

A Singularity Free Algorithm for Spatial Orientation of Bundles

Dr.-Ing. Ludger Hinsken
Institut für Photogrammetrie und Kartographie
TU Braunschweig
Gaußstr. 22
D-3300 Braunschweig
Federal Republic of Germany
Commission V

Abstract

A nonlinear algorithm is presented, which in general enables the spatial orientation of ray bundles without predefining special initial values for the chosen parameters. The major aspect of the algorithm is the parameterization of the rotation parameters. These parameters are proved to be free of any singularities. Thus no constraints concerning configuration of bundles in space exist. The algorithm converges rapidly for the rotation parameters involved in the rotation matrix are only quadratically. The algorithm is well suited to obtain initial values for bundle adjustment in close range applications. Further applications are the relative orientation of two photographs, the absolute orientation of models, the resection of photographs or directly within the bundle adjustment for the exterior orientation parameters. The contents of the presentation is subdivided into two parts. The first part deals with the development of the theoretical fundamentals of the algorithm and in the second part examples are demonstrated.

Introduction

In 1970 Alan Pope introduced a new special estimation technique for rotation parameters (Pope, A. 1970). For the derivation of the formulas he used quaternion algebra. In this paper an alternative derivation of the formulas is presented in which the quaternion algebra is omitted. Furtheron Popes' idea of parameter substitution to gain advantages in the estimation process, is transferred onto the shifting parameters. So it is possible to achieve estimation formulas for the resection of a photograph which need no special initial values for any configuration.

The Mathematical Model

The fundamental equation of analytical photogrammetry is the collinearity equation, which can be expressed in matrix terms as follows:

$$\begin{bmatrix} x_i \\ y_i \\ c \end{bmatrix} = s_i \mathbf{R} \begin{bmatrix} X_i - X_t \\ Y_i - Y_t \\ Z_i - Z_t \end{bmatrix} \quad (1)$$

x_i, y_i : observed photo-coordinates
 c : given camera constant
 s_i : unknown scaling factor
 \mathbf{R} : unknown 3x3 orthogonal rotation matrix
 X_i, Y_i, Z_i : unknown object-coordinates
 X_t, Y_t, Z_t : unknown coordinates of the perspective centre, given in object space

Any distortion and the coordinates of the principle point are neglected here, because they have no influence on items to be demonstrated. In equation (1) no special parameterization for the rotation matrix \mathbf{R} is assumed. Any kind of orthogonal 3x3 matrix is allowed. But the lateron chosen parameterization will be of special interest in this paper.

Equation (1) is a nonlinear function, which describes the transformation from points in object space into the image space. The usual way to find the unknown parameters of the functional model (1), is to use a least squares adjustment. Going that way, one has to linearize (1) with respect to the parameters of the model. To come to the derivatives the following substitutions are made.

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \mathbf{R} \begin{bmatrix} X - X_t \\ Y - Y_t \\ Z - Z_t \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} x \\ y \\ c \end{bmatrix} = s \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (3)$$

Dividing each side of equation (3) through the third vector element one obtains the classical observation equation for the bundle adjustment.

$$\begin{bmatrix} x \\ y \end{bmatrix} = c \begin{bmatrix} p/r \\ q/r \end{bmatrix} \quad (4)$$

The division in (3) eliminates the point-dependent scaling factors. The linearized observation equation can generally be obtained by the following parameter substitution.

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = \mathbf{A} \begin{bmatrix} dp \\ dq \\ dr \end{bmatrix} = \mathbf{A} \left\{ d\mathbf{R} \begin{bmatrix} X - X_t \\ Y - Y_t \\ Z - Z_t \end{bmatrix} + \mathbf{R} \begin{bmatrix} dX - dX_t \\ dY - dY_t \\ dZ - dZ_t \end{bmatrix} \right\} \quad (5)$$

with

$$\mathbf{A} = \begin{bmatrix} \delta x/\delta p & \delta x/\delta q & \delta x/\delta r \\ \delta y/\delta p & \delta y/\delta q & \delta y/\delta r \end{bmatrix} = c/r \begin{bmatrix} 1 & 0 & -p/r \\ 0 & 1 & -q/r \end{bmatrix} \quad (6)$$

Up to this point there is no difference to any other consideration of the linearized equation (5). The difference here is characterized by a special parameterization of the rotation parameters.

