Mathematical modelling to estimate the contribution to the radiance due to scattering from turbid water:case study - Landsat MSS data over the Tay Estuary

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Abstract

In this paper we present an account of an attempt to model the contribution to the radiance received by the Landsat MSS due to particle scattering from within the waters of the Tay Estuary in Scotland from measured particle-size distribution data. Some of the results are presented.

1.0 Introduction

When a body of water is viewed by a multispectral scanner, the scanner receives electromagnetic radiation which has been reflected from the water surface, from particles in the air, from particles in the water and from the seabed. The reflection from the water surface varies with Sun elevation and surface state. The contribution from air particles or atmospheric path radiance depends on the Sun elevation and atmospheric characteristics such as the amount of haze and dust. Both molecular scattering and scattering by particles of suspended matter will contribute to the intensity of light scattered upwards from within the water. Molecular scattering is so small that it may be neglected (Moore 1946). After passing through the water, light incident on the seabed is partly diffuse and partly direct, the proportion of each depending on the depth and the optical characteristics of the water. The deeper the water and the greater the amount of suspended material the more diffuse is the light. The seabed itself, then, acts as a diffuse reflector (Warne 1978).

In recent years, the use of light scattering has become a useful method for studying suspended material in water. The factors that control the scattering of light from particles in the water are (1) concentration of suspended material, (2) size distribution of suspended material, (3) index of refraction of the suspended material relative to water, and (4) particle shape (Gibbs 1974). The wavelength of the light should also be considered.

In bathymetric studies using multispectral scanner data, the effect of scattering by particles of suspended matter is important. Cracknell *et al.* (1987) have found multispectral scanner data to be useful in bathymetric studies in clear waters but in turbid waters there are difficulties involved in trying to separate the effects of reflection from the seabed and reflection from within the water column. In this paper we present an account of an attempt to model the contribution to the radiance received by the Landsat MSS due to particle scattering from the turbid waters of the Tay Estuary in Scotland from measured particle-size distribution data. However, it should be noted that the particle-size distribution data were obtained on dates that were different from the date the Landsat data were acquired over the Tay. The particle-size distribution data were obtained on 2 July 1985, 1 October 1985 and 12 November 1985 while the Landsat data that we have studied were acquired on 12 June 1981. As such, direct comparisons could not be made between the results obtained from the model and those which can be deduced from the Landsat MSS data.

2.0 Optical Scattering

The ideal case of light scattering is to consider a suspension of spherical particles. The scattering response of a suspension to light varies with suspended material of different particle sizes. The Mie theory can be used to calculate the intensity of light scattered by spherical particles of different sizes at different angles of scattering (Gibbs 1974). The Mie theory permits one to calculate the extinction and scattering cross sections for suspended spherical particles if the indices of refraction of the particles and the suspending medium are known. But these calculations involve the rather complicated Mie scattering coefficients (see Chylek 1973).

Zaneveld and Pak (1973) noted that the expressions for the cross sections may be greatly simplified if the index of refraction of the particles is close to that of the suspending medium i.e. $|m-1| \ll 1$, where $m = m_p / m_w$ and $m_p = n_p - i n_p'$ (n_p = real part of the index of refraction of particles and n_p' = imaginary part of the index of refraction, m_w is the refractive index of water). This implies that both $|(n_p/m_w) - 1| \ll 1$ and $(n_p'/m_w) \ll 1$. In the ocean, the assumption $|m-1| \ll 1$ is reasonable. The real part of the index of refraction influences refraction and reflection at the surface of the particle while the imaginary part governs the absorption in the interior of the particle.

