GENERALIZED MATHEMATICAL MODEL OF PRECISE PHOTOGRAMMETRY RECONSTRUCTION OF THE OBJECTS.

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ABSTRACT

The research presents analysis of photogrammetry data and adjustment methods, the classification scheme of mathematic processing of data in photogrammetry. The authors shaped generalized mathematic model which is universal for photogrammetry problems. Here is enlighted the theoretical solution of joint adjustment of measured quantities, their functions and control data. There are some examples of partial cases proceeding from generalized mathematical model. The authors proves that the notion of the function of loses provides with the possibility to process series of measurement with the errors division different from Gaussian law. The article provides with the generalized model of phototheodolite survey and the theoretical solution of the adjustment task when control points and linear elements of exterior orientation are known with high and almost equal accuracy (as in case with the use of GPS observation). The presented mathematical models may be used to create new algorithms, software, for extension of functional abilities of software for analytical photogrammetric equipment and digital stations.

INTRODUCTIONS

The integration of data received from various GIS sources causes the importance of fundamental task of photogrammetry, namely the reconstruction of objects and measuring its metric parameters by its photoimage. Various methods and techniques are applied to process images received in the result of stereophotogrammetric survey. The methods of the digital photogrammetry on the stage of geometrical construction of model and analyzing results of measuring apply in full the apparatus of analytical photogrammetry. Thus nowdays it still significant to elaborate research the models and algorithms for mathematical processing of photogrammetric data.

The authors have already elaborated a set of such models [3-8]. In this research the following problems are put for the discussion: Classification scheme of methods of mathematical data processing in photogrammetry, Generalized mathematical model of adjustment in photogrammetry, some partial mathematical models, mathematical model of adjustment in phototheodolite survey.
Let us discuss in detail the problems mentioned above.

1. CLASSIFICATION OF METHODS OF MATHEMATICAL PROCESSING IN PHOTOGRAMMETRY PROBLRMS

The analysis of photogrammetry data and methods of adjustment permits to make a scheme containing main characteristics of series of measured quantities, extra data and geometrical conditions that appear in different photogrammetry tasks (Fig.1).
We are going to concentrate on some explanation of this scheme, putting aside the classical points that do not need discussion.

1.1 Photogrammetry measuring suggests quantities measured directly on photos. Usually they are flat right-angled coordinate of point. When the mathematical processing is carried on the results are usually adjusted according to the classical scheme.

1Type of values subjecte. d to adjustment	**1.1** Photogrammetrical servey
	1.2 Function of Photogram metrical survey
	1.3 Control data
⇓	
2. Errors data nature	**2.1** Just occasional
	2.2 Occasional and systematical
	2.3 No errors or they are disdainfully small
⇓	
3.The possibility of including the systematic errors to adjustment	**3.1** YES
	3.2 NO
⇓	
4.Function of losses	**4.1** Square
	4.2 Unsquare
	4.3 Mixed
⇓	
5. Is the codispersive matrix known	**5.1** YES
	5.2 NO
⇓	
6. The matrix weight of photogrammetrycs data known	**6.1** YES
	6.2 NO
⇓	
7. The existance of additional geometrical conditions	**7.1** YES
	7.2 NO

PHOTOGRAMMETRIC PROBLEMS

1. Partial calibrating of the photographs
2. Complete calibrating
3. Self calibrating
4. Reverse photogrammetric intersection
5. Double photogrammetric intersection
6. Direct repeated intersection
7. Relative orientation of a couple of the photographs
8. Relative orientation of three photographs
9. Phototriangulation as to the Join-method
10. Phototriangulation as to the method of the models
11. Formation of quasiphotograph
12. Geodetic orientation of the route model
13. Geodetic orientation of the model (the couple of photographs)
14. Research stereophotogram-metrical devises

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Pic.1. The choice of the model for mathematical processing photogrammetry survey

1.2 The space photogrammetry coordinates of point of model are considered to be the functions of 1.1-type quantities. If one includes them in the adjustment he should use well-known theorem [1] to get strong theoretical solution. The theorem [1] states that if there is adjustment of F - functions of correlated measuring Y and the matrix is introduced

$$Q = \alpha \, \Sigma_Y \, \alpha', \tag{1.1}$$

then the result gained will be identical with the adjustment of values measured directly.

