Liu Chuang Department of Geography Beijing University Beijing China

Abstract

Conventional methods of land evaluation that use classical methods of hierarchical classification and logic analysis to determine land suitability classes have revealed some shortages and they often break down in GIS. One of the reasons is that it is depended on the mental model which couldn't be recognized by computer in GIS, another reason is the functions which influent the land classes are multiple and their variations are not in the same steps, that makes us often fall down in hesitation in determining the land level boundaries and mapping of land evaluation.

Fuzzy reasoning and the mathematical models of fuzzy multiple assessment can overcome these difficulties and be useful for land evaluation in GIS. It's basic idea is from a single member ship factor assessment to the multiple membership factors assessment with fuzzy reasioning and determining the weights of every factors with the AHP model.

Introduction

Based on the ideas of FAO, land evaluation should be connected with land characteristics and land use. Together with our practice in Dalian region, Liaoning Province of China, the procedures of land evaluation in GIS are as map 1. We can obtain the land evaluation parcels map and the table of the characteristics of land evaluation parcels in GIS based on the maps of land type and land use and the tables of the characteristic of land type and land use. The next step of land evaluation is to set up the mathematical models of land evaluation for different land uses or different aims, then we can obtain the maps and the tables of land evaluation for different land uses or different aims. In this procedures, the modeling of land evaluation is very importaint step, sometimes it is the key one.

The fuzzy mathematical model of land evaluation

Fuzzy set theory as a means for dealing with inexact concepts was been produced by Zadeh in 1965. It distinguishes three kinds of inexactness:

- --- generality, that a single concept applies to variety of situation;
- --- ambiguity, that a single concept embraces more than one distinguishable sub-concept;
- --- vagueness, that precise boundaries are not defined.

A fuzzy set is a class that admits the possibility of partial membership. Fuzzy sets are generalisations of the usual crisp sets of situations where the class boundaries are not, or cannot be sharply defined. Currently, applications are being found in computer science and artificial intelligence and this article present one application in land evaluation.





The fuzzy mathematical model of land evaluation is consisted of three parts:

a. A single membership function model:



Where C₀ defines the exact boundary condition which is not suitable land, C₁ defines the land evaluation exact boundary which is suitable land completely, a₁ is constant, E₁ defines the land evaluation coefficient of a single membership function of the ith land evaluation unit. b. Multiple membership function model:

$$B_{i} = W_{1}^{*}E_{1} + W_{2}^{*}E_{2} + \cdots + W_{k}^{*}E_{k}$$
(FMM2)
= $\sum_{j=1}^{k} W_{j}^{*}E_{j}$, $i=1,2, \cdots p_{*}$

Where B, defines the land evaluation coefficient of multiple membership function of the ith land evaluation unit, E, is the land evaluation coefficient of the jth membership function of the ith land evaluation unit, W, defines the weight of the jth membership function,