For orthogonal matrices the following equation holds true

$$\mathbf{R}\mathbf{R}^T = \mathbf{I} \quad (7)$$

in which \mathbf{I} denotes the identity matrix and \mathbf{R}^T the transpose of \mathbf{R} . If equation (7) is linearized with respect to the rotation parameters, one obtains the following result.

$$dRR^T + RdR^T = 0 \quad (8)$$

$$dRR^T = -(dRR^T)^T \quad (9)$$

Equation (9) expresses that the matrix product dRR^T gives a skew symmetric matrix. In the following this skew symmetric matrix will be denoted S_w .

$$dRR^T = S_w \quad (10)$$

S_w can be composed by the three independent parameters w_1, w_2, w_3 .

$$S_w = \begin{bmatrix} 0 & w_3 & -w_2 \\ -w_3 & 0 & w_1 \\ w_2 & -w_1 & 0 \end{bmatrix} \quad (11)$$

The three independent elements are involved in the vector w .

$$w = [w_1, w_2, w_3]^T \quad (12)$$

From equation (10) it follows that any differential rotation matrix dR can be expressed as the product of the skew symmetric matrix S_w and the orthogonal rotation matrix R .

$$dR = S_w R \quad (13)$$

It is very important to mention here, that the relation in (13) is independent from the parameters chosen to build the rotation matrix R . The relationship of equation (13) is now used to compose the final form of the linearized equation. For this (13) is substituted into (5).

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = AS_w R \begin{bmatrix} X - X_t \\ Y - Y_t \\ Z - Z_t \end{bmatrix} + AR \begin{bmatrix} dX - dX_t \\ dY - dY_t \\ dZ - dZ_t \end{bmatrix} \quad (14)$$

Using (2) in (14) leads to

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = AS_w \begin{bmatrix} p \\ q \\ r \end{bmatrix} + AR \begin{bmatrix} dX - dX_t \\ dY - dY_t \\ dZ - dZ_t \end{bmatrix} \quad (15)$$

The vector v and the matrix S_v will be introduced

$$v = [p, q, r]^T \quad (16)$$

$$S_v = \begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} \quad (17)$$

The following relationship

$$S_w v = -S_v w \quad (18)$$

can easily be proved by the use of the outer vector product. One obtains the final result in the form

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = L \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + AR \begin{bmatrix} dX - dX_t \\ dY - dY_t \\ dZ - dZ_t \end{bmatrix} \quad (19)$$

with

$$L = -AS_v = c/r \begin{bmatrix} pq/r & -(r^2+p^2)/r & q \\ (r^2+q^2)/r & -pq/r & -p \end{bmatrix} \quad (20)$$

In the linearized observation equation (19) a formulation is found which is very well suited for numerical computations. Note that the parameters which describe the rotation matrix are yet not given. In the estimation process only the three parameters w_1 , w_2 , w_3 are estimated. Now it will be shown how they correspond to the rotation matrix. By the way it will also be proved, that through using this kind of estimation in combination with a specific parameterization for R no singularities of the rotation parameters exist.

Setting up the Rotation Matrix

There exist several ways to parameterize a 3x3 orthogonal matrix (Schut, G.H. 1958/59; Thompson, E.H. 1959). For the application outlined here a parameterization with four algebraic parameters d , a , b and c is chosen.

$$d^2 + a^2 + b^2 + c^2 = 1 \quad (21)$$

An easy way to produce an orthogonal matrix is to introduce these four parameters into two special 4x4 matrices.

$$P = \begin{bmatrix} d & a & b & c \\ -a & d & c & -b \\ -b & -c & d & a \\ -c & b & -a & d \end{bmatrix} ; \quad Q = \begin{bmatrix} d & -a & -b & -c \\ a & d & c & -b \\ b & -c & d & a \\ c & b & -a & d \end{bmatrix} \quad (22)$$

It can be shown that the matrices P and Q are orthogonal matrices.

$$P^T P = Q^T Q = I \quad (23)$$

Building the product of the matrices P and Q one obtains another special orthogonal matrix T .

$$T = PQ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & R & & \\ 0 & & & \end{bmatrix} \quad (24)$$

The submatrix R is also an orthogonal matrix which is of order three. The orthogonality can be proved in the following way.

$$\mathbf{T}^T \mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & \mathbf{R}^T \mathbf{R} & & \\ 0 & & & \end{bmatrix} = \mathbf{Q}^T \mathbf{P}^T \mathbf{P} \mathbf{Q} = \mathbf{I} \quad (25)$$