Here $$\alpha = \frac{\partial F}{\partial Y}, \quad F = F(Y), \tag{1.2}$$

$\sum_Y$ - covariant matrix of measured values

1.3 Bearing data include the following data classes: elements of projection; elements of geodetic orientation of photography (linear and angular); coordinates of the control points; geodetic survey (angles, directions, length of lines, exceeding and so on); size and forms of objects and some others.
A separate equation is set for each type of data. This equation later is introduced into the general system of equations in the process of adjustment.

4.1-4.2. The notion of the function of losses and notion of minimization of regressive remains is widely used in the regressive analysis [1]. The function of losses for the linear model of regressive function is as follows:

$$\rho\ (\varepsilon) = |\varepsilon|^{2+d}, \tag{1.3}$$

here ε - - regressive remains, d - parameter of unsquareness $(-1 < d \le 0)$.
If d = -1, the so-called robust-method of evaluation is applied. If d = 0, we got a square function of losses or in other words classical method of least squares.
The use of the notion of the function of losses provides with the chance to process the series of surveys with errors division different from Gaussian.

5.1-5.2. To use of the hypothesis about dependence (independence) of measuring equitation it is necessary to know (or neglect) the covariant matrix of errors of measuring. On the other hand, introducing covariant matrix $\sum_Y$ into mathematical model provides for logic and correct solution. This condition is used in the present research.

2. GENERALIZED MATHEMATIC MODEL OF PHOTOGRAMMETRY SURVEY AND ITS THEORETICAL SOLUTION

In this article we are discussing the general problem of joint adjustment of the measured quantity functions, control data and direct measuring in the following order:
Let us admit that there are set the following data: n-dimensional vector Y of measured quantities, which are free from systematical errors; its covariant matrix $\sum_{Y_1}$; r - values of functions are calculated.

$$T = F(Y_1) \tag{2.1}$$

It is necessary to make the adjustment with respect to r-conditions.

$$\Phi = \Phi(T', \overline{Y'}, U') = 0 \tag{2.2}$$

here $T', \overline{Y'}, U'$ – correspondingly means adjusted values of functions

$$T' = T + \Delta T, \tag{2.3}$$

control data $$\overline{Y'} = \overline{Y} + \gamma \tag{2.4}$$

additional unknown quantities $U' = U + \Delta U$ (2.5)

Besides , the measured quantities Y_2 with the covariant matrix $\sum_{Y_2}$ are connected by the equations of corrections with the vectors $\bar{Y}, U$, so that

$$\varepsilon_2 = -B_2\gamma - S\Delta U - W_{2.} \tag{2.6}$$

The covariant matrix $\sum_{\bar{Y}}$ of vector $\bar{Y}$ is known.

We consider the errors for Y_1 and Y_2 to be divided according to the Gauss law, and for $\bar{Y}$ - they differ a bit from the Gauss. The dimensions of all the vectors do not cause the indetermination of the linear equations system, that is the matrix possess the complete rang. Let us consider as well that the vectors Y_1 , Y_2 $\bar{Y}$ are correlated with one another.

As far as $$T' = T + \Delta T = F(Y_1 + \varepsilon_1 = F(Y_1) + \frac{\partial F}{\partial Y_1}\varepsilon_1 = T + \alpha\varepsilon_1, \tag{2.7}$$

