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FUZZY SPATIAL OBJECTS AND THEIR DYNAMICS

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ABSTRACT

The determination of the spatial extent of geo-objects is generally approached through the boundaries or more precisely through the position of the boundary points. The analysis of the geometric uncertainty of the objects is therefore often based on accuracy models for the coordinates of these points. The accuracy evaluation in land surveying and photogrammetry generally refers to the mapping of crisp objects. In many other survey disciplines objects are mapped that are not crisp, in that case the geometric uncertainty is not only a matter of coordinate accuracy, but also a problem of object definition and thematic vagueness. It can not be handled by only a geometric approach such as epsilon band method. This paper proposes an approach to map the spatial extent of the objects and their uncertainties when objects are measured from field observation data.

Beyond that, this paper presents a method of detecting the dynamics of these fuzzy objects from time series. They are determined by comparing their spatial extents at successive epochs. Simultaneously, the processes through which objects evolve are identified and are represented by several types of state transition, such as shift, merge, and split of objects. The proposed method is applied in a coastal geomorphologic study of a barrier island in The Netherlands.

1 INTRODUCTION

The syntactic approach for handling spatial object information as presented in (Molenaar 1994 and 1996) makes it possible to distinguish three types of statements with respect to the existence of spatial objects:

- an *existential statement* asserting that there are spatial and thematic conditions that imply that an object exists,
- an *extensional statement* identifying the geometric elements describing the spatial extent of the object,
- a *geometric statement* identifying the actual shape, size and position of the object in a metric sense.

These three types of statements are intimately related. The extensional and geometric statements imply the existential statement and if an object does not exist it can not have a spatial extent and geometry. The existential statement often relates to the thematic information though, that is not explicit in the other two statements. The geometric statement also implies the extensional statement, often the actual geometry of the object is derived from the extensional description. These three types of statements can all have a degree of uncertainty and although these statements are related they give us different perspectives that may help us to understand the different aspects of uncertainty in relation to the description of spatial objects.

The determination of the spatial extent of geo-objects is generally approached through the boundaries, or more precisely through the position of the boundary points. The analysis of the geometric uncertainty of the objects is therefore often based on accuracy models for the coordinates of these points. The epsilon band method is well known in this context (Dunn et al., 1990). Yet the solutions for handling this problem are not found satisfactory though because the geometric uncertainty of geo-objects is not only a matter of coordinate accuracy, i.e. it is not only a problem of geometry, but it is also a problem of object definition and thematic vagueness. This latter aspect can not be handled by a geometric approach alone. This becomes apparent when mapping is not done in a crisp geometry as for land surveying and photogrammetry. The object detection through image interpretation is an example of the formulation of extensional statements. The uncertainty exists in the thematic aspect expressed by the likelihood of pixels belonging to thematic classes. Image segments can then be formed of adjacent pixels falling under the same class. If these segments represent spatial objects then the uncertainty of the geometry of these objects is due to the fact that the value of the likelihood function varies per pixel**.**

Nowadays, concepts of fuzzy set theory are being applied to model the uncertainty in geometric aspects of mapping units (Usery, 1996; Brown, 1998). Most works propose approaches to describe and represent the spatial extent or boundaries of fuzzy objects due to uncertain classification of the mapping units. However, the inter-relationships between the various types of uncertainty are not described, although Gahen & Elhers (1997) proposed a framework for uncertainty transformation between thematic data and geographic features through remote sensing interpretation. In the paper we discuss the extensional and geometric uncertainty.

Moreover, literature to date hardly discusses the dynamics of objects, particularly spatial change, in a generic way. Even less literature is available about the dynamic behavior of fuzzy objects with indeterminate boundaries. The detection of the dynamics of fuzzy objects is the second point to be addressed in this paper. The paper will elaborate an example where the dynamics of sediments along the Dutch coast are monitored.

The paper is organized as follows. Next section discusses the relationship between existential uncertainty and extensional uncertainty. An approach to identify the spatial extent of fuzzy objects is discussed. It is followed by a discussion of fuzzy spatial overlap in section 3 in order to detect the state transition of objects. Section 4 presents the identification of dynamics of fuzzy objects by linking the state transitions. The last section of the paper summarizes the major findings and further researches.