where

С。

$\sum_{j=1}^{k} W$	j = 1,	, w _j >(Ο.	
Comment	model			
	Γ^{S_1}	for	V₁≤B₁	
	S ₂	for	V ₂ <b<sub>1≤V₁</b<sub>	
Q	$\begin{cases} s_3 \end{cases}$	for	V ₃ <b<sub>i≼V₂</b<sub>	(FMM3)
~~~~	s ₄	for	V4 ^{<b< sup="">1≤V3</b<>}	
	S5	for	^v 5 ^{<b< sup="">1^{≤v}4</b<>}	
	'SO	for	^B i< ^V 5	

where S, defines the land suitability level, S₀ is the dissuitable land comment,  $V_i$  defines the exact boundary of every land suitability level.

When we apply the fuzzy mathematical model to evaluate land in GIS, we need finish below steps:

--- we need a set of possible land evaluation units, which we can finish based on the maps of land type and land use;

--- we need a set of membership functions and a set of the charicteristics of every land evaluation units about the membership functions, which we can finish based on the tables of land type and land use;

--- for each land characteristic we need a standard index to define full membership of the fuzzy set, and we need to set the values for the crossover points that determine the dispirsion indices of the set, which we can finish based on the experts' experiences, note that all these indexes or values are different for different aims of land evaluation or different regions where the land will be evaluated;

--- we need a set of weights to relate the importance of the land characteristics to each other, which we will finish by applying the AHP model (Analytical Hierachy Process).

The AHP model to determine the weights of land evaluation membership functions

The AHP model as an analytical means for the importance of every factors in a complex system which includes many factors was been produced by A.L.Saaty in 1970's. It distinguishes the factors into different levels which are related to each other. Based on the ajugement relative importance between the factors in the same level, calculate the weights of every factors, the main function of the AHP model in land evaluation is to determine the weights of every membership functions which are related with land evaluation. The basic theory of AHP as below:

where a defines the relative importance coefficient of the ith factor ^{ij} to the jth factor, that is obviously:

$$a_{ii} = 1$$
,  $a_{ij} = 1/a_{ji}$  (i, j=1,2,....n)

and

$$AW = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & & a_{nn} \end{pmatrix} \cdot \begin{pmatrix} W_1 \\ W_2 \\ \vdots \\ \vdots \\ W_n \end{pmatrix} = \begin{pmatrix} nW_1 \\ nW_2 \\ \vdots \\ \vdots \\ NW_n \end{pmatrix} = nW$$

$$\sum_{i=1}^{n} W_i = 1$$

when  $a_{ij}=a_{ik}/a_{jk}$  (i,j,k=1,2,....n) then  $AW = \lambda_{max} W$  and finally we can obtain W.

When we apply the AHP model to calculate the weights of every membership functions, we need finish below steps:

--- we need to devide the issues that different levels, the highest level is the aim level of land evaluation, the lowest level is the membership functions level, and the middle levels may be sub-aim level or the sets level which are consisted of some factors. Between the levels there are some relationship line to connect them;

--- making the adjugement matrix: the adjugement metrix shows the relative importaince of the factors in the same level and all these factors are related to one factor which is located in the higher level. Suport  $A_k$ , which is located in the A level, is related to the  $B_1, B_2$ , ....  $B_n$ , which are in the B level, the adjugement matrix is:

Ak	^B ₁ ^B ₂	B _n	Wi
B ₁	b ₁₁ b12	^b 1n	W1
^B 2	^b ₁₂ ^b ₂₂	^b 2n	^W 2
•	• •	٠	
