LEAST MEDIAN OF SQUARES MATCHING FOR AUTOMATED DETECTION OF SURFACE DEFORMATIONS

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ABSTRACT

Detecting the difference between two surfaces without the aid of control points is desirable for many industrial applications. We tackle this problem by means of robust surface matching. In presence of local deformation, conventional surface matching algorithm with least square condition would fail. Efforts have been made by some researches to robustify the surface matching algorithm using M-estimators. We use least median of squares estimator and data snooping technique to robustify the surface matching algorithm and use a M-estimator to improve the efficiency of least median of squares estimator. Evaluation and comparison of these methods are carried out using simulative data. The result shows robust matching using least median of squares estimator can detect a local deformation that covers up to 50 percents of the surface and very small deformation can detected and it is not sensitive to position of deformation, which is much superior to other methods of robustifying surface matching.

1.INTRODUCTION

There are many industrial applications where the detection of the difference between two 3-D (digital) surfaces is of great concern, such as monitoring surface deformation and soil erosion, product quality control and object recognition. The surfaces of concern could belong to one of the following three categories: (a). The same surface acquired at two different epochs, i.e. in the case of monitoring surface deformation, soil erosion etc.; (b). One surface representing a design model and the other measured from a product, i.e. in the case of quality control; and (c). The two surfaces representing two different models but for similar objects, i.e. in the case of object recognition and security analysis.

Traditional techniques for surface deformation detection involve an extensive use of control points. For example, in order to monitoring stream channel erosion, ground control points including stable stokes were established and periodically maintained by investigators to provide common reference system for DEMs acquired at different epochs [Wetch and Jordan 1983]. In applications of tool and component inspection in aircraft manufacturing by means of photogramonetry, absolute or relative monitoring networks were required to provide common datum between each measurement epoch [Fraser 1988]. However, in many circumstances, the establishment of control points is a difficult, unreliable and even impossible process. This is especially the case for surface deformation monitoring in engineering, industrial inspection and medical sciences. Techniques without the use of control points are therefore very desirable in these cases.

Digital elevation model is used in our study for surface representation. To detect the difference between two surfaces, the essential task is to transform the two DEMs representing the two surfaces respectively into a common datum. With absolute control networks, this could be done by coordinate transformation. With relative control networks, this is a procedure of deformation analysis. When no control points exist, the two DEMs have to be mutually registered by means of surface matching. However, the two surfaces cannot be precisely matched until the difference between them has been identified and removed from the data sets used for matching and the difference cannot be detected until the two surfaces have been precisely matched. Therefore, the procedure of surface matching must be performed simultaneously with the procedure of difference detection.

There have been some studies on surface deformation detection by means of surface matching. Karras and Potsa (1993) addressed medical applications of deformation detection through DEM matching. In their study, the DEM matching technique developed by Resenholm and Torlegard (1988) was applied and extended with data snooping technique to detect deformation of human body. Experiments showed such a process performs well when deformation is of large magnitude but small proportion while it gives inferior results for deformation of relatively small magnitude but large proportion. Instead of data snooping, M-estimator is used to robustify the matching procedure by Pilgrim (1991,1996) for the purpose of detecting surface difference. It is reported that the robust matching procedure is capable of detecting difference covering up to 25% of surfaces being matched.

Surface deformation can be of different forms. One form is that only parts of one surface differ from their corresponding parts of the other surface and the rest parts are identical to their corresponding parts expect for random errors. We call it local deformation this case of the difference between two surfaces. Nevertheless, surface difference could also be throughout the whole surface, i.e. global deformation. In this paper we discuss only the detection of local deformation.

2 SURFACE MATCHING

2.1 Review of Matching Methods

As mentioned above, both Karras et al. (1993) and Pilgrim (1991) used in their studies the surface matching method developed by Rosenholm and Torlegard (1988). However, to make the choice of matching method sensible, it is necessary to review various methods for surface matching and analyze their characters.

According to the abstraction level of information used for matching, matching method could be divided into two categories [Zhang 1994]: primitive-based approach and surface-based approach. Similar classification of matching methods is given by Nevatia (1996), in which primitive-based approach corresponds to feature-based method and structure-based method. The so-called structure-based method is characterized by considering not only individual feature properties but also some explicit relations between them and by employing graph matching techniques.