2 FUZZY SPATIAL EXTENT AND FUZZY BOUNDARY

This section discusses the inter-relationship between thematic and geometric aspects. The discussion will follow the procedure to identify objects from field observation data (Cheng et al., 1997) to tract the uncertainty propagation. In this procedure data is converted from a low level form (field sampling) to a high level

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form (distinct objects) through interpolation, classification and segmentation. Here we will discuss the uncertainty transformation from classification to segmentation, i.e., from thematic data to geometric aspects of objects. It is discussed that due to the vagueness of object class definitions and the errors in field sampling points, each grid cell P_{ij} will generate a membership function value vector $[L(P_{ij},C_1),L(P_{ij},C_2),...,L(P_{ij},C_N)]^T$ $(0 \le L(P_{ij},C_k) \le 1)$ after classification. Here $L(P_{ij},C_k)$ represents the membership function value of grid cell P_{ij} belonging to class C_k, N is the total number of the classes. For each class C_k regions can be identified consisting of cells with $L(P_{ij},C_k) > Threshold_k$. Each region can then be interpreted as the fuzzy extent of a spatial object belonging to a class C_k . If the classes are assumed to be spatially exclusive then each grid cell belongs to at most one class, and consequently to only one object; if the objects form a spatial partition then each grid cell belongs to exactly one object. In other applications, fuzzy spatial overlaps among objects are permitted, i.e. the objects have fuzzy transition zones that may overlap (Burrough, 1996;Usery, 1996). In the transition zones, the pixels might belong to multiple objects. The fuzzy topologic relationships of spatial objects are discussed in (Dijkmeijer & De Hoop, 1996) and (Zhan, 1997). However, here we will not discuss this issue, as in our case the objects form spatial partitions. So each grid cell belongs to exactly one class and one object, which can be determined by criteria such as we define as follows.

Let $NL[P_{ij}, C_k]$= 1- $L[P_{ij},C_k]$ represent no-membership, i.e., the certainty that P_{ij} does not belong to class C_k, and let $XL[P_{ij},C_k]$ express the membership that P_{ij} belongs exclusively to C_k and not to any other classes C_l for any $l \neq k$. Because $XL[P_{ij},C_k]$ expresses that the grid cell belongs to class C_k <u>and</u> not to any other classes, it can be derived by applying minimum operations as

$$XL[P_{ij},C_k]=MIN(\, L[P_{ij},C_k], MIN_{l\neq k}(NL[P_{ij},C_l]))\,. \qquad (1)$$

As P_{ij} can only belong to one class, it requires only one class for which the function $XL[]$ has maximum value for P_{ij}. If there are more classes with the same maximum values then additional evidences are required to come to a selection of a unique class. It can be represented as

if $XL[P_{ij},C_k]=MAX_{C_l}(XL[P_{ij},C_l]) \quad (l=1,\cdots,N)$ then let $D[P_{ij}, C_k]$ = 1,

otherwise $D[P_{ij}, C_k]$ = 0. (2)

After assigning the cells to classes, an area S_a of class type C_k will be formed by the following two conditions (Molenaar, 1996),

for all grid cells $P_{ij} \in S_a$, $D[P_{ij}, C_k]$ = 1, and

if $P_{kl} \in S_a$ and $ADJACENT[P_{kl},P_{ij}]$=1 and $D[P_{ij},C_k]$=1 then $P_{ij} \in S_a$.

(3)

$ADJACENT[P_{kl},P_{ij}]$ expresses the adjacency relationship between grid cells P_{kl} and P_{ij}, and it has value either 0 or 1. P_{ij} will only be assigned to S_a if "$D[P_{ij},C_k]$ = 1". The certainty that this assignment is correct depends on the certainty that the cell has been assigned correctly to C_k. Therefore the relationship between P_{ij} and S_a, $Part[P_{ij},S_a]$, can be wrote as

$$Part[P_{ij},S_a]=MIN(D[P_{ij},C_k],XL[P_{ij},C_k])\,. \qquad (4)$$

For example, let a grid cell has membership values of three classes:

$$L[P,C]_{t_1} = \begin{Bmatrix} 0.2 \\ 0.8 \\ 0.0 \end{Bmatrix}$$

So

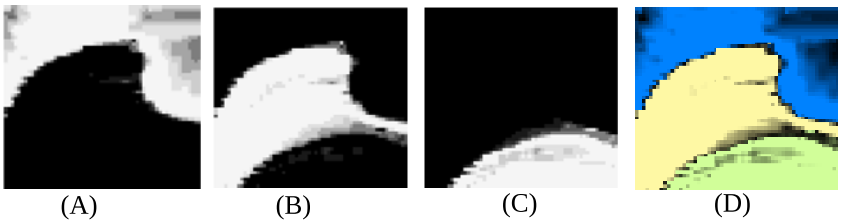
$$NL[P,C]_{t_1} = \begin{Bmatrix} 1-0.2 \\ 1-0.8 \\ 1-0.0 \end{Bmatrix} = \begin{Bmatrix} 0.8 \\ 0.2 \\ 1.0 \end{Bmatrix} \quad XL[P,C]_{t_1} = MIN \begin{Bmatrix} 0.2,0.8 \\ 0.8,0.2 \\ 0.0,1.0 \end{Bmatrix} = \begin{Bmatrix} 0.2 \\ 0.8 \\ 0.0 \end{Bmatrix}$$

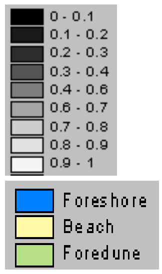
As $XL[P, C_2]=MAX_{Ci}(XL[p, C_i])$ (i=1,2,3) then $D[P, C_2]$=1.

It means that this cell is assigned to class C_2 with certainty 0.8.

A practical case is the identification of spatial extent of foreshore, beach, foredune in coastal geomorphology studies (Cheng et al, 1997). As shown in Figure 1 (A) (B) (C), the classification of grid cell P_{ij} generates a membership vector. Using the approach above, the regions of different class types, which represent the spatial extent of the objects, are shown in Figure 1 (D). The outmost grid cells of a region compose the boundary of an object, which can be considered as conditional boundary as it is formed based upon the criteria above.

Equation (4) expresses the relationship between the extensional uncertainty and the thematic uncertainty of objects. In this way the *existential* uncertainty (uncertain classification of grid cells) is converted to *extensional* uncertainty (fuzzy spatial extent) and *geometric* uncertainty (fuzzy boundary).





(A) membership value to foreshore (B) membership value to beach
(C) membership value to foredune (D) fuzzy object spatial extents

Figure 1 Fuzzy classification and fuzzy objects (1989).

3 OVERLAP OF SPATIAL EXTENT AND STATE TRANSITION OF FUZZY OBJECTS

The procedure in the previous section identifies the regions that represent the spatial extents of objects at one epoch. The regions at different epochs should be linked to form life lines of the objects. This can be realized based on the assumption that natural phenomena are changing gradually, especially the change of coastal zone can be regarded as completely continuous (Galton, 1997), so the objects are considered to be rather stable. The approach developed in this section will be designed for such cases. This implies that if two regions are the spatial extents at different epochs of one and the same object, their overlap should be larger

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than their overlaps with the region of any other object. Under this assumption we can find the successor of a region at epoch t_n by calculating its spatial overlaps with all the regions that appeared at epoch t_{n+1}. The one that has maximum overlap will be identified as the successor.

The overlap of two regions S_a and S_b can be found through the intersection of their two cell sets. It is a very simple raster-based operation.

$$Overl[S_a,S_b]=Cells(S_a)\cap Cells(S_b) \tag{5}$$

where *Cells*(S_a) and *Cells*(S_b) represents the sets of grid cells belonging to region S_a and S_b, respectively.

As the regions per epoch are uncertain, the spatial overlap between two regions at two epochs should be adopted to take care of their fuzziness. The possibility of a grid cell to be part of the overlap of two fuzzy regions can be defined as (Dijkmeijer & De Hoop, 1996),

$$Overl[S_a,S_b|P_{ij}]=MIN\{Part[P_{ij},S_a],Part[P_{ij},S_b]\} \tag{6}$$

where *Part*[P_{ij}, S_a] and *Part*[P_{ij}, S_b] as defined in equation (4). The size of P_{ij} is considered to be *1* here, so that the size of a fuzzy region *S* is defined as

$$Size(S)=\sum_{P_{ij}} Part[P_{ij},S] \text{ where } P_{ij}\in Grids(S) \tag{7}$$

The size of the overlap of two fuzzy regions is then

$$SOverl(S_a,S_b)=\sum_{P_{ij}} Overl[S_a,S_b|P_{ij}] \tag{8}$$

where $P_{ij}\in Cells(S_a)\cap Cells(S_b)$.