From this it follows

$$\mathbf{R}^T \mathbf{R} = \mathbf{I} \quad (26)$$

in which \mathbf{R} is given by

$$\mathbf{R} = \begin{bmatrix} d^2 + a^2 - b^2 - c^2 & 2(ab + cd) & 2(ac - bd) \\ 2(ab - cd) & d^2 - a^2 + b^2 - c^2 & 2(bc + ad) \\ 2(ac + bd) & 2(bc - ad) & d^2 - a^2 - b^2 + c^2 \end{bmatrix} \quad (27)$$

This form of the rotation matrix (27) is not a new one, it was already given in (Schut, G.H. 1958/59; Pope, A. 1970). Both authors agree, that the form (27) is the simplest form of any 3x3 orthogonal rotation matrix.

Fundamental Relationship between the Estimation Parameters w and the Rotation Parameters d, a, b, c

With equation (27) the rotation parameters d, a, b, c are introduced. The relationship between these parameters and the parameters w_1, w_2, w_3 of (12) will be formulated. The starting-point of this consideration is equation (13)

$$d\mathbf{R} = \mathbf{S}_w \mathbf{R}$$

in which now \mathbf{R} is a function of the rotation parameters d, a, b, c .

$$\mathbf{R} = \mathbf{R}(d, a, b, c) \quad (28)$$

Next one has to form the partial derivatives of the rotation matrix with respect to the rotation parameters

$$d\mathbf{R} = \delta\mathbf{R}/\delta d \Delta d + \delta\mathbf{R}/\delta a \Delta a + \delta\mathbf{R}/\delta b \Delta b + \delta\mathbf{R}/\delta c \Delta c \quad (29)$$

and because of relation (7) $\mathbf{R}^T \mathbf{R} = \mathbf{I}$ it follows

$$d\mathbf{R} = \delta\mathbf{R}/\delta d \mathbf{R}^T \mathbf{R} \Delta d + \delta\mathbf{R}/\delta a \mathbf{R}^T \mathbf{R} \Delta a + \delta\mathbf{R}/\delta b \mathbf{R}^T \mathbf{R} \Delta b + \delta\mathbf{R}/\delta c \mathbf{R}^T \mathbf{R} \Delta c \quad (30)$$

$$d\mathbf{R} = (\delta\mathbf{R}/\delta d \mathbf{R}^T \Delta d + \delta\mathbf{R}/\delta a \mathbf{R}^T \Delta a + \delta\mathbf{R}/\delta b \mathbf{R}^T \Delta b + \delta\mathbf{R}/\delta c \mathbf{R}^T \Delta c) \mathbf{R} \quad (31)$$

With the use of (21)

$$d^2 + a^2 + b^2 + c^2 = 1$$

it follows

$$d\Delta d + a\Delta a + b\Delta b + c\Delta c = 0 \quad (32)$$

$$\Delta d = -(a\Delta a + b\Delta b + c\Delta c)/d \quad (33)$$

A comparison of the coefficients between the elements of the skew symmetric matrix \mathbf{S}_w of (11) with the related ones of (31) leads to the following relationship.

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = 2/d \begin{bmatrix} d^2+a^2 & ab+dc & ac-db \\ ab-dc & d^2+b^2 & bc+da \\ ac+db & bc-da & d^2+c^2 \end{bmatrix} \begin{bmatrix} \Delta a \\ \Delta b \\ \Delta c \end{bmatrix} \quad (34)$$

If one denotes the coefficient matrix in (34) of Δa , Δb , Δc with C , this coefficient matrix must be inverted to achieve the final relationship.

$$\begin{bmatrix} \Delta a \\ \Delta b \\ \Delta c \end{bmatrix} = C^{-1}w \quad (35)$$

with

$$C^{-1} = 1/2 \begin{bmatrix} d & -c & b \\ c & d & -a \\ -b & a & d \end{bmatrix} \quad (36)$$

A proof for the existence of the inverse of C can be found in (Hinsken, L. 1987).

The simplicity of the transformation matrix C^{-1} in (35) is the reason why this parameterization does not possess any singularities. This is because no element of the matrix C^{-1} can become an undefined expression. The adjustment procedure works in the way, that one needs initial values for d , a , b and c to build up the matrix R . With this initial values a set of w elements is estimated and afterwards the initial values are updated by the formulas (33) and (35). This procedure is repeated until the w vector becomes the zero vector.