In the primitive-based approach, features in the sense of differential properties are first extracted and then correspondences between features are established through comparing their properties and finally transformation parameters are computed using these correspondences generally by least squares method. Features used can be points of maximal curvature (Goldgof et al.1988), contours characterizing surface variation (Rodriguz and Aggarwal 1990) (Stein and Medioni 1993) and surface patches acquired by segmenting surface according to sign variation of curvature (Lee and Milios 1990) (Fan 1990).

Generally speaking, the primitive-based matching approach is capable of dealing with large orientation differences between surfaces and is preferable for the purpose of object recognition [Zhang 1994] due to the concise information used for matching, which are invariant to orientation differences. However, the large amount of process needed for feature extraction and correspondence building makes them computationally inefficient. Further, since the procedure of extracting features is highly sensitive to random errors and the sudden variations of surface, the primitive-based approach yields only rough estimates of transformation parameters. What's more, feature-based approach relies on existence of distinctive features and may be impractical for smooth surfaces. The structure-based approach seems to be restricted to recognition of objects whose surface can be decomposed into planar and quadric patches.

Complexity of the matching task can depend greatly on what constraints from prior knowledge can be placed [Nevatie 1996]. This gives room for surface-based approach. Given that the orientation difference between the two images to be matched is small, match could be achieved by finite trials of simply moving one image relatively to the other and measuring their coincidence by some measurements. This is the basic idea of correlation matching method for intensity images. Such idea is not directly applicable to surface matching where the moving has six degrees of freedom, each three for rotation and translation, much more than two degrees of freedom considered in the case of intensity image matching. Reduction of degrees of freedom involves use of differential information like the feature-based approach.

In the correlation matching method, one surface is artificially moved around to match with the other. A procedure of automatically and directly moving to solution can serve as a substitute for artificial movement if some objective function of minimizing the distances between the two surface is imposed on the matching process as the power of movement. The energy function derived from a data compatibility constraint and a smoothness constraint is used in the work of Szeliski (1988). Sum of distances between temporarily paired points is minimized in the ICP algorithm developed independently by the Besland Mckay (1992), Zhang (1994) and Chen and Medioni (1992). Sum of Z coordinate differences between pairs of points, which lie on the two surfaces but are of the same (x, y) coordinates, was minimized in [Rosenholm and Torlegard 1988]. (For briefness, we call this method the LZD method). For other similar methods, refers to [Besl and Mckay 1992] and [Zhang 1994]. We call these matching methods least squares matching based on sample points for they are usually formulated as a least squares problem and they use directly the coordinates of all sample points of DEM or range data.

Two obvious benefits of least squares matching based on sample points are their efficiency and precision: no process for extracting features is needed; no reduction of precision will be caused by using derived information in place of original, precise information; all information are used and the redundancy is rather high. The main disadvantage is that they are only suitable for matching surfaces between which the orientation difference is small. However, we think the orientation difference can usually be limited within a small range in most applications of surface difference detection by making the surfaces of concern approximately orthogonal to range data acquiring systems without or with little extra work. Therefore, such methods are preferred in our study. Another benefit of least squares matching methods, as will be seen in the following sections, is that they can be readily reformed in order to detect surface difference.

2.2 The LZD surface matching Algorithm

The ICP and the LZD matching algorithms are employed in our study for the reasons given above. And the LZD algorithm gave better results than ICP did. Therefore only LZD algorithm is introduced here.

Resenholm and Torleghed (1988) developed the LZD algorithm originally for the purpose of absolute orientation of stereo models while the LZD algorithm is generally applicable for DEM matching.

Let $Z = F(X, Y)$ and $z = f(x, y)$ be the two DEMs to be matched and $\boldsymbol{P} = [\,X\ Y\ Z\,]^T$ and $\boldsymbol{P}' = [\,x\ y\ z\,]^T$ are a pair of corresponding points. The following equation holds:

$$[X \quad Y \quad Z] = R[x \quad y \quad z] + t \tag{1}$$

where $\boldsymbol{R}$ (rotation matrix) and $\boldsymbol{t}$ (the translation vector) define the transformation from $z = f(x, y)$ to $Z = F(X, Y)$. For a point $\boldsymbol{P}'$, $[X_0\ Y_0\ Z_0]^T$ is assigned to be the approximate corresponding point, of which $X_0 = x$, $Y_0 = y$ and Z_0 is interpolated from $Z=F(X,Y)$. Let $X = X_0 + \Delta X$, $Y = Y_0 + \Delta Y$ and $Z = Z_0 + \Delta Z$. Supposing the DEM $Z = F(X,Y)$ is continuous, we have approximately

$$\Delta Z = \frac{\partial F}{\partial X}\Delta X + \frac{\partial F}{\partial Y}\Delta Y \tag{2}$$

