# ANALYSIS OF SATELLITE LASER ALTIMETRY RANGE MEASUREMENTS OVER LAND TOPOGRAPHY USING GENERALIZED ATTITUDE PARAMETER EXTRACTION

J. R. Ridgway, J.B. Minster Institute of Geophysics and Planetary Physics University of California, San Diego U.S.A. ridgway@spot.ucsd.edu, jbminster@ucsd.edu

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# ABSTRACT

Two new laser altimeter satellites are being launched by NASA in the near future: the ICESat (Ice, Clouds, Elevation) satellite and the VCL (Vegetation Canopy Lidar) satellite. Although each satellite has a different emphasis (ICESat concentrating on ice sheets and VCL on vegetation), both will fly over land for a large portion of their orbit. The alignment of the laser is set prior to launch; however, forces on lift-off may cause a mis-alignment. Traditionally these "attitude biases" in aircraft are calculated using dedicated roll/pitch maneuvers over a flat surface such as a lake or the ocean. An alternative method to this procedure is the measurement of topography along the satellite ground-track and comparison of the measured profile to a well-known "truth grid". The residual topographic signal would then be analyzed and related to spacecraft attitude, range and other mis-alignment sources. We propose an attitude extraction approach whereby any given measured profile is related to the truth grid by a laser pointing vector. In addition, there may exist orbit and timing errors. The determination of these parameters involves a search for a minimum in a multi-variable space. For this study, we simulate roll and pitch errors and use a systematic parameter search to extract them from the topographic data. For more extensive parameter spaces, an evolutionary algorithm will be needed. We investigate the tradeoff between orbit and pointing errors and determine that use of long profiles in latitude and ascending/descending tracks can separate these sources. We examine how the accuracy of the truth grid relates to the parameter estimation. We also examine the question of whether smooth, monotonic topography or rougher wide-band topography is more suitable in calibrating the satellite laser.

### 1. INTRODUCTION

Two new laser altimeter satellites are being launched by NASA in the near future: the ICESat (Ice,Clouds, Elevation) satellite and the VCL (Vegetation Canopy Lidar) satellite. Although each satellite has a different emphasis (ICESat concentrating on ice sheets and VCL concentrating on vegetation canopies), both will fly over land for a large portion of their orbit. The alignment of the laser is precisely set prior to launch; however, mechanical forces on lift-off may cause a mis-alignment, and other effects (mainly thermal) may cause a time-dependent variation of attitude mis-alignments during the life of the mission. Thus in-flight methods to calibrate the attitude errors are necessary.

One approach by Rowlands et al. (1999) for the Mars Observer Laser Altimeter data utilizes crossovers which are modeled for short topographic segments by polynomials. An *a priori* knowledge of the topographic surface is not needed for this procedure. Another technique involves a flat surface such as a lake or the ocean, over which the vehicle performs dedicated roll/pitch maneuvers. Simulations by Luthcke et al. (1999) show that, if successful, this may be able to be used to extract pointing biased errors in roll, pitch and yaw to about 1 arcsecond accuracy. A third approach is an experiment using corner cube reflectors under the satellite track (Schutz, 1998). As the satellite flies over this known grid, an aircraft simultaneously illuminates the grid with its own laser and takes a picture of the satellites laser spots with a CCD detector.

The use of sloping land topography to determine laser pointing biases is an alternate approach to the methods described above. This relies on the surface over which the satellite flies to be very well known and used as "ground truth". The satellite laser produces a height profile which is compared to the ground truth grid, and a misfit function is calculated using some standard technique such as r.m.s. error. The misfit is related to satellite pointing/timing/location errors, and an optimal error parameter set is found, such that the misfit is minimized. This paper deals with some aspects of this "ground truth" calibration method.