Than $$\Delta T = \alpha\varepsilon_1 \tag{2.8}$$

When carrying out the linearization (2.2) we get

$$\frac{\partial \Phi}{\partial T}\Delta T + \frac{\partial \Phi}{\partial \overline{Y'}}\gamma + \frac{\partial \Phi}{\partial U}\Delta U + \Phi(T', \overline{Y'}, U') = 0,$$

or $$A\Delta T + B_1\gamma + C\Delta U + \omega_1 = 0. \tag{2.9}$$

Substituting expression (2.8) for ΔT taking into account (2.6), we get the initial system if equations

$$\begin{aligned} D\varepsilon_1 + B_1 + C\Delta U + \omega_1 = 0 \\ \varepsilon_2 + B_2\gamma + S\Delta U + \omega_2 = 0 \end{aligned} \tag{2.10}$$

where $$D = A\alpha \tag{2.11}$$

Let us make the adjustment for (2.10) under the condition of the minimization of function of losses , with

$$\rho\ (\varepsilon_1) = \varepsilon_1{}^2,\ \rho\ (\varepsilon_2) = \varepsilon_2{}^2, \rho\ (\gamma\ ) = |\gamma\ |^{2+d}\ , \tag{2.12}$$

where d - parameter of unsquareness.

Let us compose the function of Lagaran, for conditional equations (2.10) where we introduce the sum marked with Gauss symbols instead of $\gamma' \sum_{\bar{Y}}^{-1}\gamma$:

$$\Psi_\gamma = \left[p\gamma^{\ 2+d}\right] = p_1\gamma_1{}^{2+d} + p_2\gamma_2{}^{2+d} + \ldots + p_s{}^{2+d}\ , \tag{2.13}$$

$$\Psi = \varepsilon_1' \Sigma_{y_1}^{-1} + \varepsilon_2' \Sigma_{y_2}^{-1} + \Psi_\gamma - 2k_1\left(D\varepsilon_1 + B_1\gamma + C\Delta U + \omega_1\right) - 2k_2\left(D\varepsilon_1 + B_2\gamma + C\Delta U + \omega_2\right). \quad (2.14)$$

Thus we can get partial derivatives:

$$\frac{\partial\ \Psi}{\partial\ \varepsilon_1} = -2\varepsilon_1{}' \Sigma_{y_1}^{-1} - 2k_1 D = 0,$$

$$\frac{\partial\Psi}{\partial\varepsilon_2} = -2\varepsilon_2{}' \Sigma_{Y_2}^{-1} - 2k_2' = 0 \quad ,$$

$$\frac{\partial\ \Psi}{\partial\ U} = -2k_1 C - 2k_2 S = 0 \quad (2.15)$$

$$\frac{\partial\ \Psi}{\partial\ U} = \left(2 + d\right)\mathrm{B}\gamma\ ' \Sigma_{\bar{y}}^{-1} + \left(2 + d\right)\delta_\gamma \Sigma_{\bar{y}}^{-1} - 2k_1 B_1 - 2k_2 B_2 = 0,$$

$$\frac{\partial\ \Psi}{\partial\ \gamma} = \left(2 + d\right)\mathrm{B}\gamma\ ' \Sigma_{\bar{y}}^{-1} + \left(2 + d\right)\delta_\gamma \Sigma_{\bar{y}}^{-1} - 2k_1 B_1 - 2k_2 B_2 = 0,$$

where $$\delta_\gamma = \left[\gamma_1\ \Delta_1 - \gamma_2 \Delta_2 - \ldots - \gamma_s \Delta_s\right] \quad (2.16)$$

$$\Delta = \left|\gamma\ \right|^d - 1 \qquad or \qquad \Delta = \sum_{j=1}^{p} \frac{\left(d \ln|\gamma|\right)^j}{j\,!} \quad (2.17)$$

When solving (2.15) for ε_1' $, \varepsilon_2$ $, \gamma$, we get

$$\varepsilon_1 = \Sigma_{y_1} D' k_1,$$

$$\varepsilon_2 = \Sigma_{y_2} k_2,$$

$$\gamma = \frac{2}{2 + d} \sum_{\bar{y}} B_1' k_1 + \frac{2}{2 + d} \sum_{\bar{Y}} B_2 k_2 + \delta_\gamma . \quad (2.18)$$