Let ω, ϕ and κ denote rotations around X, Y and Z axis respectively and $\boldsymbol{t} = [\,t_x\ t_y\ t_z\,]^T$. Linearization of equation (1) at $\omega = \phi = \kappa = 0$, $t_x = t_y = t_z = 0$ gives:

$$\begin{aligned} X_0 + \Delta X &= x + dt_x - yd\kappa + zd\phi \\ Y_0 + \Delta Y &= y + dt_y + xd\kappa - zd\omega \\ Z_0 + \Delta Z &= z + dt_z - xd\phi + yd\omega \end{aligned} \tag{3}$$

Since $X_0 = x$ and $X_0 = y$, the first two portions of (3) become:

$$\begin{aligned} \Delta X &= dt_x - yd\kappa + zd\phi \\ \Delta Y &= dt_y + xd\kappa - zd\omega \end{aligned} \tag{4}$$

From equation (2), (3) and (4) we obtain the observation equation for a pair of corresponding points:

$$\delta = Z_0 - z + (dt_x - yd\kappa + zd\phi)\frac{\partial F}{\partial X} + (dt_y + xd\kappa - zd\omega)\frac{\partial F}{\partial Y} - dt_z + xd\phi - yd\omega \tag{5}$$

where δ is the matching residual. The following objective function is minimized to yield estimates of transformation parameters that bring the two DEMs closer:

$$F(R,t) = \sum_{j=1}^{n} p_j \delta_j^2 \tag{5.1}$$

The weight p_j is introduced to incorporating robust estimators as will be seen later. The partial derivatives needed in equation (5) are approximated by slopes over two DEM grids.

3. DETECTION OF LOCAL DEFORMATION

As analyzed in the introduction, the essential task of detecting surface difference without control points is to match the two DEMs precisely. The LZD algorithm is able to do so only if the two DEMs are identical except for random errors. In the cases that the two DEMs suffer from local deformation, if they are precisely matched according to their identical parts, then the matching residuals over all the surface (i.e. δ in (5) for the LZD algorithm after the matching procedure converges) can be divided into two categories: those coming from the identical parts, denoted by R_I, and those coming from non-identical parts, denoted by R_D. In general, elements in R_I can be assumed to obey the same normal distribution N(0, σ).The mean of elements in R_D would usually differ significantly from zero and the standard deviation would be large to some extent. When R_I is mixed with R_D, elements in R_D could be therefore considered as outliers, or gross errors, in R_I. This brings to us the idea that local deformation could be detected as outliers in the observations of matching process.

3.1 Handling Outliers

There are two major approaches for dealing with outliers: outlier detection and robust estimation. In the outlier detection approach, one first tries to detect (and remove) outliers and then perform estimation with the “clean” observations. Many statistics based on residuals (usually resulting from least squares adjustment) are designed to measure the “outlyingness” of observations. The most popular statistics are the normalized residual developed by Baarda (1968), which is applicable in the case that reference variance (variance of observation of unit weight) is known a priori, and the studentized residual proposed by Pope (1976), which is applicable in situations when reference variance is estimated from residuals. Statistical testing procedure using either of these two statistics is known as data snooping. Given a desired level of significance, a constant is determined and the observation, of which the normalized residual in absolute value is larger than this constant, is detected as an outlier. Lower bound of gross error detectable with a given probability can be estimated [Forstner 1986]. However, data snooping is only applicable under the assumption that no more than one outlier is present. Although, statistics for multiple outlier detection do exist, they are not practical because we do not know a priori how many outliers are in presence and where they are. Practical strategies for multiple outlier detection seem to fall into two categories: first, iteratively performing single-case outlier detection, of which a typical example is the iterative data snooping proposed by Kok (1984); second, combining outlier detection techniques with robust estimation, of which a typical example is the data snooping based on residuals resulting from L_1 (norm) estimation proposed by Gao et. al. (1992). If outlier detection techniques are to be used for our purpose of detecting local deformation of DEMs, a computationally efficient strategy is desired since local deformation of DEMs means hundreds to thousands of outliers to the matching process. We will return to this subject later.

