NUMERICAL PHOTOGRAMMETRY FOR GLOBAL SURVEYING OF THE LEANING TOWER OF PISA

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ABSTRACT.

This paper relates on the engineeristic photogrammetric surveying of the famous Tower.
The aim is to determine the shape and the spatial position of successive orders of the Tower by the numeric restitution of a very large number of equally spaced characteristic points. The resulting data base consists of 8800 points.
The diameters of the architectural circular elements,which limit the successive orders, the diameters of the cylinders which constitute the nucleus of the Tower and their orientation have been obtained from the computation of the above data. The data, together with direct measurements of the widths of the walls and other structural elements, may be usefull in the calculation of the stability of the tower.

1. INTRODUCTION.

The Torre Campanaria of Pisa Cathedral,which shall hereafter be referred to as Tower, which is renowned for its artistic beauty has always attracted special attention from scholars because of its exceptional slope and for the static implications which follow from this.
For a better understanding of this paper a schematic cross-section through the Tower, including the terminology that will be used, is shown in fig. 1.
Construction of the Tower began in 1173 and the architect was Bonanno Pisano. This is the only date which is certain, both because of various chronicles of that period, and because of an inscription engraved in the base of the Tower.
The construction times,instead,have various uncertainties both in the dates of commencement and completion of the various stages. The following is the summary which was put forward in the paper of the Committee of the Ministry of Public Works in 1971(LL.PP.).

1174 - 1178 work went on till half of the 4th order , when it was interrupted
1178 - 1272 a period of suspension followed which lasted almost a century
1272 - 1278 Giovanni di Simone restarted construction and, under his guide, work progressed untill the 6th repeated element of the 8th order
1278 - 1360 a further period of suspension,lasting almost 90 years,followed
1360 - 1370 Tommaso d'Andrea restarted construction and ended the work.

