MEASUREMENT AND MODELLING OF URBAN REMOTELY-SENSED DATA

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ABSTRACT

This paper lies at the interface between remote sensing, GIS, and spatial analysis. By developing an integrated framework, this interface can be used to generate detailed and frequently updated measurements and classifications of urban land coverages, as well as extensive measurements and models of urban change. Specifically, the classification is a Bayesian-modified maximum-likelihood decision rule, with *a priori* probabilities determined by GIS data, and the models of urban change are fractal-based measurements of residential development and fractal-based measurements of density profiles. All of which, when integrated and applied to the settlement of Norwich, in the United Kingdom, reveal insightful patterns within urban morphologies and detail repercussions on urban density arising from the residential configuration and effects of physical and planning constraints. The data is a single Landsat 5 (TM) image, together with population and housing data from the UK Population Censuses of 1991. This work is an important contribution to the advancement of integrated data handling and analysis systems, as well as providing a means to examining and understanding the complex arrangement of urban structures and processes.

BACKGROUND

The importance of monitoring urban areas is indisputable. With nearly 90% of the European population now living in areas designated as built-up urban* there is a tremendous need for vital information on understanding how cities expand, contract and develop. Specifically, there is a need to monitor and analyse population shifts, employment restructuring, and the layout of residential morphologies. The formulation of national monitoring, management, and planning policies needs to be based on precise and accurate source data. Effective mapping of the structure of urban areas is an essential baseline component to the assessment of the general structure and sustainability of settlements. In conjunction, a statistical framework is vital in revealing more objective measurements of urban configuration, as well as comparisons of how these urban structures change both across space and through time.

Remote sensing has long been recognized as an important technology for reproducing 'snap-shot' observations of the Earth's surface and atmosphere. In the observation of urban areas, satellite sensor data have allowed extensive areal coverage at consistent and readily updateable intervals. Given its rapid retrieval and global availability, satellite remote sensing is an ideal means for producing measurements from which to monitor various aspects of urban dynamism, particularly at the regional scale (examples in Lo, 1986; and the recent GISDATA Specialist Meeting†). However, it is also common knowledge that because of the complex heterogeneous nature of urban surfaces, once the spatial resolution of satellite images begin to approach a more local scale, more and more pixels become invariably spectrally mixed (Forster, 1985). In this paper, some of the problems of urban remote sensing will be addressed by exclusive reference to the growing debate on GIS/Remote sensing integration, in particular the role of GIS in image classification. The methodology will hinge upon the ability of GIS to handle extraneous, non-spectral data, which are then used to determine and vary the *a priori* probabilities of the standard maximum-likelihood (ML) classifier (begun by Strahler, 1980). This essentially involves the use of GIS data to first stratify urban images according to some spatial and contextual rules, and then determine the area estimates of urban classes within each stratum. Area estimates are then normalised and directly inserted into the ML classifier as prior probabilities, producing accuracy levels above classifications simply based on the standard equal prior probability assumption. In work elsewhere, favourable results have also been generated from area estimates which have been used as part of an iterative process for updating ML *a posteriori* probabilities (Mesev *et al*, 1996).

Most research treat the classified image as the end product (the spectral result) and neglect the wealth of information available on the spatial form of classified images. Along with the probabilistic modification of the ML classifier, this paper will also assess the abilities of fractal geometry to measure and summarise the highly irregular spatial patterns of urban land cover/use produced by image classification. In a similar vein to De Cola's work in 1989, fractal geometry will be used to characterise the spatial properties of classified multi-dimensional feature space. However, unlike De Cola, the derived fractal dimensions will further be used for comparative analyses which are designed to evaluate how form and density of urban land use vary within settlements. Furthermore, the assumption of classified urban classes as being fractal will also allow these classes to be represented by cumulative and density profiles generated from functions based on fractal-modified inverse power relations (see Batty and Kim, 1992; Mesev *et al*, 1995 for full description). These urban profiles will provide a means with which incremental urban development is precisely monitored and will lead to an insight on which urban processes may be in operation. It is hoped that remotely-sensed data will rejuvenate the role of urban density functions in measuring and prescribing changes in urban development (Zielinski, 1979).