#### 2. LASER POINTING GEOMETRY

Given a satellite with a nadir-pointing laser, its roll, pitch and yaw combine to mis-point the laser. This pointing vector can be described in the satellite's along-track  $(\mathbf{x})$ , cross-track  $(\mathbf{y})$ , and down  $(\mathbf{z})$  coordinate system (also called the Satellite Body-Fixed system).

$$\mathbf{p} = a\mathbf{x} + b\mathbf{y} + c\mathbf{z} \,. \tag{1}$$

The pointing vector  $\mathbf{p}$  is a unit vector, so it may be described with only 2 parameters: a heading vs. the satellite's along-track

direction ( $\beta$ ), and a deviation from nadir pointing ( $\delta$ ), where  $\beta$  and  $\delta$  are related to a, b, c as:

$$\beta = \tan^{-1} \left( \frac{b}{a} \right), \tag{2a}$$

$$\delta = \tan^{-1} \left( \frac{\sqrt{a^2 + b^2}}{\sqrt{1 - a^2 - b^2}} \right).$$
(2b)

The pointing vector  $\mathbf{p}$  is related to roll (a rotation about the  $\mathbf{x}$  axis), pitch (a rotation about the  $\mathbf{y}$  axis) and yaw (a rotation about the  $\mathbf{z}$  axis) by the rotation matrix  $\mathbf{R}$ , operating on a laser vector  $\mathbf{v}$  which points nominally along  $\mathbf{z}$ :

$$\mathbf{p} = \mathbf{R}\mathbf{v} , \qquad (3a)$$

where **R** and **v** are defined as:

$$\mathbf{R} = \begin{bmatrix} c \,\theta c \,\phi & s \,\psi s \,\theta c \,\phi - c \,\psi s \,\phi & c \,\psi s \,\theta c \,\phi + s \,\psi s \,\phi \\ c \,\theta s \,\phi & s \,\theta s \,\psi s \,\phi + c \,\psi c \,\phi & c \,\psi s \,\theta s \,\phi - s \,\psi c \,\phi \\ - \,s \,\theta & c \,\theta s \,\psi & c \,\theta c \,\psi \end{bmatrix}, \quad (3b)$$
$$\mathbf{v} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}, \quad (3c)$$

where: C is cosine, S is sine,  $\psi$  roll,  $\theta$  pitch,  $\phi$  yaw.

A roll is defined as positive if the right satellite "wing" is down, a pitch is positive if the satellite's "nose" is up, and a positive yaw is a counter-clockwise rotation when looking up at the satellite along the z axis. Using this relation, one can construct a pointing vector p, given any roll, pitch and yaw values.

The position and height of the laser bounce point on the ground is determined by knowing the satellite's height and bearing, the horizontal pointing deviation  $\beta$ , and its off-nadir angle  $\delta$ (Figure 1). This defines a line, which intersects the topographic grid at the bounce point. We can reduce this to a 2-D problem by constructing a vertical planar section along the horizontal projection of the laser path and extracting a topographic profile from the grid. The problem thus becomes finding the intersection between a line with a slope of  $-\tan(90^{\circ} - \beta)$  and the topographic profile (see Figure 1). The intersection point is easily solved for using Newton's method, but this is somewhat computer-intensive to be used for every bounce point. A good approximation for small off-nadir angles is to iteratively calculate the horizontal migration of the ground spot ( $\Delta x$  in Figure 1) as follows: 1) calculate an initial horizontal displacement  $\Delta x_0 = \rho_0 \tan(\delta)$ , 2) obtain the topographic height at  $\Delta x_0$  using interpolation, 3) calculate the satellite height above the topography (h<sub>1</sub>), 4) calculate the better approximation to  $\Delta x$ :  $\Delta x_1 = h_1 \tan(\delta)$ . This approximation will locate the bounce point to within 0.5 m for rolls up to 3° (a negligible error considering the 65 m GLAS and the 25 m VCL spot sizes), and takes much less computer time.



Fig. 1. The geometry of a laser pointing error, showing the translation of a ground spot  $(\Delta x)$  caused by a pointing error  $(\delta)$ . The range  $\rho_1$  differs from  $\rho_0$  primarily because of the effect of the topographic slope.