Another big advantage of this estimation technique with parameters w is the saving of numerical computations. If one would estimate the rotation parameters d , a , b and c directly, one would have to substitute equation (35) into (19) with following result

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = LC^{-1} \begin{bmatrix} a \\ b \\ c \end{bmatrix} + AR \begin{bmatrix} dX - dX_t \\ dY - dY_t \\ dZ - dZ_t \end{bmatrix} \quad (37)$$

If one compares (19) with (37) it comes out very clearly, that the numerical computation of the coefficients in (37) requires more work than those of (19). One has to take into account, that equation (37) is point-dependent, which means it has to be computed for each point of a photograph, while transformation (35) needs to be computed only once for each photograph within the iteration process. This item does not depend on the chosen rotation parameters.

It is also possible to use this estimation technique in combination with classical rotation parameters ω , ϕ , κ . In the case of rotation order ω , ϕ , κ , the transformation matrix C^{-1} looks like as shown below

$$\begin{bmatrix} d\kappa \\ d\phi \\ d\omega \end{bmatrix} = \begin{bmatrix} -\cos\kappa\tan\phi & \sin\kappa\tan\phi & 1 \\ \sin\kappa & \cos\kappa & 0 \\ \cos\kappa/\cos\phi & -\sin\kappa/\cos\phi & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \quad (38)$$

From this transformation it can easily be seen, that in case of $\cos\phi = 0$ the transformation will be undefined.

Examples

Two examples shall demonstrate how the idea of A. Pope to estimate rotation parameters can be used in other photogrammetric applications. For this the relative orientation of two photographs and the resection of a single photograph are chosen.

Relative Orientation

As it was outlined by (Schut, G.H. 1957/58) each mathematical formulation of the relative orientation can be reduced to the coplanarity constraint. For this the three vectors which build the epipolar plane are put together into a 3x3 determinant.

$$\begin{vmatrix} b_x & & & & & \\ b_y & \mathbf{R} & \begin{bmatrix} x' \\ y' \\ c' \end{bmatrix} & \mathbf{D} & \begin{bmatrix} x'' \\ y'' \\ c'' \end{bmatrix} & \\ b_z & & & & & \end{vmatrix} = 0 \quad (39)$$

- b_x, b_y, b_z : three components of the base vector
 \mathbf{R} : 3x3 orthogonal rotation matrix, which transforms the three axes of the right photo-coordinatesystem parallel to the model system
 \mathbf{D} : 3x3 orthogonal rotation matrix, which transforms the three axes of the left photo-coordinatesystem parallel to the model system
 x', y', c' : photo-coordinates and camera constant of right station
 x'', y'', c'' : photo-coordinates and camera constant of left station

The model-coordinatesystem is defined through $b_y=b_z=0$ which means that the x-axes coincides with the baseline. The scale is chosen arbitrarily with $b_x=1$.

$$\begin{vmatrix} 1 & & & & & \\ 0 & \mathbf{R} & \begin{bmatrix} x' \\ y' \\ c' \end{bmatrix} & \mathbf{D} & \begin{bmatrix} x'' \\ y'' \\ c'' \end{bmatrix} & \\ 0 & & & & & \end{vmatrix} = 0 \quad (40)$$

In equation (40) the six rotation parameters of the two rotation matrices \mathbf{R} and \mathbf{D} are the unknown parameters. These parameters will be estimated in the way shown in the first part of the paper. The first step to do is to linearize equation (40) with respect to the rotation parameters, this leads to:

$$\begin{vmatrix} 1 & & & & & \\ 0 & d\mathbf{R} & \begin{bmatrix} x' \\ y' \\ c' \end{bmatrix} & d\mathbf{D} & \begin{bmatrix} x'' \\ y'' \\ c'' \end{bmatrix} & \\ 0 & & & & & \end{vmatrix} = 0 \quad (41)$$

In (41) the differential rotation matrices are substituted by the relation (13).

$$\begin{vmatrix} 1 & & & & & \\ 0 & \mathbf{S}_w' \mathbf{R} & \begin{bmatrix} x' \\ y' \\ c' \end{bmatrix} & \mathbf{S}_w'' \mathbf{D} & \begin{bmatrix} x'' \\ y'' \\ c'' \end{bmatrix} & \\ 0 & & & & & \end{vmatrix} = 0 \quad (42)$$

$$\begin{vmatrix} 1 & & & \\ 0 & \mathbf{S}_w' & \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} & \mathbf{S}_w'' & \begin{bmatrix} X'' \\ Y'' \\ Z'' \end{bmatrix} \\ 0 & & & & \end{vmatrix} = 0 \quad (43)$$

with

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \mathbf{R} \begin{bmatrix} x' \\ y' \\ c \end{bmatrix} ; \quad \begin{bmatrix} X'' \\ Y'' \\ Z'' \end{bmatrix} = \mathbf{D} \begin{bmatrix} x'' \\ y'' \\ c'' \end{bmatrix}$$