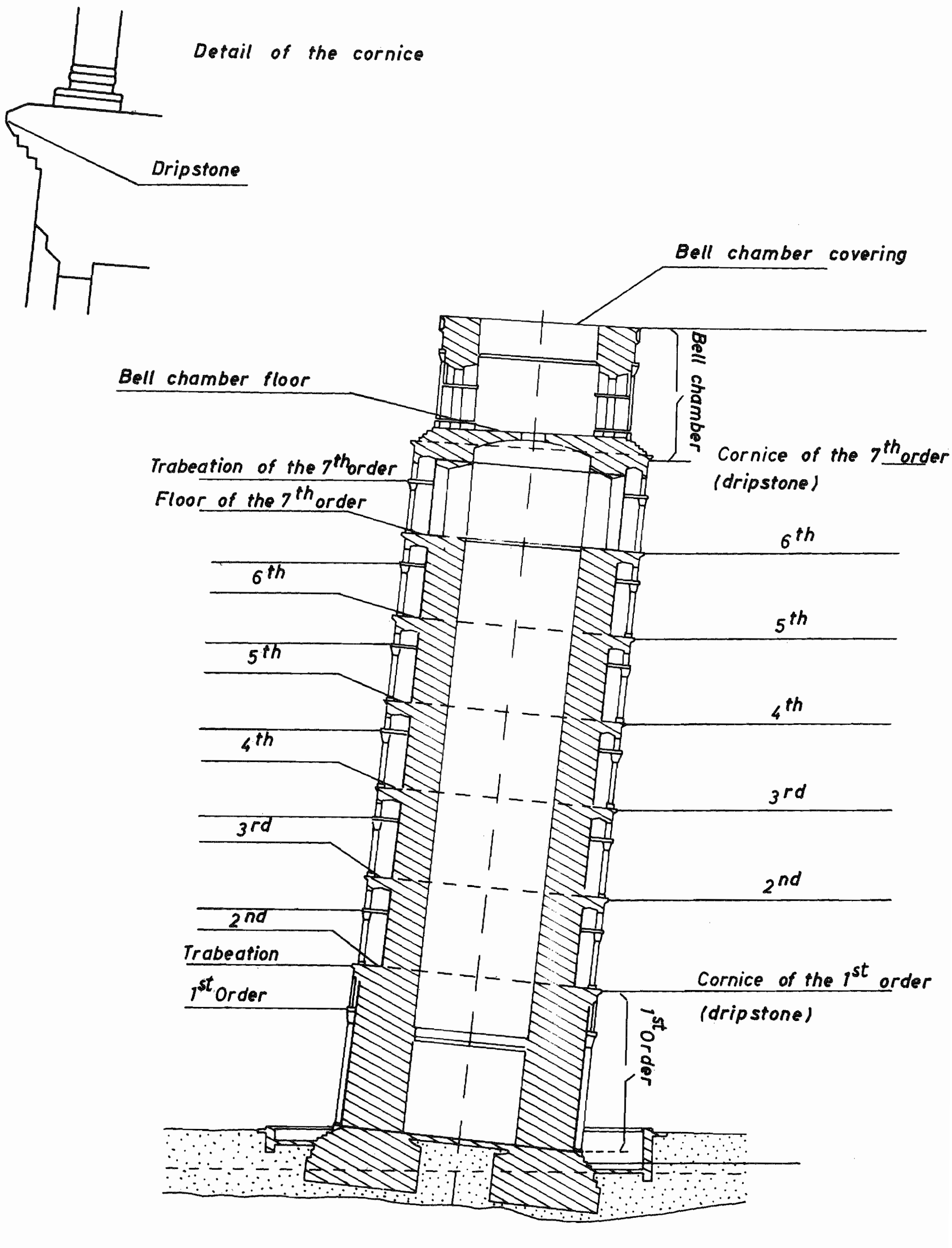


Fig. 1 - Schematic Cross-Section of the Tower and Terminology

The Institute of Geodesy, Topography and Photogrammetry of the Faculty of Engineering of the University of Pisa has annualy checked certain targets to monitor annual changes in slope since 1508.
These controls were begun by Prof. Cicconetti and have been repeated to the present day in the same way.
A systematic survey of the entire Tower was done in 1908 on the occasion of the "Study on the Conditions of the Tower," which was given by the Ministry for Public Education to a special Committee.
Another complete survey has been made as part of a static verification as a result of the studies undergone by the "Committee for the Consolidation of the Tower" nominated by the Ministry of Public Works in 1965 and chaired by Prof, Piero Locatelli.
These "complete" surveys have been done by direct methods; the possibility of a photogrammetric survey was therefore particularly interesting; because of the abundance of qualitative and quantitative data that this method produces.
The conformation of the Tower and its height require, however, the use of high precision instrumentation, with a large format camera . The Agip Photogrammetry Group , which owns and operate with this instrumentation, decided to partecipate with its Industrial Photogrammetry Group in the survey operations and so started a cooperation between the University and Agip S.p.A.

2. PHOTOGRAMMETRIC FIELD SURVEY

2.1 Instrumentation.

The instrumentation, made available by Agip, was;

a) two large format wide-angle universal UMK 10/1318 JENOPTIK--JENA metric cameras mounted on orientation system with a horizontal circle and a device to tilt the photogrammetric axis by 15° (16.66 gon), 30° (33.33 gon), 45° (50.00 gon) to a precision of 1'. The cameras have also been modified to allow takings at an inclination of 6° gon.
The cameras, in such a configuration, perform comparably to phototheodolite. Before the commencement of the work, the two cameras were checked and calibrated at the JENOPTIK/JENA Zfactory (DDR).

b) a synchronization device which allows simultaneous takings; such a device, which is indispensable for the the taking of moving objects or of still objects from unstable platforms, avoids, in our case, errors due to pseudoscopy caused by the movement of the shadows when the two stations of a stereotake are made in subsequent times.

Fairly flat plates (0,01 mm/cm), called "Selected flat glass", were used in the takings. The plates were made of optically flat crystal which was reburnt before being covered with a sensitive emulsion. Because of the characteristic shape and colour of the Tower and of the seasonal lighting, each taking was repeated using two different orthochromatic emulsions, (of sensitivities 3° and 12° DIN respectively).

2.2 Project planning

Given the particular and even exceptional characteristics of the Tower, both dimensionally and architecturally, we have attempted to maintain simultaneously two necessary conditions; that is the highest precision possible for the kind of engineeristic survey which had been decided on, using an operational technique which would allow to take photograms from the ground.
The camera was arranged to have the largest field in the vertical plane. The scheme, used for the taking, is shown in figs. 2 and 3. The Tower was taken from four principal bases; those were EAST (position 2), WEST (position 5),SOUTH (position 6) and NORTH (position 3) and from two control bases which were NORTH-EAST (position 1) and SOUTH-WEST (position 4)
The control bases were considered advisable because the principal takes give 4 models with an overlap inferior to 10%. The takes were normal , with the axes inclined by + 6 gon. to the horizontal.
The distance for the taking was to be less than 50 m (due to the surroundings, that is the presence of the Cathedral on one side and the Capuchiness Convent on the other) and the bases were 18 m long. This value was also a result of the surrounding, because it was wished for the principal bases to be symmetric with respect to the center of the tower and at the ground level. The Tower was taken with the axes at + 6 gon, up to an average height of 41 m from the ground level (45 m max,, 37 m min), that is till the seventh order base.
The plotter which Agip had at the time of the survey did not allow inclinations greater that 7 gon and it is advisable to remain below this value.
It was possible to take the entire Tower by using an inlination of 16.66 gon. However plotting of these takings requires analytical stereoplotters and appropriate software.
The standard deviation for a plotted point can, to a first approximation, be given by (fig. 4):

$$m_y = \pm Dy^2 * 0.005/(F.Bx)$$

$$m_x = m_z = \pm Dy * 0.005/F$$

where Dy is the distance from the point to the base, F is the principal (nominal) distance of the objective used in the taking and 0.005 is the intrinsic sensitivity of the high precision plotter which will be used.
In this case the standard deviations are:

$m_y = \pm$ 6.94 mm Dy = 50 m

$m_y = \pm$ 4.78 mm Dy = 41.5 m

$m_x = m_z = \pm$ 2.5 mm Dy = 50 m

$m_x = m_z = \pm$ 2.1 mm Dy = 41.5 m

These values ,unequal in x(z)and y ,can be considered acceptable.