The primary contribution to the range error  $(\Delta \rho = \rho_1 - \rho_0)$ , see Figure 1) is the topographic height change along the distance  $\Delta x$ . For small angular errors, the contribution to the range increase caused solely by the slope of the off-nadir laser pointing is negligible (for example, if  $\delta = 60$  arc-seconds, then  $\Delta \rho$  is only 2.5 cm on flat ground, well below the measurement threshold of the GLAS laser). This results in a very high correlation between the range error caused by mis-pointing and that caused by a horizontal orbit error. Conversely, for roll/pitch maneuvers over the ocean, a large laser mis-pointing error is correlated with vertical orbit error and range bias (Luthcke et al., 1999).

# 3. SIMULATION OF ATTITUDE, ORBIT ERROR

The simulation of errors was made using a Digital Elevation Model (DEM) over the Mojave Desert, in eastern California, extracted from the USGS 3-arc-second digital database. A topographic grid was produced for a strip from 116° 30' to 115° 30' W longitude, and 34° 45' to 37° N latitude (Figure 2). Simulated ascending and descending satellite tracks for the proposed ICESat calibration phase (8-day repeat, at an altitude of 600 km above sea-level) were plotted over the grid. The satellite orbit is retrograde, with an inclination of 94°. Factoring in the rotation of the earth, the ground tracks have a bearing of approximately 8° west of north for ascending tracks and 8° west of south for descending tracks. For simplicity, the nominal laser pointing was assumed to be in the nadir direction, with a -30 arc-second pointing error in roll for initial tests, and later an added 20 arc-second error in pitch. The Mojave area is a combination of smooth and rough topography, with flat plains and alluvial fans alternating with medium-to-high mountains. The 2 test tracks flying over this area yield the nominal topographic profiles (without error) shown in Figure 2.

The steepest slopes, hence largest errors, are near the mountains at  $35^{\circ}$  45' and  $36^{\circ}$  24' N latitude. For a 600 km satellite altitude, a 1 arc-second pointing error results in a ground translation of about 2.9 m. This can be mimicked by an orbit error, as is demonstrated in the "difference" plots in the lower half of Figure 2.



**Fig. 2** This region near the Mojave Desert was used to generate simulated laser altimetry profiles with attitude errors. Ascending and descending tracks will separate certain translation errors (such as in the orbit node) from pointing errors. The lower half of the figure shows the topographic profiles and difference profiles of translational vs. pointing errors.

The simulated error in this figure is a -30 arc-second roll error, which results in an 87 m horizontal offset to the right. This is matched quite nicely for the ascending track by an orbit translation to the east (accomplished by a perturbation of the longitude of the node of the orbit). However, the two error sources separate in direction on the descending track, because the roll error continues to translate the spot to the satellite's right (west) whereas the node error still produces an eastern translation. Thus the difference between the vertical topographic profile caused by the roll error and that cause by the nodal orbit error is only 0.13 m r.m.s. (10 m maximum) for the descending track.

Although a nodal orbit error can be distinguished from an attitude error by examining both ascending and descending tracks, this is not the case with an error in the orbit's

inclination. An inclination perturbation will cause a translation of the satellite's location in the east-west direction primarily, and "flips" in sign for ascending vs. descending tracksn a manner very similar to an attitude error. Thus a separation of the 2 error sources cannot be attained solely by examining both sets of tracks. However, the magnitude of the east-west translation does change dramatically and characteristically with latitude: it is zero at the equator, maximum at the poles, and flips sign going from the northern to the southern hemisphere. Conversely, an error in roll or pitch is nearly constant in latitude except near the poles. Thus the solution for translational errors in the laser ground spot over a large latitude range will separate inclination and attitude errors, whereas they are highly correlated over limited patches of latitude.

Figure 3 demonstrates the behavior of a -30 arc-second roll error and a 3.6 arc-second orbital inclination error.