On the base of (2.10) and (2.15) we get the system of the equations correlate

$$(D\Sigma_{Y_1} D' + \frac{2}{2 + d} \sum_{\bar{y}} B_1')k_1 + \frac{2}{2 + d} B_1 \sum_{\bar{Y}} B_2' k_2 + C\Delta U + \omega_1 + B_1\delta_\gamma = 0,$$

$$\frac{2}{2 + d} B_2 \sum_{\bar{Y}} B'_1 k_1 + (\sum_{y_2} + \frac{2}{2 + d} B_2')k_2 + C\Delta U + \omega_2 + B_2\delta_\gamma = 0. \quad (2.19)$$

The solution completed by means of the method of consecutive exclusion of unknown quantities leads to finding out the vector ΔU and correlate

$$\Delta U = -\left(C' M_{11} - S' R_{11}^{-1} R_{12}\right)^{-1} \left(C' M_{12} - S' R_{11}^{-1} R_{13}\right), \quad (2.20)$$

$$k_2 = -R_{11}^{-1}\left(R_{12}\Delta U + R_{13}\right), \tag{2.21}$$

$$k_1 = -N_{11}^{-1}N_{12}k_2 - N_{11}^{-1}C\Delta U - N_{11}^{-1}\overline{\omega_1} \tag{2.22}$$

Here are introduced the following symbols

$$N_{11} = D\Sigma_{Y_1}D' + \frac{2}{q}B_1\Sigma_{\overline{Y}}B_1', \; q = 2 + d, \qquad N_{12} = N_{21}' = \frac{2}{q}B_1\Sigma_{\overline{Y}}B_2', \qquad N_{22} = \left(\Sigma_{Y_2} + \frac{2}{q}\Sigma_{Y_2}B_2'\right),$$

$$\overline{\omega_1} = \omega_1 + B_1\delta_\gamma, \; \overline{\omega}_2 = \omega_2 + B_2\delta_\gamma, \qquad R_{12} = S - N_{21}N_{11}^{-2}C, \; R_{13} = \overline{\omega_2} - N_{21}N_{11}^{-1}\overline{\omega_1},$$

$$M_{11} = N_{11}^{-1}N_{12}R_{11}^{-1}R_{13} - N_{11}^{-1}\omega_1, \qquad R_{12} = -N_{21}N_{11}^{-1}N_{12} + N_{22}. \tag{2.23}$$

The covariant matrix of adjusted vector ΔU is equal to $\sum_{\Delta U} = \left(C'M_{11} - S'R_{11}^{-1}R_{12}\right).$ (2.24)

Basing on the task described we can formulate new variants of aerotriangulation. For example when one part of the net is constructed by the method of models and another part is constructed by the method of connections. Such interpretation makes it possible to fulfill in a new way the densibleness of the net limited by the couple of the photographs. Besides it is possible to construct models of locality and relief.

3. SOME PARTIAL PROBLEMS AND MODELS

Let us discuss some partial tasks that proceed from generalized model (2.10) and which are of practical interest for photogrammetry.

3.1.Measuring Y_2 was not made.

It leads the general statement to the task of joint adjustment of functions of measured quantities and the control data with the mixed division of errors. Instead the system (2.10) we get

$$D\varepsilon_1 + B\gamma + C\Delta U + \omega_1 = 0, \tag{3.1}$$
$$\left(\varepsilon_2 = B_2 = S = \omega_2 = \Sigma_{Y_2} = 0\right).$$

Then
$$\Delta U = -\left(C'N_{11}^{-1}C\right)^{-1}C'N_{11}^{-1}\overline{\omega_1}, \tag{3.2}$$

$$\sum_{\Delta u} = \left(C'N_{11}^{-1}C\right)^{-1} \tag{3.3}$$

3.2.The task is analogous to the previous but with the condition that for the errors ε_2 and γ the law of the division is normal (Gaussian).