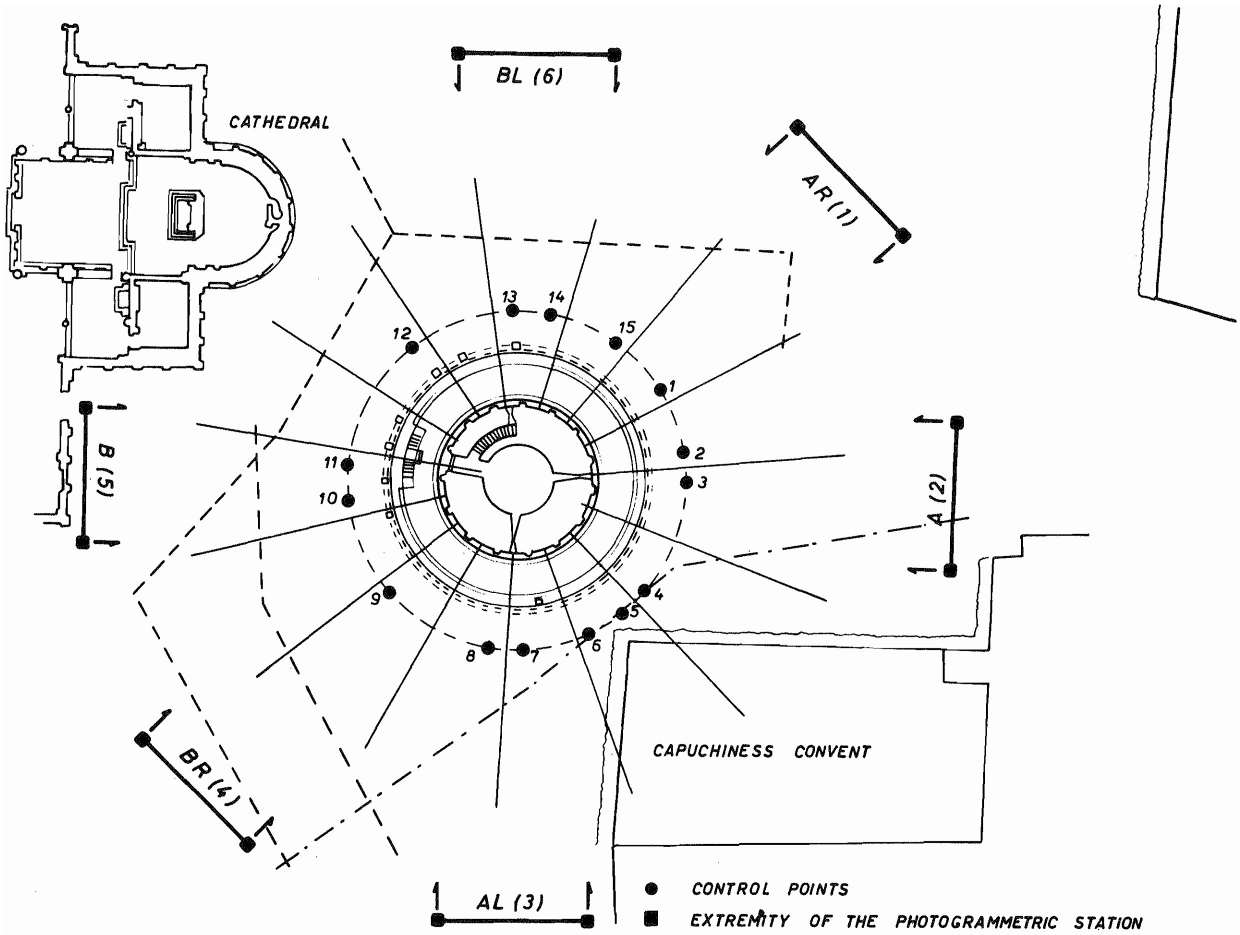


Fig. 2 - Planimetric Scheme of the Takes

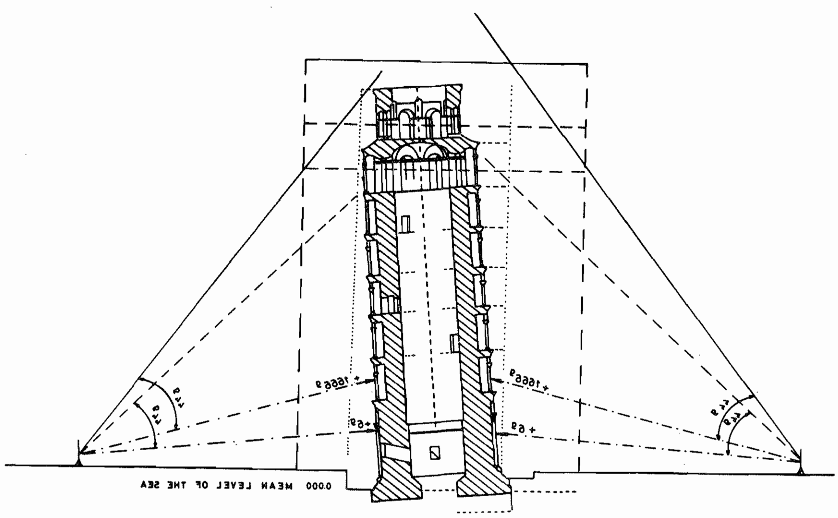


Fig. 3 - Altimetric Scheme of the Takes

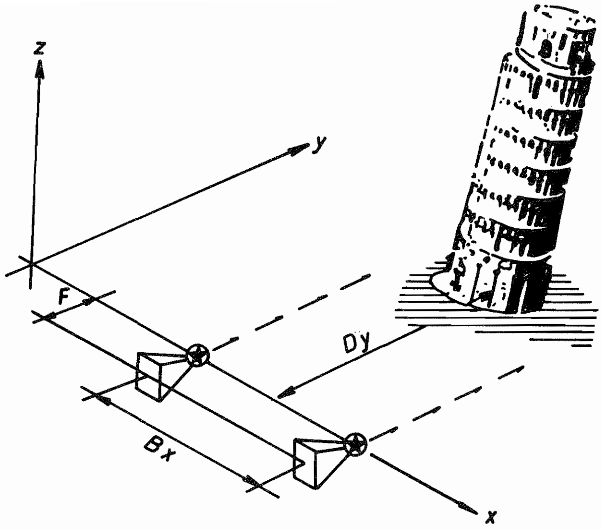


Fig. 4 - Frame of Reference

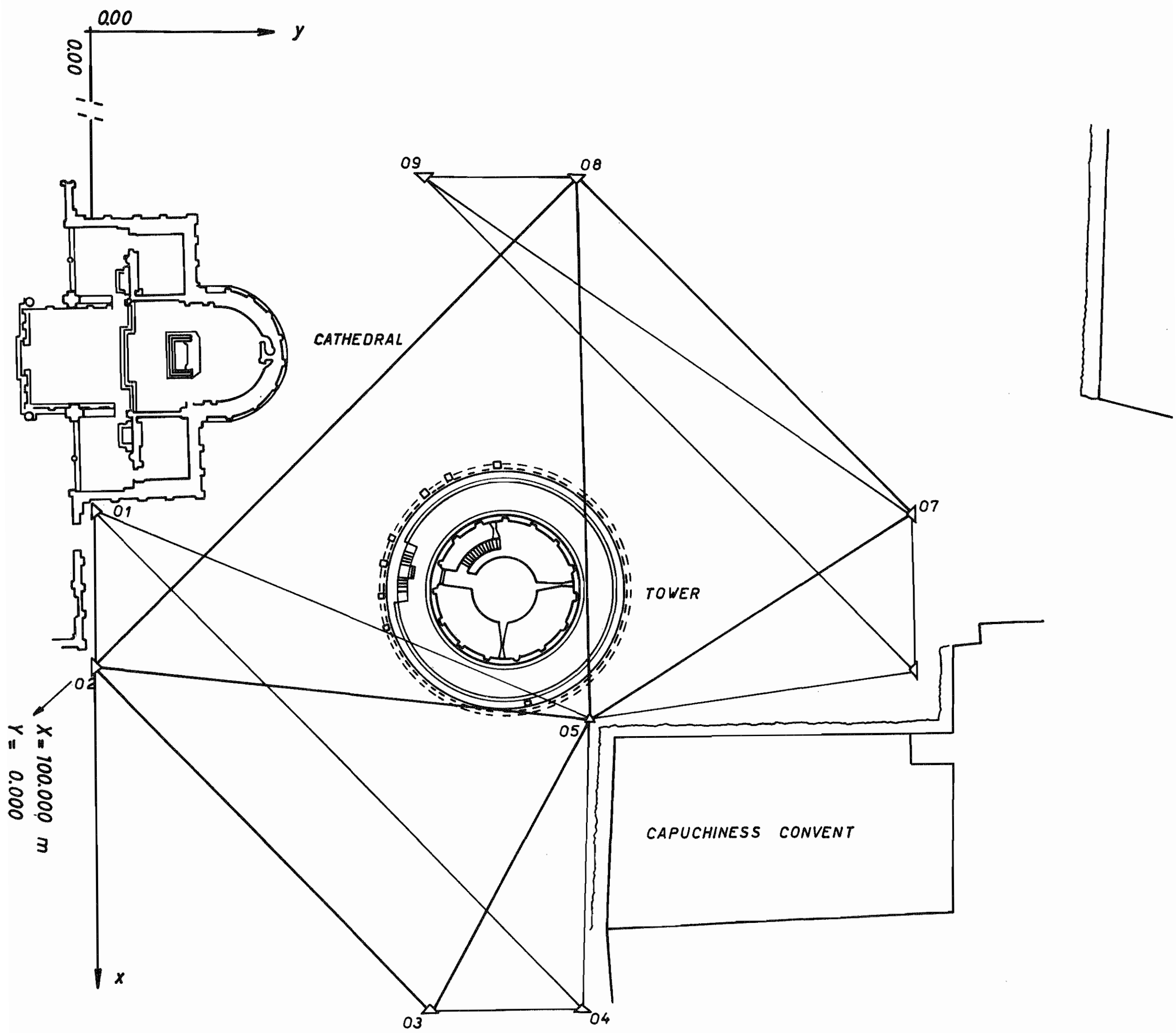


Fig. 5 - Network

2.3 Targeting

Thirty two steel nails were placed on the Tower as permanent points each one of diameter Ø 4 mm and length 40 mm; onto each nail was placed a selfadhesive square retro reflector targets of side 80 mm, corresponding to the 1st , 3th , 6th, 7th orders. 24 of the 32 points mentioned above are positioned on the central core. 15 points have been placed along a perimeter concentric with the Tower and about 10 meters from it, each recognizable and identified, at the time of the taking, by a geodetic target supported by a tripod. . Their position was chosen after consideration of the limits of the plotter.
The coordinates of the points were obtained by microgeodetic measurements, in order to create a spatial grid of fixed points into which the various sectors of the Tower, defined by the models of the stereoscopic coverage,could be locked.
Furthermore all of the 180 columns have been identified with a selfadhesive red square lebel in order to facilitate plotting of a surface which is often very homogeneous and thus makes it difficult to search for homologous points.