The robust approach employs estimators relatively insensitive to outliers to produce reliable estimates. Outliers could be discovered as those having large residuals resulting from the robust estimation. Two important concepts related with a robust estimator are the breakdown point and the relative efficiency. The breakdown point of an estimator is the smallest fraction of outlier contamination that can cause the estimates arbitrarily biased [Rousseeuw and lLeroy 1987]. For example, the breakdown point of least squares estimator is 0 since a single outlier can (but does not necessarily) make the result invalid. As another example, the median as a one-dimensional location estimator has a breakdown point of 0.5, the highest achievable value. The relative efficiency of an estimator is defined as the ratio between the lowest achievable variance (the Cramer-Rao bound) for the estimated parameters and the actual variance provided by the estimator [Meer et. al. 1991] [Huang 1990]. For examples, LS estimator has a relative efficiency of 1 in the presence of normally distributed errors and the median as a one-dimensional location estimator has a relative efficiency of 0.637. If we use a certain robust estimator to robustify the LZD algorithm for the purpose of detecting local deformation, the breakdown point and relative efficiency of the estimator are of great concern for the following two reasons. The breakdown point gives a referential value of the largest deformation percentage that can be treated like this since a larger deformation percentage could make the estimated parameters unreliable. We call this percentage the largest detectable deformation percentage. In addition, the higher the relative efficiency of the estimator is, the more precise the estimated parameters could be and therefore the smaller the size of detectable local deformation could be. It should be noted that there is a trade-off between estimators with high breakdown points and those with high efficiency [Kumar and Hanson 1994] [Huang 1990].

3.2 Robust Matching Algorithms

Three robust estimators are employed to robustify the LZD matching algorithms. The first one belongs to the M-estimators developed by Huber (1981) and other statisticians [Hampel et al. 1986]. M-estimators minimize the sum of a symmetric, positive-definite function of residuals, $\rho(r_i)$, which has a unique minimum at $r_i = 0$. The LS estimator is a special case of M-estimators, where $\rho(r_i) = r_i^2$. To be robust, the $\psi(r_i)$ function of an M-estimator, which is the derivative of $\rho(r_i)$ with respect to r_i, must be bounded. To obtain high efficiency, $\psi(r_i)$ should be linear in the range near where $r_i = 0$. For computational consideration (i.e. to be scale equivariant), residuals used in M-estimators must be standardized by means of some estimate of standard deviation of observations. Many M-estimators are constructed to be robust on the one hand and fairly efficient on the other hand, such as Huber's estimator, Hampel's three-part-redescending estimator and Turkey's biweight estimator (see Hampel et. al. 1986). These M-estimators have high efficiencies, typically more than 0.9 [Kumar and Hanson 1994] [Huang 1990]. As a result, if the matching algorithms are robustified with these M-estimators, we are likely to detect local deformation of small size. However, M-estimators have breakdown points less than $1/(p+1)$, where p is the number of parameters to be estimated [Meer et. al. 1991] (Rigorously, M-estimators have breakdown points of 0 in presence of high leverage outliers [Rousseeuw and Leroy 1987]. We assume no such outliers exist in our surface matching process. This assumption is experimentally shown to be satisfied.). In our case of surface matching, p is 6 and the breakdown point is consequently less than 1/7($\approx$0.143). As we will see, this upper bound is a little bit exceeded and we will analyze the reason. In computation, M-estimators are usually converted to iterations of reweighted least squares, where weights are determined as $w(u) = \psi(u)/u$ and are updated during iterations according to the updated residuals and estimate of standard deviation of observations.

We use Turkey's biweight estimator and the associated weight function is:

$$w(u) = \begin{cases} (1-(u/c)^2)^2 & |u| \le c \\ 0 & |u| > c \end{cases} \qquad (6)$$

where $u_i = r_i/\sigma$ and c is a tuning constant and σ is standard deviation of observations. For M-estimators, it is important to use a robust estimate of σ. In our study, σ is computed as

$$\hat{\sigma} = 1.4826 \ med|r_i|$$

which is fairly robust with a breakdown point of 0.5 [Rousseeuw and Leray 1987]. The constant 1.4826 is used to achieve consistency at normal error distribution. To make the robust estimator incorporated with the LZD matching algorithms, the only change required is to replace the weight p_j in equation (5.1) by $p_j \times w(u_j)$. The resultant matching algorithms are called LZD_m.