•	• •	•	•
•	<b>e</b> . ⊢ <b>e</b> .	•	•
Bn	^b n1 <b>n</b> 2 · · · · ·	^b nn	Wn

Where b_{ij} is the relative importaince coefficient b_i to b_j, for A_k, ^{ij} usually b_{ij} = (1,2,....9), or (1, 1/2, 173,...1/9). The more the coefficient is, the more important the factor is; --- calculate the weights of the factors in the same level:

to calculate the weights of the factors in the same level, we can use the equation:

BW = Amax W where B defines the adjugement matrix,  $\lambda_{max}$  is the maxmum characteristic root, W is the characteristic vector for  $\lambda_{max}$ .

In order to test the consistent of the adjugement matrix and whether it is satisfiable, we need to calculate it's consistent index CI and radom consistent radio CR:

$$CI = \frac{\lambda_{max} - n}{n-1}$$
,  $CR = CI/RI$ 

where RI is averange redom consistent index.

order PT	1	2	3	4	5	6	7	8	9
RI	0.00	0.00	0,00	0.90	{ ● ¹ €	1.24		1.41	1647

If CR $\langle 0.10$ , the adjugement matrix is satisfiedconsistent, and now we can finish the W;

--- calculate the weights of the factors at the tatal level, we can obtain the W_{b1}, W_{b2}, .... W_{bn}; W_{c1}, W_{c2},.... W_{cn}, .... then we can calculate the tatal weights of the factors:

$$W_{cj} = \sum_{i=1}^{n} b_i c^i_j$$
 (i=1,2,..., j=1,2,...,m)

Table 2: The tatal weights of the factors being in the level C

level B level C	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
C ₁	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
°2	$c_{2}^{1} c_{2}^{2} \cdots c_{2}^{n} \qquad W_{c2} = \sum_{i=1}^{n} b_{i} c_{2}^{i}$
C _m	$\begin{vmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ c_{m}^{1} & c_{m}^{2} & \cdots & c_{m}^{n} \end{vmatrix} \qquad \begin{array}{c} W_{cm} = \sum_{i=1}^{n} & b_{i}c_{m}^{1} \\ \vdots = 1 & \end{array}$

$$\sum_{j=1}^{m} \sum_{i=1}^{n} b_i c_j^1 = 1 ;$$

--- test the consistent of the tatal weights. The model of testing the consistent of tatal weights is similar to the step two:

$$CI_t = \sum_{i=1}^{n} b_i CI_i$$
  $RI_t = \sum_{i=1}^{n} b_i RI_i$   $CR_t = CI_t / RI_t$ 

if CR_t<0.10, then the adjugement matrix is satisfied consistent. An example to use the fuzzy mathematical model of land evaluation

In order to explain how to use the fuzzy mathematical model of land evaluation, a simplified example about land evaluation in the suburban district of Dalian City of China is produced. The Dalian City is a large city in China, there are more than 1.5 millian population in it. The main aim of the land use of the suburban district is to surve for the city. In order to use land economicly, land evaluation become very importaint. Here gives the land evaluation for agriculture land.

--- Twenty two land evaluation units where identified based on the maps of land types and land uses in GIS (Table 2);

--- The relevant land characteristics are:

U₁: the depth of soil (cm)

 $U_2$ : the slope gradient (per cent)

Uz: soil texture grade

U_A: soil organic matter content (per cent)

U₅: PH

U₆: irrigation condition

U7: the size of theparcels (per cent)

Ug: soil erosion

Uq: flood hazard

U10: management level

U₁₁: production value (yuan)

The membership function for each land characteristic is of the form given in page 2 (a single membership function model) the same form of the membership function is used for all land characteristics.

The standard indices C_i for each land characteristic are:

 $\begin{array}{c} c_{1} = 90 \quad c_{2} = 5 \quad c_{3} = 1 \quad c_{4} = 30 \\ c_{5} = 6.5 \quad c_{6} = 80 \quad c_{7} = 2 \quad c_{8} = 1 \\ c_{9} = 1 \quad c_{10} = 4 \quad c_{11} = 800 \\ \end{array}$ The values for the crossover point indices  $a_{1}$  are:  $\begin{array}{c} a_{1} = 0.0002 \quad a_{2} = 0.0055 \quad a_{3} = 0.1013 \quad a_{4} = 0.31 \\ a_{5} = 0.62 \quad a_{6} = 0.0004 \quad a_{7} = 0.0047 \quad a_{8} = 0.065 \\ a_{9} = 0.25 \quad a_{10} = 0.132 \quad a_{11} = 0.0000007 \end{array}$ 

--- Use the AHP model to calculate the weights of every factors a. devide the issues of agriculture land evaluation into three levels:

L1	hemipelagic deposit meadow soil vegetable land
1.3	fluviouls low terrance vegetable land
T.A	fluviouls low terrance orchard land
<u>1</u> 4 Т Б	fluricule high towners anahord land
T G	fluviouls high terrance orchard land
т <b>л</b>	fluviouis nigh terrance rainied land
<u>л</u> (	ituviouis nigh terrance vegetable land
ΓS	quartzite brown earth terrance vegetable land
L9	quartzite brown earth terrance rainfed land
L10	quartzite brown earth terrance orchard land
L11	loessal brown earth terrance vegetable land
L12	loessal brown earth terrance orchard land
L13	loessal brown earth terrance rainfed land
L14	calcareous brown earth terrance vegetable land
L15	calcareous brown earth terrance rainfed land
L16	calcareous brown earth terrance orchard land
L17	quartzite rhogosol brown earth low hill orchard land
L18	quartzite rhogosol brown earth low hill forest land
L19	quartzite rhogosol brown earth low hill rainfed land
L20	quartzite rhogosol brown earth high hill orchard land
L21	quartzite rhogosol brown earth high hill forest land
L22	quartzite rhogosol brown earth high hill grass land

Table 3: The charicteristics of land evaluation units

Li L1	U1 80	U2 3	U3 2	U4 2.79	U5 7.5	U6 1 00	บ7 3	U8 0	U9 0	U10 5	U11 2486
L2	120	3	1	1.64	6.5	100	5	1	2	5	3532
L3	120	3	1	2.01	7.1	0	8	1	2	4	242
L4	120	3	3	1.32	6.5	100	5	1	2	4	76
L5	80	7	2	1.22	6.5	10	10	2	0	4	251
L6	80	7	2	1.02	6.5	0	10	2	0	4	69
L7	80	7	2	1.30	6.5	80	10	2	0	4	1800
L8	60	15	1	1.01	6.5	70	3	3	0	4	1650
L9	60	15	1	1.01	6.5	0	3	3	0	-3	62
L10	60	15	1	1.22	6.5	20	3	3	0	4	247
L11	120	10	3	1.13	6.8	0	3	3	3	4	1920
L12	120	10	3	1.40	6.8	25	3	3	3	4	250
L13	120	10	3	1.04	6.8	10	3	3	3	3	65
L14	80	10	3	1.01	7.5	80	3	3	0	4	1552
L15	80	10	3	1.00	7.5	0	3	3	0	3	60
L16	80	10	3	1.31	1.5	25	3	3	2	4	228
L17	60	15	4	0.98	6.5	0	6	4	0	2	160
L18	60	15	4	1.31	6.5	0	8	4	0	2	0
L19	60	15	4	1.21	6.5	0	10	5	0	2	25
L20	50	25	4	1.32	0.5	0	Ø	4	U	2	120
L21	30	25	4	1.85	0.5	0	2	2	0	2	0
L22	20	25	4	1.65	6.5	0	13	5	U	1	0



Map 2: The level structure of land evaluation for agriculture land use

b. making the adjugement matrixes:

Table 3: The adjugement matrixes and the calculated results (1). The adjugement matrix A-B:

Α	B1	B <b>2</b>	B3	W1
B1	1	2	6	0.6
B2	1/2	1	3	0.3
B3	1/6	1/3	1	0.1

CR=0.032<0.10

(2). The adjugement matrix B4-C:

B1	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10 C11	Wi
C1	1	1/5	1	6	8	1/8	1/3	1	1/2	1/7 1/4	0.041
C2	5	1	5	9	9	1/2	2	5	3	11 1	0.151
C3	1	1/5	1	6	8	1/8	1/3	1	1/2	1/7 1/4	0.041
C4	1/6	1/9	1/6	1	1	1/9	1/9	1/6	1/7	1/9 1/9	0.012
C5	1/8	1/9	1/8	1	1	1/9	1/9	1/6	1/7	1/9 1/9	0.011
C6	8	2	8	9	9	1	3	8	4	2 2	0.241
C7	3	1/2	3	9	9	1/3	1	3	2	1/2 1	0.100
C8	1	1/5	1	6	6	1/8	1/3	1	1/2	1/7 1/4	0.038
C9	2	1/3	2	7	7	1/4	1/2	2	1	1/3 1/2	0.064
C10	7	1	7	9	9	1/2	2	7	3	1 2	0.182
C11	4	1	4	9	9	1/2	1	4	2	1/2 1	0.119

CR=0.075<0.10

(3). The adjugement matrix B2-C

B2	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	Wi
C1	1	1	2	6	6	5	5	3	1	5	1	0.173
C2	1	1	2	6	6	5	5	3	1	5	1	0.173