Then d = 0 and
$$\Delta U = -\left(C'N_{11}^{-1}C\right)^{-1}C'N_{11}^{-1}\overline{\omega_1},$$
$$N_{11} = D\Sigma_{\varepsilon_1}D' + B_1\Sigma_\gamma B_1', \tag{3.4}$$
$$\overline{\omega}_1 = \omega_1.$$

$\sum_{\Delta U}$ - is expressed by the equation (3.3)

3.3. The initial equations are analogous to (2.10) but $B_2 = 0$ **.**
It means that the measuring for the control data was not made $(Y_2 = 0)$. Then we get

$$\Delta U = \left(C'N_{11}^{-1}C + S'\Sigma_{Y_2}^{-1}S\right)^{-1}(C'N_{11}^{-1}\overline{\omega_1} - S'\Sigma_{Y_2}^{-1}\omega_2),$$
$$\Sigma_{\Delta U} = \left(C'N_{11}^{-1}C + S'\Sigma_{Y_2}^{-1}S\right)^{-1}. \quad (3.5)$$

3.4. The task is analogous to the third, but the functions of losses for ε_1 , ε_2 **and** γ **are square.**

The solution will be as follows: $\Delta U = \left(C'N_{11}^{-1}C + S'\Sigma_{Y_2}^{-1}S\right)^{-1}(C'N_{11}^{-1}\overline{\omega_1} - S'\Sigma_{Y_2}^{-1}\omega_2).$ (3.6)

The matrix $\sum_{\Delta U}$ is analogous to (3.3).

3.5 It is necessary to fulfill the joint adjustment of the measured quantities functions under condition when the errors of control data can be neglected.

The initial system (2.10). Will be as follows:

$$D\varepsilon_1 + C\Delta U + \omega_1 = 0,$$
$$\varepsilon_2 + S\Delta U + \omega_2 = 0. \quad (3.7)$$

Here is $B_1 = B_2 = 0,$ $\sum_{\overline{Y}} = 0$ and

$$\Delta U = \left(C'N_{11}^{-1}C + S'\Sigma_{Y_2}^{-1}S\right)^{-1}(C'N_{11}^{-1}\overline{\omega_1} - S'\Sigma_{Y_2}^{-1}\omega_2),$$
$$\Sigma_{\Delta U} = \left(C'N_{11}^{-1}C + S'\Sigma_{Y_2}^{-1}S\right)^{-1}. \quad (3.8)$$

Where $N_{11} = D\Sigma_{Y_1}D'.$

3.6. If there is no measuring Y_2 **in task five we shall have the adjustment of the measured functions of quantities with the additional unknown quantities U.**
Then

$$\varepsilon_1 = S = \omega_2 = \Sigma_{Y_2} = 0,$$
$$\Delta U = -(C'(D\Sigma_{Y_1}D')^{-1}C)^{-1}(C'(D\Sigma_{Y_1}D')^{-1}\omega_1), \quad (3.9)$$
$$\Sigma_{\Delta U} = (C'(D\Sigma_{Y_1}D')^{-1}C)^{-1}.$$

The cases (1 - 6) described above do not restrict the list of the partial problems that can be gained by the change of coefficients A, D, B_2, B_1, C, S, correlation matrix. $\sum_{\overline{Y}}, \sum_{Y_1}, \sum_{Y2}$ and quantity d. It should be noticed that it is not diffucult to get formulas when the functions of losses for the vectors ε_1 and ε_2 are unsquare and are analogous to $\rho(\gamma)$ from (2.12).

4. MATHEMATICAL MODEL OF ADJUSTMENT IN PHOTOTHEODOLITE SURVEY

The problem of adjustment of data of phototheodolite survey is another partial case of generalized partial model.
The latter is based on the following data : photogrammetric surveys (monocular or stereoscopic) of terrestrial photograph, photo station coordinates fixed by different means with the special accent on the use of GPS observation, slope (Shift) angle of photos, space coordinates of control (correction) points, photography base data, ccorrection directions (vertical, horizontal).