2.4 Microgeodetic structure for connection of photogrammetric bases.

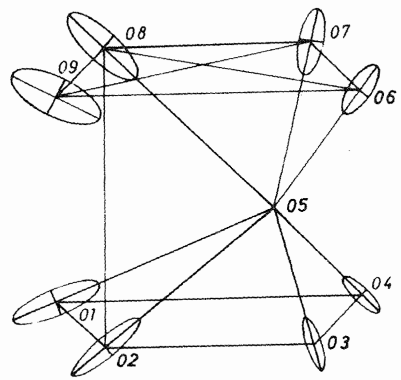
The structure of the triangulation (fig. 5), was also conditioned by the presence of the Cathedral and the Convent. The triangulation was set up so as to have the least number of vertices to be determined with precision and so that they were uniformly distributed around the Tower.
The structure consists of a principal triangulation made up of 3 nearly regular triangles (4 vertices coincide with one of the ends of the principal bases), and of a secondary triangulation which is also made up of triangles whose vertices coincide with the other end of the principal bases mentioned above. All the angles and two of the bases, that is bases 3-2 and 6-8 of these structures, were measured using the following instruments:

- Universal Theodolite Jena Theo 010 B
- Distance meter AGA Mod. 122

The angle of the principal and secondary triangulations were reiterated three times and the bases were measured by the method of differentiated measurements using the AGA distance meter.
The triangulation was adjusted and the errors for the vertices are tabulated in fig. 6.
Furthermore a precision geometric levelling was done between the vertices of the triangulation using a Zeiss (Ni2) automatic level.
The photogrammetric points on the Tower were surveyed by direct intersections reiterating the measurement three times and always using the Jena Theo 010 B Universal Theodolite.

3. NUMERICAL RESTITUTION FOR COMPUTER PROCESSING OF THE STEREO MEASUREMENTS.

The plotting system we used was a digital one, even though an

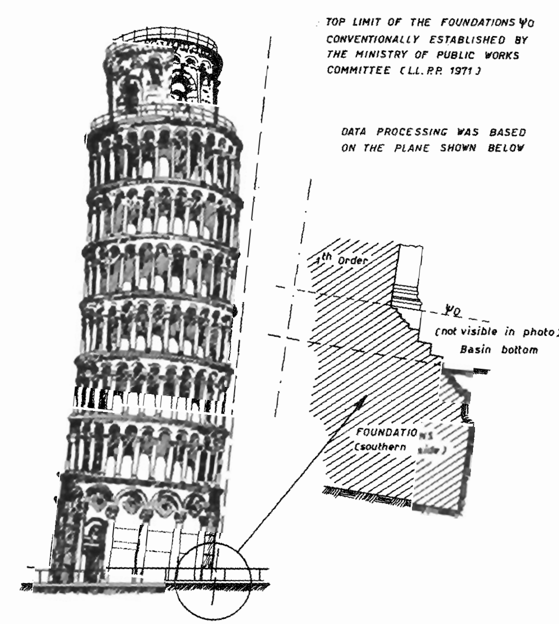


CHARACTERISTIC ELEMENTS OF ERROR ELLIPSES

	INCLINATION OF THE MAJOR AXIS (in GON) (clockwise from the N axis)	SEMIAXIS MAJOR (in meters)	SEMIAXIS MINOR (in meters)
02	57.2031	.0060	.0012
03	182.1049	.0041	.0008
04	151.1507	.0036	.0009
06	39.9301	.0040	.0014
07	14.4211	.0046	.0016
08	150.1412	.0064	.0019
09	131.6059	.0067	.0025
01	76.4064	.0065	.0017

Fig. 6 - Error Ellipses

(approximated NORTH) (approximated SOUTH)



PISA: THE LEANING TOWER - A PERSPECTIVE VIEW, WITH MODIFIED SLOPE, ILLUSTRATING THE CORRECTIONS MADE DURING THE CONSTRUCTION

Fig. 7 - Reference Planes

analogical stereoplotter was available, because of the engineeristic use which it was aimed for.
In fact the purpose of the survey was to determine the shape of the Leaning Tower and its position in the space. To this end it is important to calculate, for example: the lying of significant planes in order to define their structure and their position in the space; the diameters of elements that can be compared to circumferences; the inclination of the axes that, at the various orders, can be considered of symmetry. These calculations and othesr, which prove to be very interesting, can be methodically performed if you have the possibility of using a well organized and easily accessable Data Bank.
The organization of the data recording, specifically when it regards an object with a complex shape such as the Leaning Tower, needs a particular and precise approach so that the data are suitably coded,and that they are homogeneous as far as their localization and their arrangement on the whole construction are concerned. In this case, we are dealing with a construction that is certainly unusual in its predominant and repeated architectural elements: it is made up of 7 orders (6 with a central core and an arcade) with each order being circumscribed by circular cornices (dripstones) and of a bell chamber. What is more , during the construction of the Leaning Tower, which spanned several centuries, many modifications were carried out probably as a consequence of the subsidences that took place in the meantime. Such modifications have changed the inclination of the floor of each order; therefore, in the same order we find a different height in the arcade and in the trabeation. Thus, each order must be regarded, at first, as a separate element, limited at the top and at the bottom by the plane identified by the points forming the perimeter of the cornice's overhang (dripstone). By assuming that the overhangs are circular, we calculated their diameter. Moreover, inside each order, in the portion placed between the cornice's overhang (dripstone) and the arches (which is called trabeation) we studied a series of sections defined by points belonging to the same construction or architectural element that can be easily recognized and identified along the entire perimeter. By assuming that such sections can be compared to a circumference, we calculated their diameter, their position and their tilt in the space. Once suitably coded, it is possible to compare these sections with other sections, near or far, that architecturally correspond to the former ones. By grouping the various sections, we can obtain cylinders or other bodies of different heights, which can be compared one to the other.
As for the central core, since there are no continuous architectural elements along the entire perimeter, it is not possible to define any section. Therefore we studied a series of points, equally arranged and easy to be identified , and we checked if they could be regarded as belonging to a single solid surface. On the basis of the data obtained, for each order we took into consideration other fundamental geometrical elements, such as the position and the direction (with reference to the chosen coordinate system to the vertical) of the joining line of the centers of the circumferences defined by the hangovers (drip -stones) and of the core's axis.
The Data Bank (which holds 8830 points) facilitates the