Based upon the spatial overlap between regions, we can match the regions that are spatially related. Let R_i be the set of regions at epoch T_i, and let $S_a\in R_1$ and $S_b\in R_2$. The following indicators can be used to evaluate the types of relationship between regions at two epochs.

The relative fuzzy overlap between two regions can be defined as

$$ROverl(S_b|S_a)=SOverl(S_a,S_b)\,/\,Size(S_a) \tag{9}$$

$$ROverl(S_a|S_b)=SOverl(S_a,S_b)\,/\,Size(S_b) \tag{10}$$

where *ROverl*(S_b|S_a) represents the ratio of the overlap to the size of S_a (relative fuzzy overlap to S_a); *ROverl*(S_b|S_a) is the ratio of the overlap to the size of S_b (relative fuzzy overlap to S_b).

The similarity of two fuzzy regions can be defined as

$$Similarity(S_a,S_b)=\frac{SOverl(S_a,S_b)}{\sqrt{Size(S_a)\cdot Size(S_b)}} \tag{11}$$

Using these indicators, object state transitions can be identified between two epochs. Seven fundamental cases are shown in Table 1. The combinations of indicator functions behave differently for these seven cases. State transition can be identified by the following process.

For all $S_b\in R_2$ compute Size (S_b)
For all $S_a\in R_1$ do
> compute Size (S_a)
For all $S_b\in R_2$
> compute *SOverl*(S_a, S_b)
> compute *Roverl*(S_b/S_a), *Roverl*(S_a/S_b), *Similarity*(S_a,S_b)
> evaluate *shift*(S_a; S_b), *expand*(S_a;S_b), *shrink*(S_a;S_b)
> evaluate *split*(S_a; ...S_b, ...), *appear*(S_b)
> evaluate *merge*(..., S_a, ...; S_b), *disappear*(S_a)

The split process implies that one region $S_a\in R_1$ splits in several regions $S_b\in R_2$ and the merge process implies that many regions $S_a\in R_1$ merge into one region $S_b\in R_2$. The evaluation is made by identifying the type of state transition between S_a and S_b based upon the indicators according to the situations indicated in Table 1.

4 DYNAMICS OF FUZZY OBJECTS - FROM STATE TRANSITIONS TO PROCESSES

The procedure of the previous section identified possible dynamic relationships between regions at two different epochs. Regions thus related can be linked to form life lines of objects that may have "shifted", "expanded" or "shrunk" between two successive epochs. The regions that appeared at a specific moment represent a newly appeared object, and regions that disappeared at some moment represent disappearing objects. Furthermore, "merging" and "splitting" objects can be identified. The procedure to identify the dynamic object can be illustrated by the following case study.

Table 2 presents the fuzzy sizes of regions and fuzzy overlap of regions of three successive years. The indicators of section 3 can now be evaluated; with these we can link the regions by several lines (as shown in Figure 2) which indicate that the regions connected by these lines are most likely the representations of the spatial extent of an object in successive years. For example, region 1 has been linked with 4, 4 with 8; region 3 has been linked with region 6, 6 with 10. We also found that there is a new region in 1990 (region 7). By checking the overlap of this region with the regions at 1989 and 1991, we found it has overlap with region 3 and 10; these regions are linked by a line also.

Table 2 Fuzzy overlaps and links among fuzzy regions.