The theory of photogrammetry provides for well-elaborated and described mathematical models, which admit that control data are accurately known. Besides it is almost always admitted that elements of the central projection are accurately known too, which means that making system gauge for phototheodolite survey is not executed. This is caused by the high metrics characteristics of surveying system in general.
We think that some practical and theoretical interest may be caused by other models for which control data are considered to be known with some certain preciseness and some corrections to photogrammetrical and control data are found in the process of these data adjustment.
Under such condition the mathematical model looks as follows:

$$
\begin{aligned}
\varepsilon &= -(B\delta S + C\delta\psi + D\delta\Gamma) + Y_\Phi, \text{ weight } P_\Phi \\
\gamma_S &= -\delta S + Y_S, \text{ weight } P_S \\
\gamma_\psi &= -\delta\psi + Y_\psi, \text{ weight } P\psi \\
\gamma_\Gamma &= -\delta\Gamma + Y_\Gamma, \text{ weight } P_\Gamma \qquad (4.1) \\
\gamma_? &= -B_?\delta S + Y_?, \text{ weight } P_? \\
\gamma_\beta &= -B_\beta\delta S - D_\beta\delta\Gamma + Y_\beta, \text{ weight } P_\beta
\end{aligned}
$$

where

γ_S - vector of corrections to measured coordinates of photo stations, γ_Γ -– vector of corrections to measured coordinates of control points, γ_ψ - vector of corrections to measured angle elements of exterior orientation,

Y , Y_S, Y_ψ , Y_Γ , $Y_?$, Y_β - correspondingly vectors of measurements: photogrammetrical, coordinates of photo stations, angle elements of exterior orientation, control points, basis, correction directions.

ε - vector of corrections to the measured quantities: $\delta E, \delta S, \delta\Psi, \delta\Gamma$ - vectors of corrections to elements of central projection, linear elements of exterior orientation (photo station coordinates), angle elements of exterior orientation, space coordinates of the point of the object.
P_i – – weights of corresponding measured quantities.

In this case it is necessary to explain two last equations out of [4.1] which are connected with the basis and correction directions.

The length of photography basis and its angle orientation is the function of the left and right centers of photography. Therefore this type of equation is always reduced to the model with correction $\gamma_?$, an absolute term $Y_?$ and matrix of partial derivatives $B_?$.

The same situation is noticed for correction directions (horizontal and vertical angles) which may be presented as space coordinates functions of the center of projections and the point of the object. Thus this type of equation is reduced to the model with the correction γ_β, an absolute term Y_β and matrixes of partial derivatives B_β and D_β.

System (4.1) presented in general terms will look as follows:

$$\varepsilon = Y - AX - DZ, \qquad (4.2)$$
$$\gamma = \bar{Y} - TX - FZ.$$

Vectors X and Z are to be found due to certain priorly defined conditions, that characterize stochastic nature of probabilistic model. This conditions may be as follows:
1)errors ε and γ are influenced by the Gaussian law of division. They are mutually uncorrelated and uncorrelated inside groups ε and γ (this is classical method of the smallest square; in this case the diagonal matrix of weighs is known for vectors ε and γ :

$$P = \begin{bmatrix} p_\varepsilon & \\ & p_\gamma \end{bmatrix}; \qquad (4.3)$$

2. errors ε and γ are influenced by the division law which is different from Gaussian. They are mutually uncorrelated but correlated inside groups ε and γ , thus co-variant matrix of measurement errors is known –

$$\sum = \begin{bmatrix} \sum_\varepsilon & \\ & \sum_\gamma \end{bmatrix} \qquad (4.4)$$

The correction minimization model is adopted for errors with Gauss division and matrix of weighs (4.3)

$$\varepsilon^T \cdot P_\varepsilon \cdot \varepsilon + \gamma^T \cdot P_\gamma \cdot \gamma = \min \ , \qquad (4.5)$$