Van de Hulst (1957) derived approximate formulae for the extinction and absorption efficiencies derived from Mie theory for the case $|m-1| \ll 1$ and for particles larger than the wavelength of light. The efficiencies are obtained by dividing the cross sections for the extinction or absorption by the geometrical cross section of the particle. The equations for the extinction efficiency, Q_{ext}, and the absorption efficiency, Q_{abs}, are given by equations (1) and (2) respectively in Zaneveld and Pak (1973). By integrating the extinction cross section over the particle-size distribution $f(D)\,dD$ we obtain the extinction coefficient, c_p, for particles in the medium,

$$c_p = \int_0^{\infty} f(D)\, Q_{ext} (\pi D^2/4)\, dD \qquad (1)$$

where D is the diameter of a particle. Similarly, the absorption coefficient, a_p, for particles is given by,

$$a_p = \int_0^{\infty} f(D)\, Q_{abs} (\pi D^2/4)\, dD\,. \qquad (2)$$

In the Tay Estuary, the particles in suspension are composed of inorganic and organic material. The organic matter content increases with the total suspended sediment concentration and its proportion varies systematically during the year with higher values during the spring. Both the inorganic and organic fractions are important in the behaviour of the suspension (Dobereiner 1982). The organic fraction mainly consists of detritus, decaying tissues of plants and algae. From samples taken along the estuary, Weir (1986) notes that the main minerals in suspension are chlorite, muscovite/illite, quartz, plagioclase, kaolinite and feldspar. From the known values of the index of refraction (real) for these minerals, a mean value of 1.55 for n_p (1.16 relative to water) was used in the calculations in this paper. As for the imaginary part of the index of refraction n_p', a value of 0.04 was used. These values approximately satisfy the relationship $a_p = 0.43\, b_p$ found by Jerlov (1974) by using a large number of samples from turbid to clear ocean waters; i.e. particles scatter about twice as much as they absorb (b_p is the scattering coefficient of particles). These values also satisfy the requirement $|m-1| \ll 1$ to enable equations (1) and (2) to be used. Morel and Bricaud (1981) noted that the value of n_p' for phytoplankton is about 0.01.

3.0 Particle-size distribution data

The particle-size distribution data were obtained by Weir (1986) in his study of suspended sediment dynamics and characteristics in the upper Tay Estuary using a Coulter Counter TAII. Some of the data have been used in this work. The data used were from samples taken on 2 July 1985, 1 October 1985 and 12 November 1985.

From the Coulter Counter measurements, plots of particle size versus number of particles give an exponential relationship of the form,

$$f(D)\,dD = NAe^{-AD}\,dD\,. \qquad (3)$$

This form of the distribution permits the cumulative particle-size distribution $g(D)$ to be written as,

$$g(D) = Ne^{-AD} \qquad (4)$$

The cumulative particle-size distribution (the number of particles per unit volume with diameters larger than D) is the distribution which is measured. N is the total number of particles and A is a parameter characterizing the shape of the size distribution. A least-squares fit of equation (4) was performed on the size distribution data at the locations shown in Figure 1 to solve for the values of N and A.