C3	1/2	1/2	1	3	3	3	3	2	1/2	2	1/2	0.091
C4	1/6	1/6	1/3	1	1	1	1	1/2	1/6	1	1/6	0.031
C5	1/6	1/6	1/3	1	1	1	1	1/2	1/6	1	1/6	0.031
C6	1/5	1/5	1/3	1	1	1	1	1/2	1/5	1	1/5	0.032
C7	1/5	1/5	1/3	1	1	1	1	1/2	1/5	1	1/5	0.032
C8	1/3	1/3	1/2	2	2	2	2	1	1/3	2	1/3	0.059
C9	1	1	2	6	6	5	5	3	1	3	1	0.173
C10	1/5	1/5	1/2	1	1	1	1	1/2	1/5	1	1/5	0.033
C11	1	1	2	6	6	5	5	3	1	5	1	0.173

CR=0.042<0.10

C,

(4). The adjugement matrix B3-C:

	Carlo and a second second second				Constitution of the second second							and the second se
B3	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	Wi
C1	1	1/2	3	6	8	9	2	1	1	2	1	0.135
C2	2	1	6	9	9	9	4	2	2	4	2	0.237
C3	1/3	1/6	1	2	3	3	1/2	1/3	1/3	1/2	1/3	0.041
C4	1/6	1/9	1/2	1	2	2	1/3	1/6	1/6	1/3	1/6	0.025
C5	1/8	1/9	1/3	1/2	1	1	1/4	1/8	1/8	1/4	1/8	0.015
C6	1/9	1/9	1/3	1/2	1	1	1/4	1/9	1/9	1/4	1/9	0.016
C7	1/2	1/4	2	3	4	4	1	1/2	1/2	1	1/2	0.057
C8	1	1/2	3	6	8	9	2	1	1	2	1	0.135
C9	1	1/2	3	6	8	9	2	1	1	2	1	0.135
C10	1/2	1/4	2	3	4	4	1	1/2	1/2	1	1/2	0.069
C11	1	1/2	3	6	8	9	2	1	1	2	1	0.135
CR=0.097 < 0.10 c. Calculate the tatal weights of the factors being in the level C: Based on the equation $W_{ij} = \sum_{j=1}^{n} b_{i}c_{j}^{1}$ (j=1,2,m)												
1=1 we can ca;ci;ate the tatal weights of the factors in the level ( the results are:												
fact	or Ci	C	1 (	C <b>2</b>	C3	C4	C5	C6	C7	C8	C <b>9</b>	C10 (
the weig Ci	tatal ht of	0.0	090 0.1	0 166	.056	0.01	0.01' 9	7 0.15	0.07	75 0.05	0.104 54	0.1 0.126

--- Based on the fuzzy mathematical model of land evaluation including equation FMM1, FMM2 and FMM3, the results of land evaluation can be finished. (V1=0.95, V2=0.90, V3=0.80, V4=0.76, V5=0.66)

Table 4. The results of land evaluation including the results of a single factor, multiple factors and the comments.

land evalu- ation units	C1	C2	C3	C4	C5	C6	C7	C8	C9	C1 0	C11	M	з
L1 L2 L3 L4 L5 L6 L7 L8 L9 L10 L11 L12 L13 L14 L15 L16 L17 L18 L19 L20 L21 L22	.98 1 1 .98 .98 .98 .98 .98 .85 .85 1 1 .98 .98 .85 .85 .98 .98 .85 .85 .85 .85 .98 .98 .98 .98 .98 .98 .98 .98 .98 .98	1 1 98 98 98 65 65 65 88 88 88 88 88 88 88 88 65 65 .65 .65 .31 .31	•91 1 •91 •91 •91 •91 •91 1 1 •71 •71 •71 •71 •71 •71 •	•99 •93 •82 •80 •76 •81 •76 •76 •76 •76 •76 •76 •76 •81 •75 •81 •90 •87	•62 1 •82 1 1 1 1 1 1 95 •95 •62 •62 •62 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 28 1 28 1 28 41 28 45 34 1 28 45 28 28 28 28 28 28 28 28 28 28	.98 .96 .96 .97 .77 .77 .77 .99 .99 .99 .99 .99 .99	1 1 94 94 94 94 94 94 79 79 79 79 79 79 79 79 63 63 63 63 9 63 9	1 .80 .80 1 1 1 1 1 .50 .50 1 1 .80 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 .88 1 .88 1 .88 1 .88 .65 .65 .88 .65 .46	1 .87 .79 .87 .79 1 1 .78 .87 .79 1 .78 .87 .79 1 .78 .86 .83 0 .77 .81 0 0	•98 •97 •83 •93 •84 •82 •96 •90 •75 •80 •75 •80 •75 •94 •75 •94 •76 •81 •86 •67 •65 •59 •50	<b>S1</b> <b>S3</b> <b>S2</b> <b>S3</b> <b>S3</b> <b>S1</b> <b>S2</b> <b>S3</b> <b>S3</b> <b>S1</b> <b>S2</b> <b>S3</b> <b>S3</b> <b>S1</b> <b>S2</b> <b>S3</b> <b>S3</b> <b>S1</b> <b>S2</b> <b>S3</b> <b>S3</b> <b>S3</b> <b>S3</b> <b>S3</b> <b>S3</b> <b>S3</b> <b>S3</b>

From the comments Si, we can seen the land L1,L2 and L7 are the first-level lands for agriculture land use, the land units L4, L8 and L14 are the second-level lands, the land units L3, L5, L6, L10, L16 and L17 are the third-level lands, the land units L11, L12 and L15 are the forth-level lands, the land units L9, L13 and L19 are the fifth-level lands, the land units L18, L20, L21 and L22 are not suitale lands.

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