Year	Region	Area	Overlap with regions in next year				Class Type
1989	1	1108.1	937.5	81.8	0.0	0.0	Foreshore
	2	1246.8	106.3	1104.8	9.2	0.0	Beach
	3	644.3	0.0	12.7	572.5	27.5	Foredune
1990	4	1138.7	975.0	76.0	0.0		Foreshore
	5	1229.7	76.0	1129.5	2.6		Beach
	6	586.8	0.0	0.0	564.3		Foredune
	7	28.0	0.0	0.0	26.3		Beach
1991	8	1101.3	862.7	116.9	6.4	0.0	Foreshore
	9	1260.1	87.3	1146.6	0.0	0.5	Beach
	10	609.8	0.0	3.3	0.0	605.7	Foredune

For example, the spatial overlap of region 3 in 1989 (S_a) and region 6 in 1990 (S_b) is 572.5 (*Soverl*(S_a,S_b)), and here S_a=644.3, S_b=586.8. So

ROvel(S_b|S_a)= 572.5/644.3 =0.819
ROvel(S_a|S_b)= 572.5/586.8 = 0.976
Similarity(S_a,S_b)=0.894

Therefore, these two regions are very similar to each other and can be considered as instances of a same object (here we call it object 3) at two epochs. As there are differences between the boundaries of these two regions, we considered that object 3 shifted from region 3 in 1989 to region 6 in 1990.

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We also calculated the similarities between region 3 (as S_a) and region 7 (as S_b),

$ROvel(S_b|S_a)$= 27.5/644.3=0.043
$ROvel(S_a|S_b)$= 27.5/28.0 = 0.982
$Similarity(S_a,S_b)$=0.205

Therefore, we can conclude that these two regions are not similar to each other, but region 7 is more or less contained in region 3. It can be identified as a new object appearing in 1990, and is split from object 3 (region 3 represents its spatial extent in 1989). By analyzing the overlap between regions of 1990 and 1991, we found that region 7 disappeared in 1991, it was merged into object 3 (region 10 in 1991). Using the above approach, the objects and the processes involved in object developments are identified as illustrated by Figure 4. The icons represent the regions (states) of objects at different times. The symbols represent the types of state transition. It can be seen from the figure that object 4 split off from object 3 between 1989 and 1990; it is merged again into object 3 between 1990 and 1991.

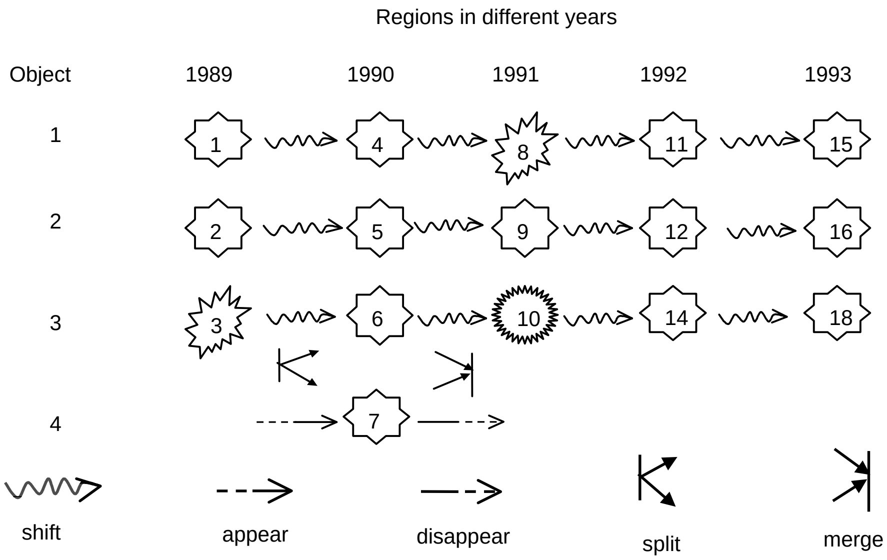

Figure 2 Identified fuzzy objects and processes.

5 CONCLUSIONS

This paper presented a method to identify fuzzy objects and their dynamics from field data sampled at different times. The methodology has been demonstrated by an empirical example in a coastal geomorphological study of Ameland. It will also be applicable to modeling natural environments and physical processes in other fields.

It is revealed in our experiment that the uncertainties in the field observation data and in the definition of object classes have obvious influences on the identification of the spatial extent of objects at different epochs. Therefore, the geometric uncertainty of objects is due to the uncertainties of thematic aspect and semantic domain. It means that the extensional, existential and geometric aspects of objects all have a degree of uncertainty and they are related to each other.

The dynamics of fuzzy objects are revealed through the spatial extents (states of objects) at different epochs. They are determined by comparison of the relationship of these spatial extents. Simultaneously, the processes through which these objects evolve are identified.

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