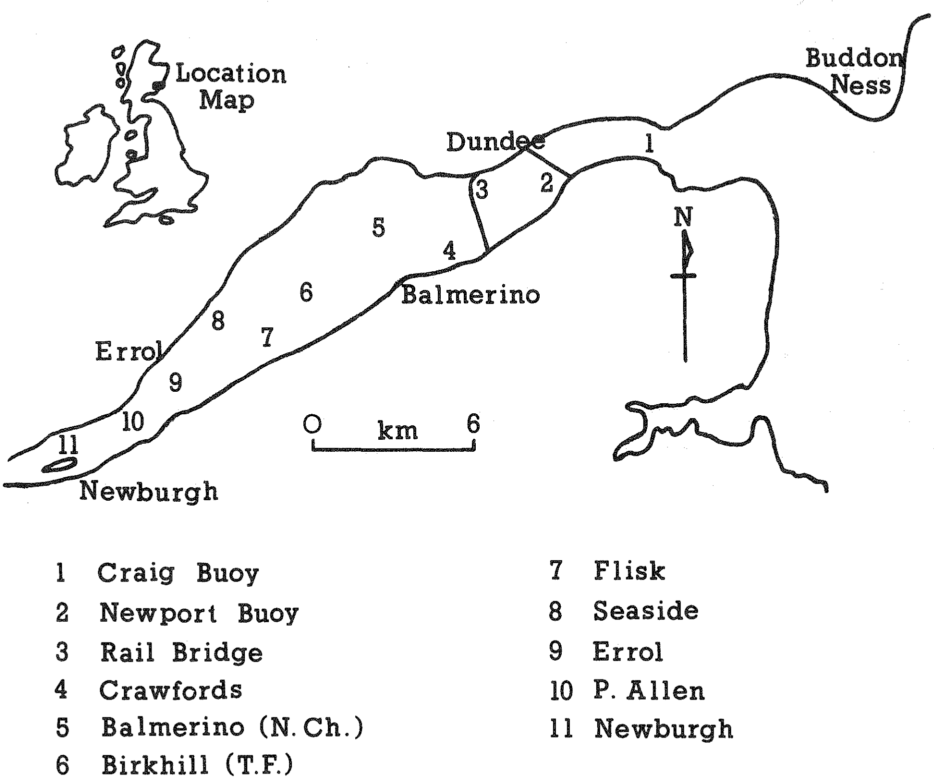

Figure 1: Location map of sample points on the Tay Estuary

4.0 Determination of extinction, absorption and scattering coefficients

From equations (1) and (2), Zaneveld and Pak (1973) give the equations to calculate the extinction and absorption coefficients of particles.Substituting equation (3) and the equation for the extinction efficiency, Q_{ext}, into equation (1) and integrating, the particle extinction coefficient, c_p, can be obtained (see equation (8) of Zaneveld and Pak (1973)). Similarly, by substituting equation (3) and the equation for the absorption efficiency, Q_{abs}, into equation (2) and integrating, the absorption coefficient, a_p, of particles can be obtained (see equation (10) of Zaneveld and Pak (1973)).

Having calculated the extinction and absorption coefficients of particles, the scattering coefficient of particles may be obtained from $b_p=c_p-a_p$. The scattering coefficient of water should be added to the scattering coefficient of particles to obtain the total scattering coefficient. Morel (1974) gives the scattering coefficient of pure sea water at 0.55 μm wavelength as 0.00193 m^{-1}. This wavelength, which is the mid wavelength for Landsat MSS band 4, is used throughout this work. The total absorption coefficient can be obtained by adding the absorption coefficient of water to the absorption coefficient of particles. Morel (1977) gives the absorption coefficient of water at 0.55 μm wavelength as 0.064 m^{-1}. The total extinction coefficient is then the sum of the total scattering and total absorption coefficients. The contribution from yellow substance is not taken into account. Yellow substance effects can be ignored for wavelengths greater than 0.625 μm (Gordon *et al.* 1979).

5.0 Determination of radiance emergent from below the water surface due to scattering from within the water

For determining the radiance emergent from below the water surface due to scattering, the following quantities were calculated.

1. Reflectance due to volume scattering
2. Irradiance attenuation coefficient
3. Volume-scattering function and scattering angle
4. Global irradiance.

Lyzenga *et al.* (1976) give the equation for the reflectance due to volume scattering and bottom reflection. Since it was intended to calculate the contribution to the radiance from volume scattering only due to the suspended material, the last term in that equation which is the bottom reflectance term is ignored.

If E_g is the global irradiance incident on the water surface, then the radiance emergent from below the water surface due to volume scattering alone, L_w, is given by,

$$L_w=E_g R/\pi \qquad (5)$$

where, R is the reflectance due to volume scattering.

The equation for calculating the irradiance attenuation coefficient in Lyzenga's equation is given by Gordon *et al.* (1975). The volume-scattering function may be expressed as a series of Legendre polynomials of the following form, as expressed by Lenoble (1960) and Jerlov (1976),

$$\{b/(4\pi)\}\sum_{n=0}^{N}\alpha_n P_n(\cos\theta). \qquad (6)$$

From the knowledge of the Sun and satellite positions at each location along the estuary, it is possible to determine the scattering angle in the water at these locations.These values of the scattering angle are used in the equation for the volume-scattering function. The global irradiance, E_g, incident on the water surface was determined by using Turner's model as expressed by Sturm (1979). Equation (5) can therefore be solved to determine the radiance emergent from below the water surface due to volume scattering.