With the use of (17) the parameter exchange is done

$$\begin{vmatrix} 1 & & & \\ 0 & \mathbf{S}_v' & \begin{bmatrix} w_1' \\ w_2' \\ w_3' \end{bmatrix} & \mathbf{S}_v'' & \begin{bmatrix} w_1'' \\ w_2'' \\ w_3'' \end{bmatrix} \\ 0 & & & & \end{vmatrix} = 0 \quad (44)$$

In (44) both skew symmetric matrices have the form

$$\mathbf{S}_v' = \begin{bmatrix} 0 & Z' & -Y' \\ -Z' & 0 & X' \\ Y' & -X' & 0 \end{bmatrix} ; \quad \mathbf{S}_v'' = \begin{bmatrix} 0 & Z'' & -Y'' \\ -Z'' & 0 & X'' \\ Y'' & -X'' & 0 \end{bmatrix} \quad (45)$$

If now the determinant (44) is computed and the coefficients are ordered as usual one finally obtains the linearized coefficients of the parameters w_1', \dots, w_3'' , which are put together in the coefficient matrix \mathbf{L} .

$$\mathbf{L} = [Z''Z' + Y''Y', -X''Y', -X''Z', -(Z''Z' + Y''Y'), Y''X', Z''X'] \quad (46)$$

As can be seen easily there is a linear dependency between the first and fourth element of \mathbf{L} , which was expected, because the problem of relative orientation has only five degrees of freedom. After the elimination of the linear dependency one reaches a coefficient matrix, which is well suited for numerical computation.

$$\mathbf{L} = [Z''Z' + Y''Y', -X''Y', -X''Z', Y''X', Z''X'] \quad (47)$$

If in this context the form (27) is chosen for the parameterization of the rotation matrices, one obtains an algorithm which is free of singularities, because transformation (35) would then be used again. Due to the fact, that the rotation parameters d, a, b , and c are involved in (27) only quadratically the estimation process converges very rapidly, even with very poor initial values for d, a, b, c . In practice the identity matrix is usually good enough as starting point for the rotation matrix, even in close range applications with highly convergent exposure station configurations.

Resection of a Single Photograph

As the mathematical formulation of the resection of a photograph, the collinearity equation (1) is usually used. Here a modified version of this equation will be introduced, which leads to some advantages in comparison with the form given in (1)

$$\begin{bmatrix} x_i \\ y_i \\ c \end{bmatrix} = s_i \mathbf{R} \begin{bmatrix} X_i - X_t \\ Y_i - Y_t \\ Z_i - Z_t \end{bmatrix}$$

Equation (1) can be splitted up in the following way

$$1/s \begin{bmatrix} x \\ y \\ c \end{bmatrix} = \mathbf{R} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \mathbf{R} \begin{bmatrix} X_t \\ Y_t \\ Z_t \end{bmatrix} \quad (48)$$

$$1/s \begin{bmatrix} x \\ y \\ c \end{bmatrix} = \mathbf{R} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} \quad (49)$$

In equation (48) the vector $[X_t, Y_t, Z_t]^T$ represents the coordinates of the perspective centre given in object space. In equation (49) the vector $[x_t, y_t, z_t]^T$ represents the related vector expressed in the image space. The unknown parameters in (49) are the three rotation parameters in \mathbf{R} and the three shifting parameters x_t, y_t, z_t . The reason for the form (49) will become clear later, when talking about initial values for the six parameters. First one has to form the partial derivatives of the image-coordinates with respect to the unknown parameters.