The second robust estimator employed is a combination of M-estimator and data snooping. An important consideration in data snooping is that one should look not just for large residuals, but for residuals that are large in comparison to their own standard deviations [Schwarz and Kok 1993], i.e. the redundancy numbers of individual observations should be taken into account. Noting this, we replace u_i in equation (6) by $u_i=r_i/\sigma_{ri}$, which is the Baarda's statistic. The resultant estimator belongs to the GM-estimators (generalized M-estimators) [Rousseeuw and Leroy 1987] which hope to bound the influence of leverage points. This estimator is here called MD estimator. Experiments on simple regression problem [Rousseeuw and Leroy 1987] showed that GM-estimators tolerate more outliers than M-estimators do. From a different point of view, the MD estimator is similar to the iterative data snooping [Kok 1984] while they differ in two aspects. First, more than one observations can be identified as potential outliers after each iteration. This is necessary for detecting local deformation of DEMs where there could be too many outliers that deleting only one of them after each iteration makes the procedure computationally infeasible. Second, potential outliers are not deleted permanently but are less weighted temporarily. Incorporating the MD estimator with the LZD algorithm, we call the resulting matching algorithms LZD_md.

The last robust estimator employed is the LMS (least median of squares) estimator developed by estimator Rousseeuw and Leroy (1987), which is given by

$$\text{Minimize} \ \underset{i}{med} \ r_i^2 \qquad (7)$$

That is, the estimates must yield the smallest value for the median of squared residuals computed for all observations. The LMS estimator has a breakdown point of 0.5, the highest possible value, which is desirable for our application. The high breakdown point of the LMS estimator results from the that the estimates need only to fit well half of the observations and ignore the other half. However, its relative efficiency is abnormally low. This can be intuitively understood by noting that half of the observations have no actual influence on estimates. Actually, the LMS estimator converges as $n^{-1/3}$ and is not asymptotically normal. To compensate for this deficiency, it is proposed to perform an M-estimation or a one-step weighted least squares using the LMS estimates as starting values. In our study, we use the same weight function in equation (6) to perform a successive M-estimation. As we will see, this compensation works only if some condition is fulfilled. Robustifying the LZD algorithms with the LMS estimator, we call the resultant matching algorithms LZD_lms.

We use the random sampling algorithm to resolve the LMS problem given by (7). The algorithm proceeds by repeatedly drawing subsamples of p different observations. In most cases of our experiments, we let p=7. For such a subsample, we compute the 6 transformation parameters using LS estimation and denote the solution by Θ_i (trial estimate). For each Θ_i, we determine the objective function with respect to all available observations, i.e. the value $med\ r_i^2$ is calculated. Finally, the trial estimate for which this value is minimal is retained as the LMS estimate. In principle, we should make trials with all possible subsamples, of which there are C_n^p (n is number of observations). However, this is infeasible for our application. The random sampling technique is therefore introduced. The basic idea is that one performs a certain number (m) of random selections such that the probability that as least one of the m subsamples contains no outliers is almost 1. The expression of this probability is

$$1-(1-(1-\varepsilon)^p)^m \qquad (8)$$

where ε is the contamination fraction and ε is at most 0.5. In most of our experiments, with ε=0.5 and p=7, we let m=600 and therefore the above probability is approximately 0.991.

3.3 Experiments and Analysis

We now test the capabilities of the robust matching algorithms developed above in detecting local deformation. The surface examined here is a 50×50 grid DEM, called Pulse, as shown in figure 1. Random errors and deformation quantities are added to the Z coordinates of Pulse to generate a simulated DEM with local deformation relative to the original one. Then the simulated DEM is rotated and translated by a parameter vector Θ. The robust matching algorithms are required to compute the parameter vector accurately.

Several matters about our experiments are explained before we present experiment results. Since local deformation with various sizes, shape, deformation percentage and position may occur in practice, we can not simulate all the possible cases. The simulated deformation quantities in our experiments take the form of random errors distributing as N(c, σ) while the added random errors distribute as N(0, σ). With varying c, local deformation of different size can be simulated. When c is as small as only several times of σ, the simulated local deformation is of very flat shape and small size. The spatial distribution of local deformation is always grouped together within a square area and the local deformation can locate at any position within planar range of Pulse.