In order to obtain the radiance reaching the satellite sensor, L_{sat}, due to scattering from within the water, the atmospheric transmittance, T, should be applied to the radiance emergent from below the water surface as given in equation (5). The equation for T as given by Sturm (1981) may be used for this purpose. Therefore, the radiance reaching the satellite sensor due to scattering from within the water is given by,

$$L_{sat}=L_w T. \qquad (7)$$

The results from the 1 October 1985 data are presented in Table 1.

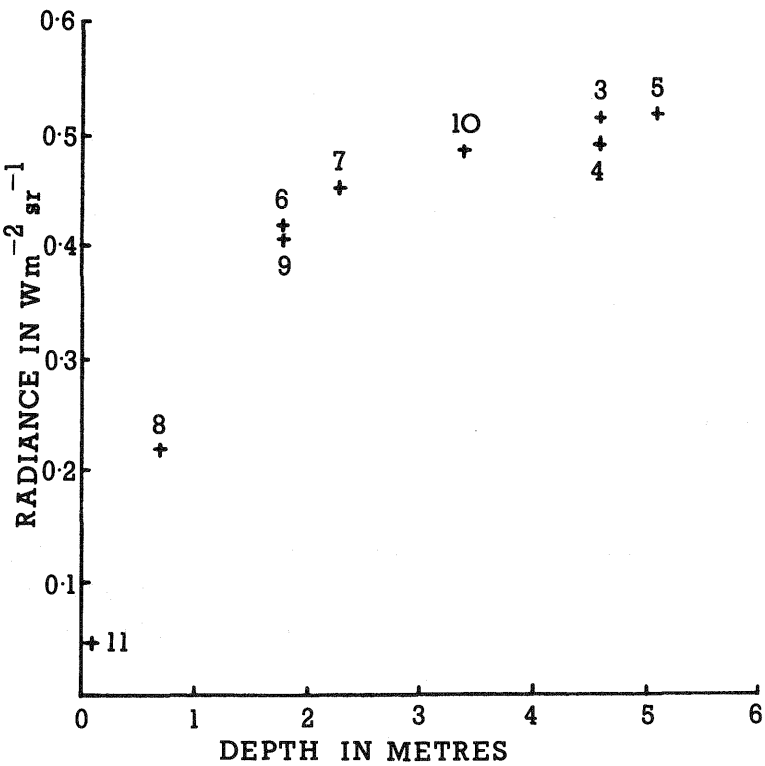

Figure 2: Plot of radiance versus depth, from 2 July 1985 data

6.0 Results, discussion and conclusions

In the Tay Estuary, the total scattering coefficient ranges from about 0.64 to 0.84 m^{-1} while the total extinction coefficient ranges from about 1.04 to 1.23 m^{-1}. This is expected in relatively turbid waters such as in the Tay where the suspended sediment concentration ranges from about 8 to 50 *mg/l* at the time the samples were taken. At some points, much higher values have been recorded. Smith *et al.* (1974) have given a value of 1.824 m^{-1} and 2.190 m^{-1} for the scattering and extinction coefficient respectively in turbid San Diego harbour waters, while in moderately productive Californian coastal