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \mathbf{R} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} \quad (50)$$

With this substitution the observation equation can be rewritten

$$\begin{bmatrix} x \\ y \end{bmatrix} = c \begin{bmatrix} p/r \\ q/r \end{bmatrix} \quad (51)$$

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = \mathbf{A} \begin{bmatrix} dp \\ dq \\ dr \end{bmatrix} = \mathbf{A} \left\{ d\mathbf{R} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \begin{bmatrix} dx_t \\ dy_t \\ dz_t \end{bmatrix} \right\} \quad (52)$$

with

$$\mathbf{A} = \begin{bmatrix} \delta x / \delta p & \delta x / \delta q & \delta x / \delta r \\ \delta y / \delta p & \delta y / \delta q & \delta y / \delta r \end{bmatrix} = c/r \begin{bmatrix} 1 & 0 & -p/r \\ 0 & 1 & -q/r \end{bmatrix} \quad (53)$$

With the relationship (13), equation (52) becomes the form

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = \mathbf{A} \mathbf{S}_w \mathbf{R} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \mathbf{A} \begin{bmatrix} dx_t \\ dy_t \\ dz_t \end{bmatrix} \quad (54)$$

Using equation (50), this leads to

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = AS_w \begin{bmatrix} p + x_t \\ q + y_t \\ r + z_t \end{bmatrix} - A \begin{bmatrix} dx_t \\ dy_t \\ dz_t \end{bmatrix} \quad (55)$$

Introducing the vector v , with

$$v = [p+x_t, q+y_t, r+z_t]^T \quad (56)$$

gives

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = -AS_v \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} - A \begin{bmatrix} dx_t \\ dy_t \\ dz_t \end{bmatrix} \quad (57)$$

In (57) S_v has the following form

$$S_v = \begin{bmatrix} 0 & r+z_t & -(q+y_t) \\ -(r+z_t) & 0 & p+x_t \\ q+y_t & -(p+x_t) & 0 \end{bmatrix} \quad (58)$$

The linearized observation equation for the resection of a single photograph is finally given by the following equation

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = L \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} - A \begin{bmatrix} dx_t \\ dy_t \\ dz_t \end{bmatrix} \quad (59)$$

with

$$L = -AS_v$$

$$L = -c/r \begin{bmatrix} -(q+y_t)p/r & r+z_t+(p+x_t)p/r & -(q+y_t) \\ -(r+z_t)-(q+y_t)q/r & (p+x_t)q/r & p+x_t \end{bmatrix} \quad (60)$$

If the form (27) is used for the parameterization of the rotation matrix R , then transformation (35) must also be used during the estimation process, which leads to a singularity free algorithm here too.

Last but not least it can be shown that the parameterization of the shifting vector in (49) has some advantages. A temporarily shift of the origin of the object-coordinatesystem to point j brings equation (49) into the following form

$$1/s_j \begin{bmatrix} x \\ y \\ c \end{bmatrix}_j = R \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_j - \begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} \quad (61)$$

From equation (61) it becomes clear, that one can obtain very good initial values for the shifting vector, because the image coordinates and the camera constant are known. Also one needs to have a good approximation of the scaling factor s_j . This may be obtained by a mean value of all scaling factors, or a priori known from the distance between exposure station and object. With this initial values one can start the iteration process in which it is advisable to fix the shifting parameters during the first iterations. The identity matrix is usually a good approximation for the rotation matrix, from where the iteration process can be started. The algorithm behaves very robust in the case of very poor approximation values, because of the form of the coefficient matrices L and A in (59). The most important advantages of the algorithm are - it is free of singularities - it has a large ratio of convergence - it converges rapidly. The algorithm is also very fast, because the numerical effort is not very high. The computation of L and A requires only a few multiplications.

Conclusions

The algorithms presented here are realized in computer programs. The application of the new developed software using this estimation technique ranges from relative orientation over absolute orientation and single photo orientation to the new Combined Adjustment Program named CAP. All programs run on Personal Computers effectively. The implemented software is mainly used for close range applications.

Literature

Hinsken, L. Algorithmen zur Beschaffung von Näherungswerten für die Orientierung von beliebig im Raum angeordneten Strahlenbündeln, Dissertation 55 Pages Deutsche Geodätische Kommission Reihe C Nr.333 München, **1987**

Pope, A. An Advantageous, Alternative Parametrization of Rotations for Analytical Photogrammetry, Symposium on Computational Photogrammetry of the American Society of Photogrammetry, Alexandria, Virginia Jan. 7-9, **1970**

Schut, G.H. An analysis of methods and results in analytical aerial triangulation, Photogrammetria, pp.16-32, **1957/58**

Schut, G.H. Construction of Orthogonal Matrices and their Application in Analytical Photogrammetry, Photogrammetria XV No. 4 p.159, **1958/59**

Thompson, E.H. A method for the construction of orthogonal matrices, Photogrammetric Record Vol. III No.13 p.55 Apr. **1959**