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* Official Journal of the European Communities, C138/52, Paragraph 5.5 (1993)

† Conference on "Remote Sensing and urban analysis" given by the European Science Foundation, Strasbourg, France, 11 June, 1995

Both the measurement of urban areas using modified image classifications, and the modelling of urban areas using fractal-based analysis, will be applied to Norwich, a medium-sized settlement in the United Kingdom, approximately at the time of the 1991 UK Population Census.

MEASUREMENT OF URBAN REMOTELY-SENSED DATA

In this paper, measurement will relate to image classification, in particular the conventional maximum-likelihood (ML) algorithm, its Bayesian modification, and links to GIS data.

Conventional Maximum-Likelihood Classification

As a parametric classifier, the ML algorithm relies on each training sample being represented by a Gaussian probability density function, completely described by the mean vector and variance-covariance matrix using all available spectral bands. Given these parameters, it is possible to compute the statistical probability of a pixel vector being a member of each spectral class (Thomas *et al*, 1987). The goal is to assign the most likely class w_i, from a set of N classes, $w_1, \ldots, C_N$, to any feature vector $\mathbf{X}$ in the image. A feature vector X is the vector $(\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_M)$, composed of pixel values in M features (in most cases, spectral bands). The most likely class w_i for a given feature vector $\mathbf{X}$ is the one with the highest posterior probability $\Pr(w_i|\mathbf{X})$. Therefore, all $\Pr(w_i|\mathbf{X})$, $i \in [1 \,..\, N]$ are calculated, and w_i with the highest value is selected. The calculation of $\Pr(w_i|\mathbf{X})$ is based on Bayes' formula,

$$\Pr\left(w_i|\mathbf{X}\right) = \frac{\Pr\left(\mathbf{X}|w_i\right) \times \Pr\left(w_i\right)}{\Pr\left(\mathbf{X}\right)} \tag{1}$$

On the left hand side is the *a posteriori* probability that a pixel with feature vector $\mathbf{X}$ should be classified as belonging to class w_i. The right hand side is based on Bayes formula, where $\Pr(X|w_i)$ is the conditional probability that some feature vector $\mathbf{X}$ occurs in a given class, in other words, the probability density of w_i as a function of $\mathbf{X}$. Supervised classifications, such as the ML, derive this information from training samples. Often, this is done parametrically by assuming normal class probability densities and estimating the mean vector and covariance matrix. Also on the numerator and coupled with the conditional probability is what is known in Bayes' formula as the prior probability of w_i, shown as $\Pr(w_i)$. This is the *a priori* probability of the occurrence of w_i irrespective of its feature vector, and as such is open to estimation by prior knowledge external to the remotely-sensed image. External prior knowledge will typically include information on the distribution and relative areas covered by each class in the study scene and is most readily generated from GIS data. It follows that the accuracy of class priors is at best equal to the quality of GIS prior knowledge. In image classification terms, prior probabilities can be visualized as a means of shifting decision boundaries to produce larger volumes in M-dimensional feature space for classes that are expected to be large and smaller volumes for classes that are expected to be small (Mather, 1985). The denominator in (1), $\Pr(\mathbf{X})$ is the unconditional probability density which is used to normalise the numerator such that

$$\Pr(\mathbf{X}) = \sum_{i=1}^{N} \Pr(\mathbf{X}|\, w_i) \times \Pr(w_i) \tag{2}$$

Normally, ML classifiers assume prior probabilities to be equal and assign each $\Pr(w_i)$ a value of 1.0. However, it would seem intuitively more sensible to suggest that some classes are more likely to occur than others. By taking account extraneous information on the areal properties of each spectral class it will be possible to generate thematic per-pixel classifications that are more accurate than those produced from conventional ML techniques (Barnsley *et al*, 1989; Maselli *et al*, 1992). The paper will now examine how prior probabilities may be modified to incorporate external GIS information on class area estimates.