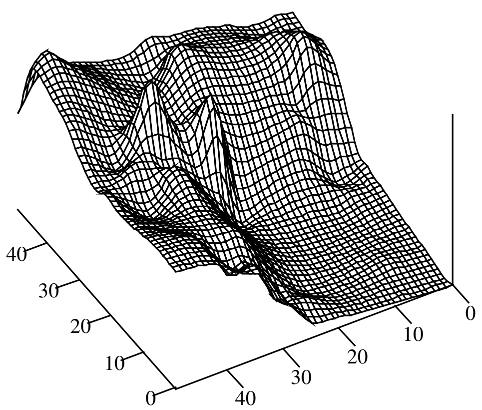

Figure 1. The Pulse surface

To decide whether a robust matching algorithm computes the transformation parameters accurately, we set some thresholds as 3.5 times of standard deviations of estimated parameters obtained from deformation-free matching cases. If the error of any computed parameter exceeds the associated threshold, the matching process is decided to have failed. The loose thresholds are used rather than more restrictive ones, such as 2.5 or 3.0 times of standard deviations, because most robust estimators are not as efficient as least squares estimator at normal error distribution.

Finally, it should be noted that we should perform robust matching after the LZD matching algorithm converges. Since at the beginning of matching process the distances or Z differences between the temporarily paired points are caused almost totally by orientation difference not by random errors and local deformation, it is senseless to detect outliers in this stage.

Now we inspect a matching case with local deformation in presence. In this case, the Pulse DEM is added with random errors as N(0,20) and local deformation as N(200,20) to generate a deformed version of Pulse surface. The deformation percentage is 9% and the local deformation lies at the up-right corner (in the XY plane) of Pulse. The deformed DEM is then rotated and translated. Table 1 shows the true values of transformation parameters and their estimates computed by different versions of LZD algorithm in a set of tests. Estimates from the conventional LZD algorithm are significantly affected by the local deformation, i.e. the deformed DEM is tilted and elevated to match well with the Pulse. All the three robust versions of LZD algorithm yield reasonable results in spite of the presence of local deformation. The success of LZD_m algorithm in this case shows that local deformation causes no outliers of high leverage to the matching process.

Table 1 The True Values of Transformation Parameters and Their Estimates Computed by Different Versions of LZD Algorithms

	ω	ϕ	κ	t_x	t_y	t_z
True values	5° 00′ 00″	5° 00′ 00″	5° 00′ 00″	500	500	500
LZD	4° 43′ 51″	4° 46′ 17″	5° 08′ 32″	491.6	493.8	507.7
LZD_m	5° 01′ 36″	4° 57′ 40″	5° 02′ 18″	498.4	503.9	500.9
LZD_md	5° 01′ 43″	4° 58′ 02″	5° 02′ 10″	497.7	503.1	500.7
LZD_lms	5° 01′ 44″	4° 57′ 51″	5° 01′ 39″	498.2	503.3	500.6

e ~ N(0,20), d ~ N(200,20) ε =9%

As the size, percentage and position of local deformation change, the three robustified LZD algorithms behave quite differently. The deformation percentage plays a dominant role in making these robust matching algorithms effective or not. In most cases that deformation percentage is over 16%, the LZD_m algorithm fails. This results from fact that M-estimators have low breakdown point for our applications (less than 0.143) as pointed out above, although the upper bound of 0.143 is exceeded a little. Note that the definition of breakdown point is needed to be theoretically rigorous while in practice outliers of smallest contamination fraction that can cause estimates arbitrarily unbiased would be unlikely to occur in some situations. In fact, this is why the upper bound of breakdown point is exceeded in our application since the outliers caused by local deformation would not be arbitrarily large. The LZD_md algorithm always succeeds in cases that deformation percentage is less than 22%. The better performance of LZD_md compared with LZS_m is of course a result of using the Baarda's statistic since this is the only difference between them. The LZD_lms algorithm performs reasonably well even if the deformation percentage is as large as 40%. We will return to the LZD_lms algorithm later.

Both the LZD_m and LZD_md algorithms are sensitive to the position of local deformation. The more far the local deformation locates from the center (in the XY plane) of Pulse DEM, the less the largest detectable deformation percentage of LZD_m and LZD_md are. The largest detectable deformation percentages of LZD_m and LZD_md are both about 32% if the local deformation locates at center of Pulse DEM. This can be understood by noting that observations far from geometric center of observations, which have therefore lower redundancy numbers, contribute more to estimates than those near the center, which therefore have higher redundancy numbers. The LZD_lms algorithm is not sensitive to the position of local deformation because the LMS estimator does not faver leverage observations (observation of abnormally low redundancy number) at all (see [Rousseeuw and Leroy 1987]).