Table 1: Results from the 1 October 1985 data

Pt.	Particle-size Distribution Parameters		Totsca Coeff.	Totext Coeff.	Irat Coeff.	Depth	VSF	R	L_{sat}
	$N(ml^{-1})$	$A(\mu m^{-1})$	(m^{-1})	(m^{-1})	(m^{-1})	(m)	$(m^{-1}sr^{-1})$		$(Wm^{-2}sr^{-1})$
1	396680	1.0963	0.7749	1.1615	0.4954	14.0	0.0095	0.0173	0.5008
2	390671	1.0991	0.7595	1.1392	0.4865	6.0	0.0093	0.0173	0.4999
3	370773	1.0713	0.7566	1.1398	0.4902	4.6	0.0093	0.0170	0.4919
4	384129	1.0836	0.7672	1.1527	0.4936	4.6	0.0095	0.0171	0.4964
5	364567	1.0646	0.7527	1.1354	0.4894	5.1	0.0093	0.0170	0.4935
6	436990	1.1421	0.7894	1.1746	0.4950	1.8	0.0098	0.0151	0.4377
7	290446	0.9723	0.7099	1.0908	0.4842	2.3	0.0088	0.0148	0.4297
8	440372	1.1448	0.7919	1.1776	0.4958	0.7	0.0098	0.0093	0.2701
9	299264	0.9802	0.7206	1.1047	0.4886	1.8	0.0090	0.0140	0.4043
10	343245	1.0320	0.7512	1.1391	0.4951	3.4	0.0093	0.0165	0.4782
11	503969	1.2042	0.8214	1.2100	0.5014	0.1	0.0102	0.0019	0.0539

Totsca Coeff. = Total scattering coefficient (m^{-1})

Totext Coeff. = Total extinction coefficient (m^{-1})

Irat Coeff. = Irradiance attenuation coefficient (m^{-1})

VSF = Volume-scattering function $(m^{-1}sr^{-1})$

R = Reflectance due to volume scattering (dimensionless)

L_{sat} = Radiance reaching satellite sensor due to scattering from below the water surface $(Wm^{-2}sr^{-1})$

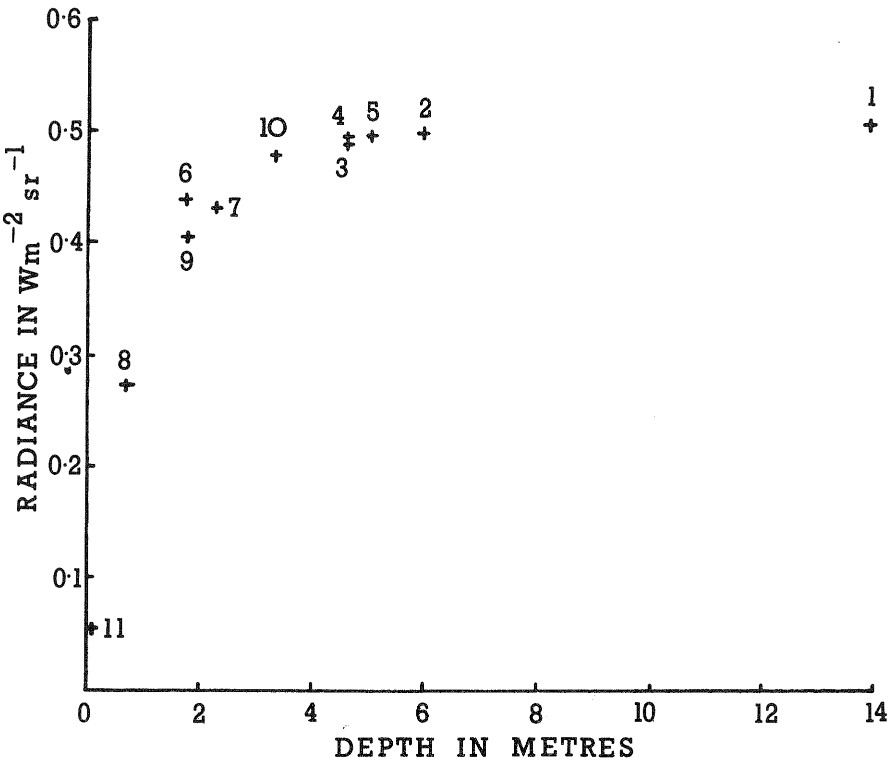

Figure 3: Plot of radiance versus depth, from 1 October 1985 data

waters the value is 0.291 m^{-1} and 0.398 m^{-1} respectively. Also, the total scattering and extinction coefficients increase linearly with an increase in the number of particles (see Table 1).