Modification of Prior Probabilities

Before we examine precisely how prior probabilities may be modified, it is important to stress from the outset that our modifications can only be conducted within the more general framework of GIS/RS integration. This requires a systematic strategy which can co-ordinate the flow and coupling of GIS data within image classification procedures. In the worked example, prior probabilities will be modified using a hierarchical stratification strategy based upon data from the United Kingdom Population Census. The stratification will essentially allow census data to assist in the selection and hierarchical partition of spatial features from a satellite image. This hierarchical partition is critical to the statistical assumptions of ML prior probabilities, of which the most important being that all multi-dimensional feature space is subdivided between weighted classes. In other words, for prior probabilities to function most efficiently they need to be applied to inclusive feature space but mutually-exclusive classes. This essentially means that for the classification of mutually-exclusive residential dwelling classes, an image must only be composed of residential feature space. Census tract data have already been shown to be amenable to the generation of pseudo-surfaces of urban representations, especially residential surfaces (Martin and Bracken, 1991) from which such stratification is possible. These surfaces have been used by Mesev (1995) to enhance per-pixel classifications through training sample selection and post-classification sorting. The result is that satellite images have been routinely segmented into "urban" and "non-urban", as well as "residential urban" and "non-residential urban" (Mesev *et al*, 1995). Using the "residential urban" category it will now be shown how prior probabilities of the surrogate residential density categories, "detached", "semi-detached", "terraced", and "apartment" blocks, may be generated by census data and then inserted into the ML classifier.

Consider z_k as the census variable, "residential building type" (where k: 1 = "detached", 2 = "semi-detached", 3 = "terraced", and 4 = "apartments"). When stratified into exclusively residential feature space, the four classes will have A pixels with feature values X_i, where, $X_1, \ldots, X_A$ are not necessarily mutually-exclusive. The objective is to find the probability that a random pixel (within the "residential" stratum of the image) will be a member of a spectral class w_i (where i: 1 = detached, 2 = semi-detached, 3 = terraced, 4 = apartments), given its density vector of observed measurements $\mathbf{X}$, in m-dimensional feature space *and* that it belongs to ancillary class z_k, described as

$$\Pr\left(w_i|\mathbf{X}, z_k\right) \tag{3}$$

It is also assumed that the effects of z_k are external to the original generation of the mean vector and covariance matrix of w_i. As a result the likelihood function $\Pr(w_i|\mathbf{X})$ is unaltered by the introduction of z_k, but is simply modified by the conditional probability

$$\Pr\left(w_i|z_k\right) \tag{4}$$

This is a process of identifying the association between spectral class w_i with census variable z_k. For example, the spectral class labelled as "low density residential" would be directly associated by a conditional probability with the census variable, "detached dwellings". In effect, w_1 is weighted by the probability of z_1, producing the prior probability of w_1. In the example its assumed that the prior probabilities of each of the four dwelling types exist in inclusive m-dimensional feature space, so that,

$$\Pr\left(w_1\right) + \Pr\left(w_2\right) + \Pr\left(w_3\right) + \Pr\left(w_4\right) = 1.0$$

The probability densities $d_{i1} = \Pr(\mathbf{X}_i|w_1)$, $d_{i2} = \Pr(\mathbf{X}_i|w_2)$, $d_{i3} = \Pr(\mathbf{X}_i|w_3)$, $d_{i4} = \Pr(\mathbf{X}_i|w_4)$, are known for each pixel. Let l_{i1} be the shorthand for the posterior probability $\Pr(w_1|\mathbf{X}_i,z_1)$ that pixel i belongs to class w_1, and p_j as the shorthand for the prior probabilities. The Bayesian modified ML is now represented as

$$l_{i1} = \frac{d_{i1}p_1}{d_{i1}p_1 + d_{i2}p_2 + d_{i3}p_3 + d_{i4}p_4} \tag{5}$$

Likewise, $l_{i2} = \Pr(w_2|\mathbf{X}_i,z_2)$, $l_{i3} = \Pr(w_3|\mathbf{X}_i,z_3)$ and $l_{i4} = \Pr(w_4|\mathbf{X}_i,z_4)$ may also be found, and of course, the sum of the four posterior probabilities equals 1.0,

$$l_{ij} = \frac{d_{ij}p_j}{\sum_{j=1}^{k} d_{ij}p_j} \tag{6}$$

Empirical Application

Let's look at one application of the Bayesian-modified ML classifier, the case of the settlement, Norwich in eastern England (others may be found in Mesev 1995; Longley and Mesev, 1996). The aim was to classify the four residential dwelling types from a Landsat 5 (TM) image, taken on the 15th July 1989, using census data from the April 1991 UK Population Census (the 21 month discrepancy was unavoidable and does not represent a period of high residential development in eastern England). Norwich is a free-standing medium-sized city located on land that is relatively flat and unaffected by serious impediment to urban development.