calculation of all the elements that are considered necessary, or useful, to know.

4. STEREOSCOPIC MEASUREMENTS

4.1 Instruments

For the plotting we used a WILD/HEERBRUGG Apparatus that is made up of an AUTOGRAPH A 10 (a high precision analogical plotter) directly connected to electronical peripheral units which can store, transform and identify the coordinates (EK 22), as well as rotate and translate them, record them on magnetic tape (DPTS 1630) and produce a print-out (TPl 6042).The coordinates, by the"on line"connection AIO-CED, can also be transmitted, via MODEM, from a VT 100 to a PDP 11/14 to be stored into the Data Bank from which they can be drawn whenever necessary to be processed by an IBM computer.

4.2 Plotting

For the plotting we chose the plates 3 DIN, taken with the axes inclinated at + 6 gon.
On the basis of the project approach previously described, we proceeded as follows:

1) Photogrammetric measurements were restricted to details that could be stereoscopically collimated with certainty, or they were performed along vertical lines of the stone to improve the accuracy. In the latter case, when possible, the measurements were taken at the intersection with horizontal lines.

2) The Tower was surveyed following sections (cornices, etc.) formed by repeated architectural or ornamental motifs clearly visible across the physical points of which they are composed.
These sections are generally inaccessable and therefore they are less subject to damage. As for the cornice's overhang, we surveyed both the upper and the lower edges. However, we noticed that some parts of the upper edge were damaged and therefore the order was considered limited by the lower edge (dripstone).

3) As far as the outer surface of the central core is concerned we surveyed a series of points that were conveniently arranged and that could be well identified by markings and traces in the stone.
The plotted points were from the base:

1	1289 points;	2	1342 points;
3	1177 "	4	1404 "
5	1819 "	6	1799 "

These totalled 8.830points and were recorded by using their three space coordinates and a 16 digit identification code.
The synchronized takes were very useful for the plotting.
In fact, by fixing the shadows, areas of light and shade were formed on the outer wall of the core and this facilitated the collimation, in particular of the core, where, without the

synchronization , it would have not always been possible to perform the collimations.
By repeating the collimation of the clearly visible details,we obtained, when compared to the mean values, maximum differences of 1 mm in X and in Z, and of 2 mm in Y. This seems to confirm: a good level of stereoscopy (due to the base to distance ratio of the take) a good image quality and the absence of mechani - cal clearances. Figures 7 and 8 show the reference planes, the lines and the points that were surveyed, and their code.
In order to analyze the accuracy of our results, we compared the coordinates of the same point plotted by two different pairs. Preliminary verifications showed maximum differences of ± 2 cm, compatible with collimations on the stone, in y (x) , i.e. in the depth direction, and of a few mm in z.
In the future it will be possible to obtain better results, and probably with greater accuracy, by plotting the pairs having an inclination of + 16.66 Gon (which will allow the entire Tower to be seen, even its top), and by using a high precision analy-tical stereo-plotter. These kinds of experiments have already been carried out using a Planicomp ZEISS/OBERKOCHEN, a Kern DSR1 and an ACI WILD/HEERBRUGG and it was noticed that such procedure could be performed. In the near future it will also be possible to execute integral plotting.

5. FIRST RESULTS *

The data processing made possible the definition of some fundamental elements of the Tower such as:

a) the diameters at each level, calculated in mm, of the cir - cumferences of the cornices dripstone (mentioned in section 3.1) from level 20 to level 80 (Fig. 9). At each level more than 100 points were surveyed, except for level 80 as it could only be partially seen (33 points);
b) the standard deviation of the plotted points in relation to the circumference itself;
c) the coordinates X, Y, Z of the circumferences' centers mentioned above in point a);
d) the direction angles of the normals to the circumferences' plane;
e) the histograms illustrating the frequencies of the residuals, in relation to the circumference itself, of the plotted points.

The values of points a), b), c) and d) are shown in Appendix 1. In Appendix 1 you can also find the diameters obtained in 1971 by the Committee of the Ministry of Public Works (see point f). A preliminary comparison between the values of the diameters mentioned in point a) and those taken in 1971 by the Committee of the Ministry of Public Works mentioned in point f) shows a difference of a few cm (from 2 cm to 7 cm).
Then, from the examination of the values that were obtained, we registered the following planimetrical shiftings of the circum-ferences' centers from level 20 upwards.