According to the optical classification of Jerlov (1976) of coastal water types 1, 3, 5, 7 and 9, the types 7 and 9, which are relatively more turbid waters, have irradiance attenuation coefficients of 0.46 m^{-1} and 0.63 m^{-1} respectively at a wavelength of 0.55 μm for a depth range of 0 - 10 m. The values obtained in the Tay range from about 0.48 m^{-1} to 0.51 m^{-1} which is quite reasonable.

The volume-scattering function which is a function of the scattering coefficient and the scattering angle ranges from about 0.008 to 0.01 $m^{-1}sr^{-1}$. This is quite comparable with a value of about 0.006 $m^{-1}sr^{-1}$ deduced from the plots of Duntley (1963) for the same scattering angle in lake waters and a value of about 0.005 $m^{-1}sr^{-1}$ deduced from the plots of Petzold (1972) for the San Diego harbour waters.

The reflectance due to volume scattering is dependent upon depth because the angular distribution of the light field changes with depth. It will also vary with the wavelength and type of water but will generally range from about 0.01 to 0.1 (Austin 1974). In the case of the Tay, it ranges generally from about 0.008 to 0.018 with a lower value at very shallow depths.

As the plots of depth versus radiance received by the satellite sensor due to volume-scattering

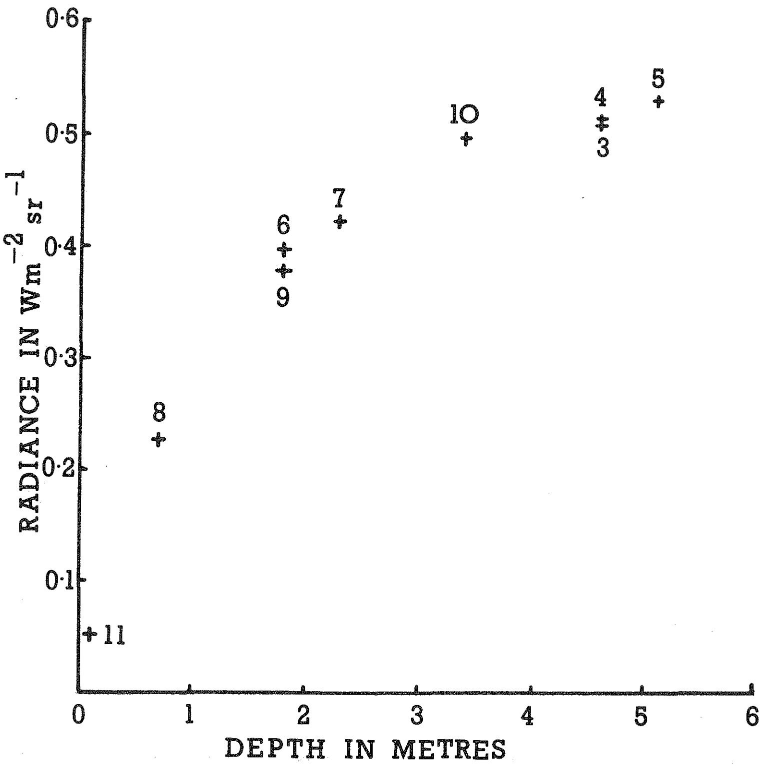

Figure 4: Plot of radiance versus depth, from 12 November 1985 data

show, there is an exponential decrease of radiance for depths down to about 5 *m* on the 3 days (see Figures 2, 3 and 4). For depths in excess of about 5 *m* the increase in the radiance is very small as shown by the plot in Figure 3 for a point 14 *m* deep near Craig Buoy. This indicates that for the optical properties during these 3 days, the signal received from scattering within the water comes from depths within about 5 *m*. Secchi disc depths recorded at various locations along the Tay in 1980 range from about 1.5 - 5.5 *m* (McManus 1984). Secchi disc depths recorded on 1 October 1986 at various locations between the Tay road and rail bridges range from about 3.0 - 5.0 *m*. There appears to be some agreement between the results obtained from the particle-size distribution data and the Secchi disc depth data. However, it should be noted that the dates of these data and of the Sun and satellite positions differ since, under the circumstances of this study, it was not possible to have these data at the same time.

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