The image was first geometrically corrected and enhanced before classified into a binary distinction of "residential" and "non-residential" using training sample selection and postclassification sorting based on census probability pseudo-surfaces within a GIS (see Mesev 1995; Mesev *et al*, 1995). The "residential" stratum was then exposed to the modified ML classifier, with *a priori* probabilities from the 1991 census (figure 1). Before equation (6) could be implemented, a size ratio between each relative dwellings type had to be calculated. This would help to preserve the relative areal proportions of each dwelling type, where for instance "detached" dwellings occupy larger areas than "terraced" households. Using stereoscopic photographs, 20 samples of dwelling type sizes were generated and average relative size ratios between dwelling types were constructed. The ratios were 1 detached dwelling to 1.5 semi-detached, 1 detached to 2.25 terraced, 1 semi-detached to 1.5 terraced, and 1 detached to 10 apartments. Although these were approximations they are still more realistic than assuming absolute linear relationships.

The results are thematic classifications of the four dwelling density types based on maximum *a posteriori* probabilities. together with area estimates. Table 1 quantifies how classifications based on adjusted *a priori* probabilities produced areal estimates that were in most cases closer to observed census data than classifications assuming equal prior probabilities. The Bayesian modified classifier performed best for the detached category due perhaps to its larger size on the ground and hence least spectrally mixed. The worst category was apartments were the estimated size-ratio may not have truly been representative.

MODELING OF URBAN REMOTELY-SENSED DATA

The second part of this paper will examine how ideas from fractal geometry can be instigated within an urban modeling approach. Specifically, the use of fractal geometry, and density functions based on fractal properties, to describe and summarize the spread of urban development in terms of size, form, and density. For this, thematic urban categories generated by image classification will be used as the source data. This represents a departure from established work where census tract data and derived residential data have been the usual baselines for urban modeling (summary in Zielinski, 1979; Batty and Xie, 1994). It will be argued here that urban data from classified images represent the most appropriate source for the measurement of fractal properties. More appropriate in the sense that the inherent spatial irregularities associated with urban areas and assumed as fractal, are represented by classified remotely-sensed of varying spatial resolutions that exhibit a similar amount of spatial irregularity (De Cola, 1989).

Estimating fractal dimensions and density functions

This paper will add support to the contention that cities exhibit generalized fractal properties (Batty and Longley, 1994; Frankhauser, 1994), and that fractal geometry provides a much deeper insight into urban density functions than has so far been recognized. In particular, emphasis is given to the ways the form of urban development can be linked to its spread and development (Batty and Kim, 1992). Measurement in this sense is restricted to the manner and rate at which space is filled with respect to distance from the CBD (Mesev *et al*, 1995). The suggestion here is that the inverse power function,

$$p(R) = \zeta \quad R^{-\alpha} \tag{7}$$

where, ζ is a constant of proportionality, but not defined where radius $R = 0$, and α is the parameter on distance, is able to accommodate scale independence observed in urban systems through the notions of fractal geometry (Batty and Kim, 1992). Given, the limits on the range of (7), the cumulative population $N(R)$ associated with the density of $p(R)$ can be modeled as

$$N(R) = \varsigma \; R^{2-\alpha} \tag{8}$$

from which the area $A(R)$ over which density is defined with respect to distance R from the centre is given as

$$A(R) = \gamma \; R^2 \tag{9}$$

where a perfect circle of area would have $\gamma = \pi$. Both ς and γ are constants of proportionality. Applying the principle of self-similarity evident in fractal geometry, it is possible to show that the density parameter α is related to the fractal dimension D, such that $D = (2 - \alpha)$ and that the cumulative population relation can now be rewritten as

$$N(R) = \varsigma \; R^D \tag{10}$$

where D is the fractal dimension measuring both the extent and the rate at which space is filled by urban development with increasing distance from the urban centre. From here there are two sets of techniques which have been developed for estimating the fractal dimension. The first set emphasis space-filling (Batty and Kim, 1992), and the second set focuses on density attenuation (Mesev *et al*, 1996). Both are based on squared lattices which are easily derived from classified remotely-sensed data, but only the second set also generates density profiles and will be the one examined in this paper.