Table 2 Smallest Detectable Size of Local Deformation at Different Deformation Percentage

Deformation Percentage	9%	16%	20%	22%	25%	36%	42%	46%	49%
LZD_m	3.5 σ	4 σ							
LZD_md	3 σ	3.5 σ	4 σ	5 σ					
LZD-lms	3 σ	3.5 σ	3.5 σ	3.5 σ	4 σ	7 σ	10 σ	15 σ	25 σ

The last factor affecting the effectiveness of these robust matching algorithms is the size of local deformation. Experiments shows that all the three robust versions of LZD algorithm yield accurate estimates of transformation parameters in presence of local deformation of large size. What happens if we decrease the size of local deformation? Given the standard deviation of random errors (σ), the deformation percentage (ε) and the distribution of deformation quantities as N(c, σ), we call c the smallest detectable size of local deformation of a robust matching algorithm such that a smaller c will make the robust matching algorithm fail. Through experiments at different level of random errors (i.e. σ =5, 10, 20), we find out the overall smallest detectable size of local deformation (as shown in Table 2) of the three robust versions of LZD algorithm.

As we can see, when the deformation percentages are no larger than their own largest detectable deformation percentages, the LZD_m and LZD_md are both able to deal with local deformation of very small size. This is attributed to the high relative efficiencies of M-estimators, the extremely high redundancy we have, and the good geometry of observations (i.e., observations are nearly evenly distributed in space). It can also be noticed that the smallest detectable size increases with the deformation percentage. At a given deformation percentage, the smallest detectable size of LZD_md algorithm is smaller than that of the LZD_m algorithm. This is another result of using the Baarda's statistic which approximately takes into account the spatial positions of individual observations.

The performance of the LZD_lms algorithm varies with respect to the combined effect of deformation percentage and the deformation size. When the deformation percentage is less than 36%, the LZD_lms algorithm is quite well behaved. With the robust initial estimates provided by the LMS estimator, the M-estimator does improve greatly the relative efficiency of the LMS estimator. Therefore, the LZD_lms algorithm is superior to the LZD_m and the LZD_md algorithms even in the cases that local deformation is of small size. However, as the deformation percentage is reaching to 50%, the smallest detectable size grows acutely. This is due to the fact that the M-estimator improves the efficiency of the LMS estimator only if the LMS estimator does not break down. In cases that the deformation percentage is near 50% and the size of local deformation is small enough, we find that subsamples consisting of non-outliers generated by the random sampling procedure of the LZD_lms algorithm can not yield a $med\ r_i^2$ (the value of objective function) less than that resulting from the conventional LZD algorithm, i.e. the LMS estimator breaks down.

Conclusively, the LZD_lms algorithm performs better than the LZD_m and the LZD_md algorithms and it is able to precisely match two DEMs differing by local deformation of large percentage (up to 50%) but small size (for details, see Table 2).

4. CONCLUSIONS AND DISCUSSIONS

We have addressed the problem of detecting local deformation between two surfaces without the aid of control points. This problem is resolved by means of robust surface matching in our study. The LZD algoritm are employed for their high precision and computational efficiency.

We have developed several robust matching algorithms, employing an M-estimator, a combination of M-estimator and the data snooping technique and a combination of LMS estimator and M-estimator. The differences between robust estimators in breakdown point and relative efficiency result in the different performances of the several robust matching algorithms. The LZD_lms is able to detect local deformation covering up to 50% of the surfaces being matched. If deformation percentage is less than 40%, local deformaiton of very small size can be detected. We believe the LZD_lms developed here are suitable for many applications of detecting surface difference.

It seems that the percentage of 50% is the upper bound of largest detectable deformation percentage for any matching process which employs some robust estimator to detect simultaneously surface difference, because 0.5 is the highest achievable breakdown point. Stewart (1995) developed a new robust estimator, called MINPRAN, which can tolerate more than 50% outliers, by assuming that outliers are uniformly distributed. The suitability of this estimator for our purpose needs to be investigated. The problem would be whether outliers, which in our case are caused by local deformation, can be assumed to distribute uniformly. To improve the performance of the LZD_lms algorithm in cases that deformation percentage is near 50%, one may apply the so-called S-estimators [Rousseeuw and Leroy 1987], which have relative efficiency much higher than that of the LMS estimator and are still as robust as the latter. However, this would result in great increase of computation.

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Reference

Baarda, W.: 1968, A test procedure for use in geodetic networks, *Netherlands Geodetic Commission, New Series*, Delft, Netherlands, 2(5).

Besl, P.J. nd mckay, N.D.: 1992, A method for registration of 3-D shape, *IEEE Trans on PAMI* 14(2).