**Fig.3.** An error in the orbit's inclination varies with latitude, which can be used to separate it from an attitude error. The use of ascending/descending tracks cannot separate the two different error sources, as they behave similarly.

The error source becomes apparent as the domain of the problem is extended to large latitude ranges, including comparisons to DEMs performed in the southern hemisphere. For satellites such as ICESat, whose predicated orbit accuracy is 0.05 m radially and 0.20 m horizontally (Rim and Schutz, 1999), aliasing by orbit error not a pressing issue, but for vehicles with greater orbit error such as the Shuttle Laser Altimeter (SLA) the problem is significant.

# 4. EXTRACTING ATTITUDE ERRORS FROM RANGE DATA

Many different methods to inversely find the laser pointing error given an observed topographic profile and a ground-truth grid are possible; this study utilized a search through the parameter space of the pointing errors. Combinations of errors are systematically explored, the forward calculation of the expected topographic profile is made, and the r.m.s. fit of the measurement vs. the theoretical topography is compiled. The combination which minimizes the misfit is chosen. For this study, only roll and pitch errors were simulated. Other errors could include orbit errors as discussed above, a range scale factor, and timing errors in the roll/pitch/yaw signals. When the parameter space expands, more sophisticated methods are needed, as a systematic search suffers from a "combinatorial explosion." One such method is evolutionary programming (Fogel, 1994), whereby different "parent" populations generate "offspring" which are competed against each other using a fitness function (such as goodness-of-fit). The most fit offspring then re-constitute the new parent population, and the process is iterated until the fittest parameter population is attained. This method will be utilized in future simulations with more parameters in the error space.

Figure 4 is a contour map of the r.m.s. mis-fit between the observed topographic track with simulated roll and pitch errors, and the truth grid over the Mojave Desert.



**Fig.4.** A contour plot of the r.m.s. difference between the measured profile and a truth grid. The 2 meter contour interval spans approximately 10 arc-seconds in pointing

The contours in this 2-parameter space are very smooth, even though the topography is often rough. The parameter space search easily "finds" the minimum misfit (at -30 arc-seconds roll, 20 arc-seconds pitch). The contours give an indication of how errors in the DEM map into the solution for pointing errors. The 2 meter r.m.s. contour describes a roughly circular function around the solution, with a diameter of 10 arc-seconds. Thus a reasonable estimate of the accuracy needed for 1 arcsecond resolution of the pointing error is 0.2 meters. This estimate encompasses all error sources. Given that the total radial error budget for GLAS is 0.14 m (Bae and Schutz, 1999), the accuracy of the DEM must also be 0.14 m to attain the required 0.2 m r.s.s. error level. This level of accuracy has been attained using current scanning laser altimetry systems from aircraft over flat topography (Huising and Gomes Pereira, 1998) and over ice sheets (Krabill et al., 1995).

## 5. SMOOTH VS. ROUGH TOPOGRAPHY

One might ask what type of topography is optimal for doing the ground-truth calibration: a smooth, monotonic surface with low slopes and low amplitude, or a rougher, wide-band surface? One advantage of a smooth, low-amplitude surface is that it is easier to make an accurate ground-truth DEM there. Huising and Pereira (1998) indicate that errors in aircraft scanning laser surveys increase significantly over hilly terrain vs. flat terrain. Also, the large diameter satellite spot (25 m for VCL, 65 m for GLAS) will create an averaged height value within its footprint, and will have a larger associated error for rugged vs. smooth terrain (Harding et al., 1994). The disadvantage of a very uniform, monotonic terrain such as sand dunes, is that it is possible for large attitude errors to be aliased, making it impossible to determine unambiguously the optimal pointing error through profile matching. This idea was tested by constructing a two-dimensional sinusoidal ground-truth grid with wavelength 2000 m and amplitude 100 m. The satellite track and simulated roll/pitch errors were the same as before.