Dimensions and profiles from linear regression

A well known method which takes into account variance within the distributions is to first, linearize the power laws in both (7) and (8), and then perform regression to calculate values for ζ and α, ς and D respectively. Linearized forms can be expressed for both discrete densities p_i and cumulative populations, N_i, in this way

$$p_i = \ln \zeta - \alpha \ln R_i \tag{11}$$

$$N_i = \ln \varsigma - D \ln R_i \tag{12}$$

The slope parameters for both equations measure the rate at which density attenuates and population increases, each with respect to distance, respectively. Fractal dimensions are generated by the intercept parameters, ζ and ς which are, in turn, affected by the slope parameters, α and $2 - D$, in (11) and (12) respectively. It has duly been noted that slope parameters may become volatile when confronted with abnormal data sets, leading to fractal dimensions that could lie beyond the logical limits associated with generalized space-filling, i.e. $1 < D < 2$. These are considered abnormal, in the sense that data do not conform to established linear relationships in both cumulative population and discrete density with respect to distance. An example of abnormal data would result if physical barriers restrict development near the central business district (CBD), and in this case, fractal dimensions of over 2 may be possible. Similarly, values of less than 1 are possible if "reversals in the norm" are encountered. This is when discrete density actually increases with distance from the urban centre. Work on "constraining" linear regression can be found in Batty and Kim (1992) and Mesev *et al* (1995).

Empirical application

The Norwich case study may now be re-started by applying the linearized fractal modeling equations (11) and (12) to the thematic residential dwelling types produced through classification. Hence, *the modeling of the measured urban remotely-sensed data.*

Table 2 shows the fractal dimensions and the coefficient of determination (goodness of fit) for both cumulative and density profiles. The dimensions are within the hypothesised $1 < D < 2$ range but much lower than those produced using more traditional data sources (lists in Batty and Longley, 1994). These lower dimensions are due to the fact that the higher spatial resolution of remotely-sensed data have been able to determine "pockets" of undeveloped residential land within settlement boundaries. It means that residential areas are not treated as continuous areas of development but as more discrete entities of incremental growth. The residential patterns produced are therefore more in line with the way land is incrementally apportioned into residential use by planners and echoed by idealised fractal growth (Fotheringham *et al*, 1989).

Between the four dwelling types, terraced has the highest dimension and mirrors conventional centralized tendency in British cities. When examining the cumulative and density profiles (figure 3), it quickly becomes apparent that the density gradients are less than linearized and this is reflected in the coefficients of determination in Table 2. Lower r-squared values for the density profiles are a symptom of the degree of constraints to urban development. These may be physical impediments or, as is more likely in the case of Norwich, planning restrictions which are an important aspect of British settlements. Nevertheless, the profiles in figure 3 are good representations of the ability of remotely-sensed data to measure the way urban development varies within a settlement. The finer spatial resolutions of satellite images allow more detailed intricate variabilities to be highlighted than have yet been possible. With temporal comparisons it may be possible to evaluate urban changes with respect to estimated levels of suburbanisation, decentralisation of economic functions, and segregation of urban land uses.

CONCLUSIONS

This paper has demonstrated how interaction between image processing, GIS data, and spatial analysis can be applied to the measurement and modeling of urban development. What it has shown is that residential development can be measured using the standard ML classifier, as long as reliable extraneous data, preferably handled by a GIS, can be incorporated as representative *a priori* probabilities. Once measured, the structure of residential development and the way residential attenuates with distance, can be modeled by responsive fractal geometry and fractal-based density functions.

It is hoped that research into links between remote sensing, GIS, and spatial analysis will continue and more applications will be developed. The urban application in this paper has produced valuable insights into the manner in which residential development can be measured and modeled. In particular the increased amount of detail now possible, and the effect of development constraints on density profiles. It has produced results which may be used for urban monitoring management, as well as for prescribing planning decisions.

REFERENCES

Barnsley, M.J., Sadler, G.J. and Shepherd, J.W., 1989. Integrating remotely sensed images and digital map data in the context of urban planning. Proc. of 5th Annual Conference of the Remote Sensing Society University of Bristol, UK (Nottingham Remote Sensing Society) pp. 25-32.

Batty, M. and Kim, K.S., 1992. Form follows function: reformulating urban population density functions. Urban Studies, 29, 1043-1070.

Batty, M. and Xie, Y., 1994. Modelling inside GIS: part 1. Model structures, exploratory spatial data analysis and aggregation. International Journal of Geographical Information Systems, 8(3), pp. 291-307.

De Cola, L.. 1989. Fractal analysis of a classified Landsat scene. Photogrammetric Engineering and Remote Sensing 55, pp. 601-610.

