially considered by applying this general theory to the analysis of overlapped photographs.

The proposed method in its present form is applicable only to the general case and to a special case where we have measured distances as object space controls (in this special case the distances are fixed and the first six linearly independent vectors in Equation (11) are introduced in the free network adjustment). In other cases where we have constraints among the eleven coefficients of the general collinearity equations for each photograph we may find linearly independent vectors satisfying the conditions by rearranging the 15 linearly independent vectors in Equation 10. It should, however, be noted that such constraints are object space controls in a wide sense and that the eleven coefficients of the general collinearity equations are independent in all cases in photogrammetry, even in the case of metric photography. Thus, if we have more than five control points, both the general orientation calculation and the general free network adjustment discussed in this paper is mathematically applicable to all cases in photogrammetry by neglecting the constraints among the eleven coefficients.

The method presented has been tested with some simulated examples. Through this investigation the proposed theory has been shown to be mathematically sound and useful in the analysis of overlapped photographs. However, in order to use this method effectively, various practical algorithms may be developed.

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