Chen, Y. and medioni, G.: 1992,Object modeling by registration of multiple range images, *Image and Vision Computing,* 10(3):145-155.

Fan, T.J.: 1990, describing and recognizing 3-D objects using surface properties, *Springer-Verlag*, New York.

Forster, W.:1986, Reliability, gross error detection and self-calibration, ISPRS Commission III Tutorial on Statistical Concepts for Quality Control, *International Archives of Photogrammetry* 26(3/4)1,34p.

Fraser, C.S.: 1988, Periodic inspection of industrial tooling by Photogrammetry, *Photogrammetry Engineering and remote Sensing*, 54(2), 211-216.

Gao, Y., Krakiwsky, E.J. and Czompo, J.: 1992, Robust testing procedure for detection of multiple blunders, *Journal of Surveying Engineering* 118(1), Feb. 1992, 11-23.

Goldgof, D.B., Huang, T.S. and Lee, H.:1988, Feature extraction and terrain matching, *IEEE Proceeding CSC on CVPR*. 899-904.

Hample, f.R., ronchetti, E.M., Rousseeuw, P.J., and Stahel, W.A.:1986, *Robust Statistics: An Approach Based on Influence Functions*, Wiley, New York.

Huang, Y.C.: 1990, *Data Snooping and Robust Estimation* (in Chinese), Press of Surveying and Mapping, Peking, China.

Huber, P.J.: 1981, *Robust Statistics,* Wiley, New York.

Karras, G.E. and Pesta, E.: 1993, DEM matching and detection of deformation in close-range photogrammetry without control, *Photogrammetric Engineering and Remote Sensing* 59(9).

Kok, J.J.: 1984, On data snooping and multiple outlier testing, *NOAA Technical Report NOS NGS 30,* National Geodetic survey, Rockville, MD.

Kumar, R. and Hanson, A.R.: 1994, Robust method for estimating pose and a sensitivity analysis, *CVGIP: image Understanding* 60(3), Nov. 1994, 313-342.

Lee. J.C. and Milios, E.: 1990, Matching range image of human faces, *IEEE Proceeding 3rd International Conference on Computer Vision.* 722-725

Li, D.R.: 1988, *Error processing and Reliability Theory* (in Chinese), Press of Surveying and Mapping, Peking, China.

Meer, p., Mintz, D. and Rosenfeld, A.: 1991, Robust regression method for computer vision: a review, Int. J. of Computer Vision 6(1), 57-70.

Nevatia, R.: 1996, matching in 2-D and 3-D, *International Archives of Photogrammatry and Remote Sensing*, Vol.XXXI Part B3, 567-574.

Pilgrim, L.J.: 1991, Simultaneous three dimensional object matching and surface difference detection in a minimally restrained envirnment, *Research Report, University of Newcastle*.

Pilgrim, L.J.:1996, surface matching and difference detection without the aid of control points, *Survey Review* 33(2)

Rodríguez, J.J. and Aggarwal, J.K.: 1990, Terrain matching by analysis of aerial images, *IEEE Proceedings 3rd International Conference on Computer Vision.*

Rosenholm, D. and Torlegard, K.: 1989, Three dimensional absolute orientation of stereo models using digital elevation models, *Photogrammetric Engineering and Remote Sensing* 54(10).

Rousseeuw, P.J. and Leroy, A.M.: 1987, Robust regression and outlier detection, Wiley, New York.

Schwarz, C.R. and Kok, J.J.: 1993, Blunder detection and data snooping in LS and adjustments, *Journal of Surveying engineering* 119(4), Nov 1993, 127-136.

Stein, F. and Medioni: 1990, TOSS---a system for efficient three dimensional object recognition, *Proceeding DAPRA Image Understanding Workshop*.

Stewart, C.V.:1995, MINPRAN: anew robust estimator for computer vision, *IEEE Trans on PRMA* 17(10), 925-938.

Szaliski, R.:1988, Estimating motion from sparse range data without correspondence, *Proceedings of 2nd Int. Conf. Computer Vision*, DEC.1988, 207-216.

Wetch, R. and Jordan, T.R.:1983, Analytical non-metric close-range photogrammetry for monitoring stream channel erotion, *Photogrammetric Engineering and Remote Sensing* 49(3), Mar 1983, 367-374.

Zhang, Z.Y.:1994, Iterative point matching for registration of free-form curves and surfaces, *Int. J. of Computer Vision* 13(2), 119-152.