Forster, B.C., 1985. An examination of some problems and solutions in monitoring urban areas from satellite platforms. International Journal of Remote Sensing, 6, pp. 139-151.

Fotheringham, A.S., Batty, M. and Longley, P.A., 1989. Diffusion-limited aggregation and the fractal nature of urban growth. Papers of the Regional Science Association, 67, pp. 55-69.

Frankhauser, P., 1994. La Fractalite des Structures, Urbaines. Collection Villes, Anthropos, Paris, France.

Lo, C.P., 1986. Applied Remote Sensing. Longman, Harlow and London, UK.

Longley, P.A. and Mesev, T.V., 1996. The use of diverse RS-GIS sources to measure and model urban morphology. Geographical Systems, (in press).

Martin, D.J. and Bracken, I., 1991. Techniques for modelling population-related raster databases. Environment and Planning A, 23, pp. 1069-1075.

Maselli, F., Conese, C., Petkov, L. and Resti, R., 1992. Inclusion of prior probabilities derived from a nonparametric process into the maximum-likelihood classifier. Photogrammetric Engineering and Remote Sensing, 58, pp. 201-207.

Mather, P.M., 1985. A computationally-efficient maximum likelihood classifier employing prior probabilities for remotely-sensed data. International Journal of Remote Sensing 6, pp. 369-376.

Mesev, T.V., 1995. Urban Land Use Modelling From Classified Satellite Imagery. Unpublished PhD thesis, British Library Catalogue, pp 279.

Mesev, T.V., Gorte, B. and Longley, P.A., 1996. Modified maximum-likelihood classifications and their application to urban remote sensing. In: Remote Sensing and Urban Analysis, J.-P. Donnay and M.J. Barnsley (editors), Chapter 3, (forthcoming).

Mesev, T.V., Batty, M., Longley, P.A. and Xie, Y., 1995. Morphology from imagery: detecting and measuring the density of urban land use. Environment and Planning A, 27, pp. 759-780.

Mesev, T.V., Longley, P.A. and Batty, M., 1996. RS/GIS and the morphology of urban settlements. In: Spatial Analysis: Modelling in a GIS Environment, P. Longley and M. Batty (editors), Chapter 7. (forthcoming).

Strahler, A.H., 1980. The use of prior probabilities in maximum likelihood classification of remotely-sensed data. Remote Sensing of Environment, 10, pp. 135-163.

Thomas, I.L., Benning, V.M. and Ching, N.P., 1987. Classification of Remotely-Sensed Images, IOP, Bristol, UK.

Zielinski, K., 1979. Experimental analysis of eleven models of population density. Environment and Planning A, 11, pp. 629-641.

Table 1. Classification results using equal and unequal prior probabilities

Dwelling Type	Census		Equal Priors			Unequal Priors		
	census tracts	Area %	Pixels	Area %	Error	Pixels	Area %	Error
Detached	37 364	42.40	12 486	38.75	-3.65	13 910	43.17	+0.77
Semi-Detached	26 675	30.27	10 311	32.00	+1.75	9 161	28.43	-1.84
Terraced	21 088	23.93	8 030	24.92	+0.99	7 440	23.09	-0.84
Apartments	2 987	3.39	1 395	4.33	+0.94	1 712	5.31	+1.92
Totals	*88 114*	*100.00*	*32 222*	*100.00*	*7.33*[‡]	*32 223*	*100.00*	*5.37*[‡]

[‡]total error in absolute terms

Table 2. Fractal dimensions from linear regression

Dwelling Type	Fractal Dimension (D)	r-squared (cumulative)	r-squared (density)
Detached	1.423	0.953	0.770
Semi-Detached	1.408	0.954	0.787
Terraced	1.661	0.919	0.320
Apartments	1.176	0.950	0.903