	I (30 - 20)	597 mm		I (60 - 20)	2,242 mm
"	I (40 - 20)	1,182 "	"	I (70 - 20)	2,788 "
"	I (50 - 20)	1,737 "	"	I (80 - 20)	3,259 "

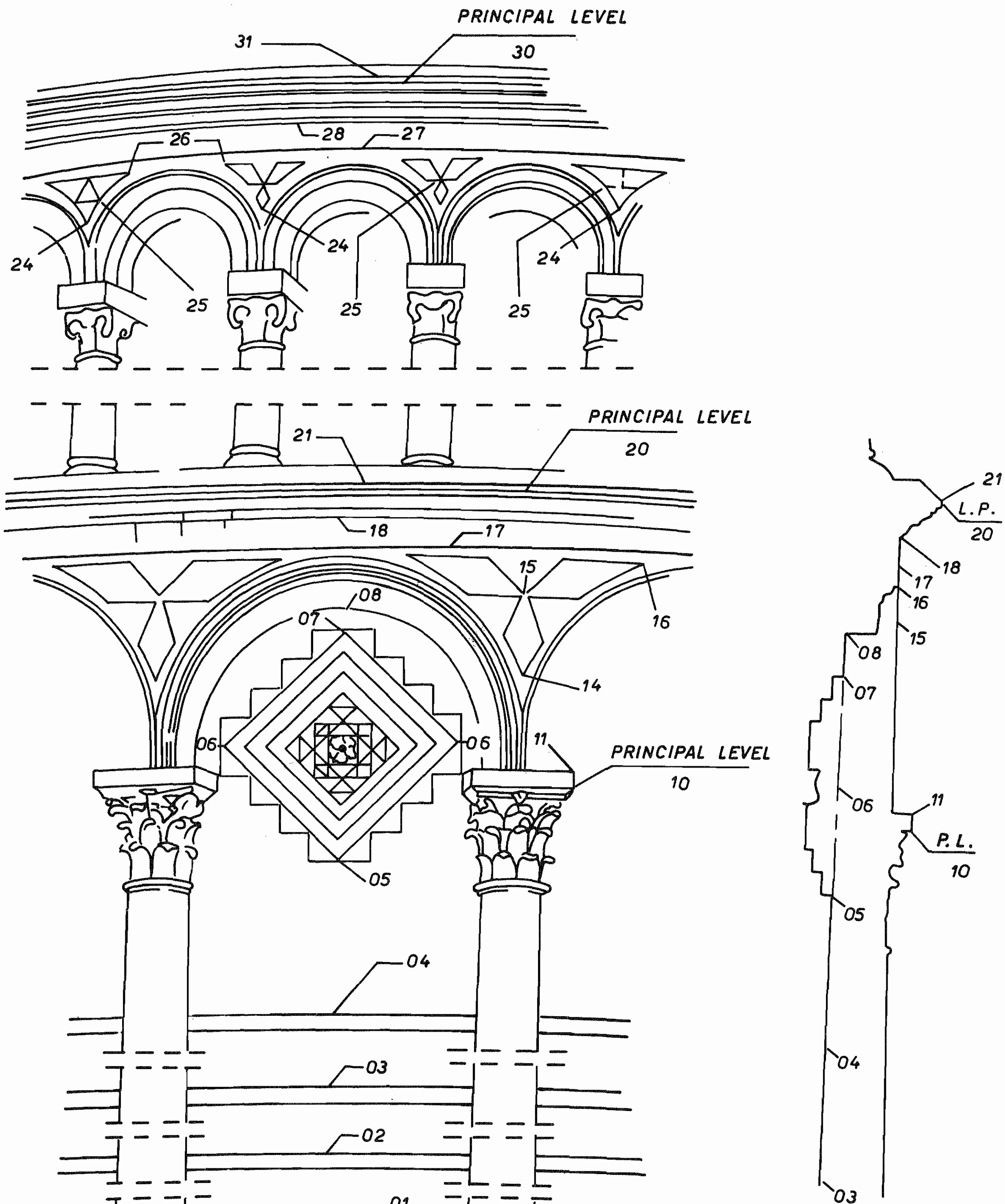


Fig. 8 - Surveyed Detailes and Codification

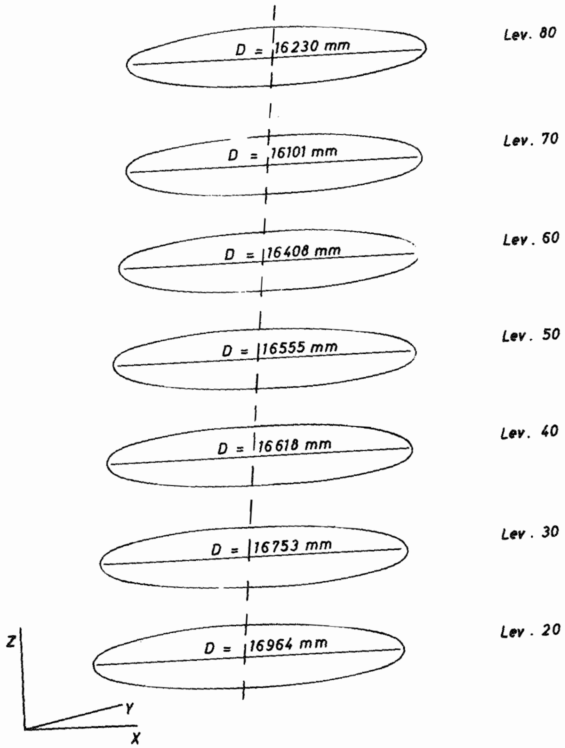


Fig. 9 - Axonometric Representation of the Circumferences of the Dripstones

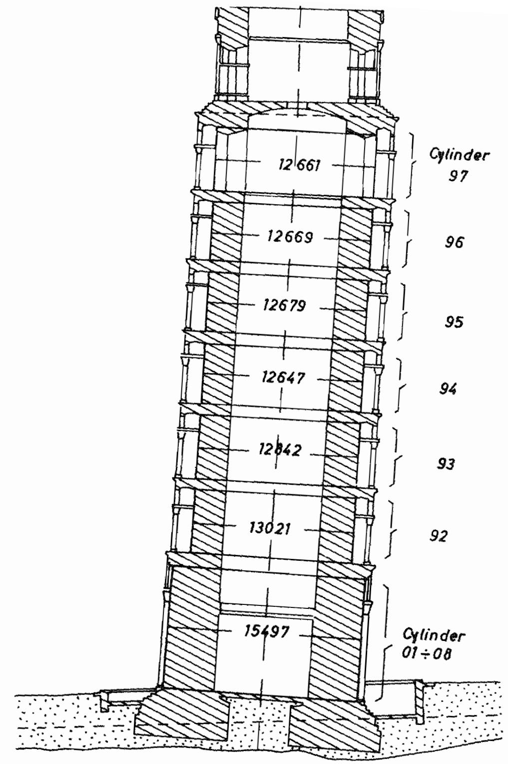


Fig. 10 - Core Coding for each Order

These vectors do not have the same direction. In fact, from level 20 to level 70 there is a variation of more than 1 . It can therefore be said that a slight torsional phenomenon is present in the Tower.
Subsequently, by regarding the core as having a cylindrical shape, we determined (see Figure 10 and Appendix 2):

g) the diameter of the core at each order. For each order more than 600 points were surveyed, except for the top level (440 points).

h) the corresponding standard deviations

i) the direction angles of the core's axis at each order.

l) the coordinates of the cylinder axis' extremes at each order.

In Appendix 2 one can find also the corresponding values of the diameters calculated by the Committee of the Ministry of Public Works in 1971 (point m).
At cylinder 97 you can notice a change in the values of the direction angles. These data need to be checked by plotting the takes having an inclination of 16 gon, also considering that in those having an inclination of 6 gon the cylinder can be only partially seen.
It is foreseen that the Data Bank will be able to provide in - formation that can be used to define the shape and the features of the Tower. Such information will certainly be interesting, if not indispensabile, in any structural intervention that is to be carried out on the Tower.