Figure 5 is an example of the difference functions along the satellite track, for a sinusoidal grid (upper profile) and for rougher, "real" topography. When the parameter space for the simulated roll/pitch errors was searched and contoured for the smooth grid, a picture similar to Figure 4 was attained, with smoothly-varying contours and a unique minimum point. Another test was made, but the ground-track error was increased to 1000 m (vs. < 100 m for the previous roll/pitch This dramatically changed the outcome for the errors). sinusoidal topography. Searching the parameter space yielded multiple minima with identical r.m.s. error. When applied to the wide-band topography, only one minimum point was attained. We can conclude from this simulation that nearly monotonic topography is acceptable for use as a ground-truth grid in satellite pointing-error calibrations, as long as the expected error translation of the ground spot is a small fraction of the dominant wavelength of the grid. Using wide-band topography will ensure that ground-truth ambiguities will not happen, at the cost of reduced accuracy in both the ground-truth DEM and in the satellite topographic data.



Fig. 5. Two different error profiles: for ideally sinusoidal topography (top) and for actual, wide-band topography in the Mojave (bottom).

### 6. CONCLUSION

Satellite laser attitude biases can be extracted from measurements of land topography over accurate "truth grids." The accuracy in the pointing errors so derived is strongly dependent on the accuracy of the DEM used as the groundtruth. The satellite pointing errors are highly correlated with certain orbit errors, and can be separated only by examining ascending/descending tracks and a large range in latitude. Smooth, monotonic topography has the advantage of being easier to accurately measure, but care must be taken that it does not alias the measurement profile for large attitude errors.

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### REFERENCES

Bae., S., B.E. Schutz, 1999. Precision attitude determination (PAD). NASA ATBD, Version 2.0, Goddard Space Flight Center, Greenbelt, Md.

Fogel, D. B., 1994. An introduction to simulated evolutionary optimization. IEEE Transactions on Neural Networks, 5(1), pp. 3-14.

Harding, D.J., J. L. Bufton and J. J. Frawley, 1994. Satellite laser altimetry of terrestrial topography: vertical accuracy as a function of surface slope, roughness, and cloud cover. IEEE Transactions on Geoscience and Remote Sensing, 32 (2), pp. 329 – 339.

Hofton, M. A., J. B. Minster, J. R. Ridgway, J. B. Blair, D. L. Rabine, J. L. Bufton, N. P. Williams, 1997. Using laser altimetry to detect topographic change at Long Valley caldera, California. SPIE, 3222, pp. 295-305.

Huising, E.J. and Gomes Pereira, L.M., 1998. Errors and accuracy estimates of laser data acquired by various laser scanning systems for topographic applications. ISPRS Journal of Photogrammetry and Remote Sensing, 53, pp. 245 - 261.

Krabill, W.B., R.H. Thomas, C.F. Martin, R.N. Swift and E.B. Frederick, 1995. Accuracy of airborne laser altimetry over the Greenland ice sheet. International Journal of Remote Sensing, 16, pp. 1211 - 1222.

Luthcke, S. B., D. D. Rowlands, J. J. McCarthy, E. Stoneking, D. E. Pavlis, 1999 (in press). Spaceborne laser altimeter pointing bias calibration from range residual analysis. The Journal of Spacecraft and Rockets.

Ridgway, J. R., J. B. Minster, N. Williams, J. L. Bufton, W. B. Krabill, 1997. Airborne laser altimeter survey of Long Valley, California. Geophysical Journal International, 131, pp. 267-280.

Rim, H. J., B. E. Schutz, 1999. Precision orbit determination (POD). NASA ATBD, Version 2.0, Goddard Space Flight Center, Greenbelt, Md.

Rowlands, D. D., D. E. Pavlis, F. G. Lemoine, G. A. Neumann, S. B. Luthcke, 1999. The use of laser altimetry in the orbit and attitude determination of Mars global surveyor. Geophysical Research Letters, 26(9), pp. 1191-1194.

Schutz, B.E., 1998. GLAS Validation Plan. NASA ATBD, Version 0.1, Goddard Space Flight Center, Greenbelt, Md.