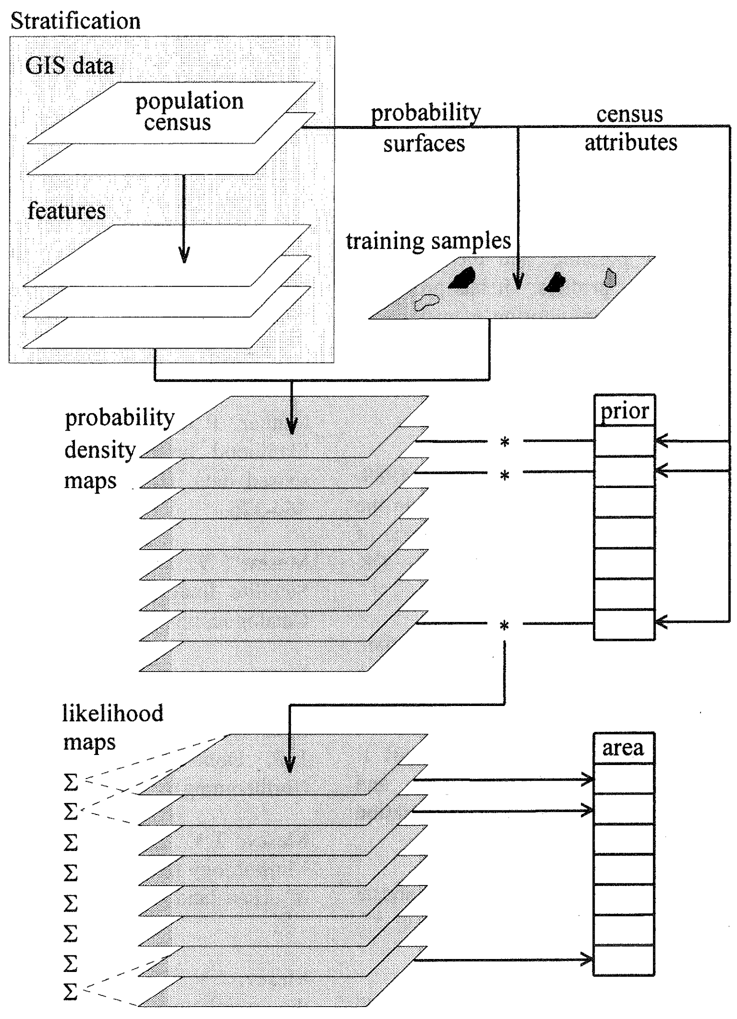

Figure 1. Insertion of prior probabilities in the maximum likelihood classifier

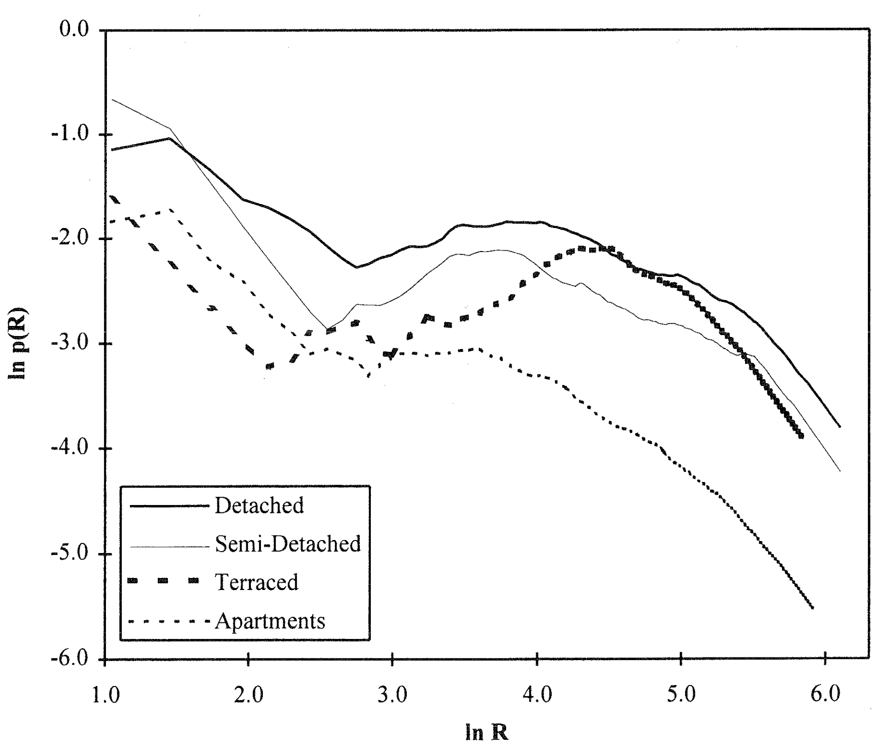

Figure 3. Density profiles of the four residential dwelling types of Norwich, England