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* The data processing was carried out at AGIP S.p.A., Department for the Study and the Processing of Geophysical Data, by Mr. Barattini.

Level	Point a) Diameter mm	Point b) standard deviation mm	Point c) X Y Z circumference center	Point d) Direction angles of the normal to the circumference	Point f) Diameter LLPP 71 mm
20	16964	7.2	X = 91674 Y = 48116 Z = 18263	= 84°32′22″ = 89°38′13″ = 5°28′22″	16980
30	16753	8.5	X = 92269 Y = 48161 Z = 24146	= 84°21′59″ = 89°33′42″ = 5°39′ 2″	16820
40	16618	5.2	X = 92852 Y = 48213 Z = 30078	= 84°28′ 6″ = 89°27′20″ = 5°33′30″	16640
50	16555	7.2	X = 93406 Y = 48258 Z = 35852	= 84°19′37″ = 89°34′ 5″ = 5°41′22″	16540
60	16408	9.4	X = 93908 Y = 48314 Z = 41603	= 84°51′21″ = 89°36′11″ = 5°09′34″	16460
70	16101	9.0	X = 94452 Y = 48361 Z = 47238	= 84°57′25″ = 89°36′ 4″ = 5°03′32″	16130
80	16230	1 4	X = 94921 Y = 48396 Z = 53457	= 85°26′37″ = 89°49′31″ = 4°33′35″	16200

APPENDIX 1

Cylinder	Point g) Diameter mm	Point h) standard deviation mm	Point i) Direction angles of the cylinder axis	Point l) Coordinates of cylinder axis' extremes X mm	Y mm	Z mm	Point m) Diameter LLPP 71 mm
01 + 08	15493	7	= 84°26′32″ = 89°30′45″ = 5°34′45″	90667 91682	48008 48097	7836 18263	15540
92	13021	6	= 84°33′23″ = 89°30′32″ = 5°27′57″	91713 92270	48099 48149	18304 24146	13030
93	12842	6	= 84°42′27″ = 89°39′01″ = 5°18′15″	92294 92840	48162 48198	24187 30078	12860
94	12647	6	= 84°16′26″ = 89°34′06″ = 5°44′83″	92836 93411	48204 48248	30118 35853	12650
95	12679	7	= 84°51′09″ = 89°51′31″ = 5°08′57″	93460 93973	48321 48335	35901 41603	12680
96	12669	6	= 83°39′01″ = 89°47′46″ = 6°21′11″	93894 94518	48319 48339	41636 47238	12650
97	12661	13	= 86°25′34″ = 97°51′10″ = 8°38′14″	94529 94919	48820 47965	47270 53456	12710

APPENDIX 2