And for errors with division different from Gaussian and matrix (4.3) we may apply the condition of minimization of unsquare function of loses, or the condition of minimization of mixed function of loses [5]:

$$\varepsilon^2 + |\gamma|^{2+d} = \min \qquad (4.6)$$

Where d – parameter of square - $-1 < d \leq 0$
Model (4.2) with condition (4.6) and covariant matrix (4.4) is the most general from theoretical view.
Let us show (4.2) in the following way

$$\begin{bmatrix} \varepsilon \\ \gamma \end{bmatrix} = \begin{bmatrix} Y \\ \bar{Y} \end{bmatrix} - \begin{bmatrix} A & D \\ T & F \end{bmatrix} \cdot \begin{bmatrix} X \\ Z \end{bmatrix} \qquad (4.7)$$

or η = R - S U

According to square functions of loses $\eta^T \cdot \eta = \min$ and after transformations, analogic to shown above [6], we get adjusted value of the unknown $\hat{U} = (S^T \cdot \Sigma^{-1} \cdot S)^{-1} \cdot S^T \cdot \Sigma^{-1} \cdot R$ or

$$\hat{U} = \begin{bmatrix} \hat{X} \\ \hat{Z} \end{bmatrix} = \left(\begin{bmatrix} A^T & T^T \\ D^T & F^T \end{bmatrix} \cdot \begin{bmatrix} \Sigma_\varepsilon^{-1} & \\ & \Sigma_\gamma^{-1} \end{bmatrix} \cdot \begin{bmatrix} A & D \\ T & F \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} A^T & T^T \\ D^T & F^T \end{bmatrix} \cdot \qquad (4.8)$$

$$\begin{bmatrix} \Sigma_\varepsilon^{-1} & \\ & \Sigma_\gamma^{-1} \end{bmatrix} \cdot \begin{bmatrix} Y \\ \overline{Y} \end{bmatrix}$$

and

$$\hat{U} = \begin{bmatrix} \hat{X} \\ \hat{Z} \end{bmatrix} = \begin{bmatrix} A^T \Sigma_\varepsilon^{-1} A + T^T \Sigma_\gamma^{-1} T & A^T \Sigma_\varepsilon^{-1} D + T^T \Sigma_\gamma^{-1} F \\ D^T \Sigma_\varepsilon^{-1} A + F^T \Sigma_\gamma^{-1} T & D^T \Sigma_\varepsilon^{-1} D + F^T \Sigma_\gamma^{-1} F \end{bmatrix}^{-1} \cdot$$

$$\begin{bmatrix} A^T \Sigma_\varepsilon^{-1} Y + T^T \Sigma_\gamma^{-1} \overline{Y} \\ D^T \Sigma_\varepsilon^{-1} Y + F^T \Sigma_\gamma^{-1} \overline{Y} \end{bmatrix}$$

In the reduced form it looks as follows:

$$\begin{bmatrix} \hat{X} \\ \hat{Z} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \qquad (4.9)$$

The valuation of probability maximum for dispersion σ^2 Is equal to

$$\hat{S}^2 = (R - S\hat{U})^T \cdot \Sigma^{-1}(R - S\hat{U}) \,/\, (n - r)\,, \qquad (4.10)$$

Where n – the number of equations, included into system (4.2),
r – the number of the unknowns in the system (4.2).

In mixed function of loses $\varepsilon^T \cdot \varepsilon + |\gamma|^{2+d} = \min$. On the ground of [4] we get

$$\begin{bmatrix} \tilde{X} \\ \tilde{Y} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} b_1 + \Delta_1 \\ b_2 + \Delta_2 \end{bmatrix} \qquad (4.11)$$

where $\Delta_1 = T^T \cdot \Sigma_\gamma^{-1} \cdot \overline{Y} \cdot \Sigma(\gamma \cdot \Delta)$, $\Delta_2 = F^T \cdot \Sigma_\gamma^{-1} \cdot \overline{Y} \cdot \Sigma(\gamma \cdot \Delta)$,

$\Delta_i = \sum_{j=1}^{n} \frac{(d \cdot \ln|\overline{G} \cdot \gamma_i|)^j}{j!}$, where j=1,2,3……,p – the number of series members (usually $p \le 4$).

$$G = (K^{-1})^T, \quad K^T \cdot K = \Sigma_\gamma \qquad (4.12)$$

The valuation of the dispersion σ^2 is equal to:

$$\tilde{S}^2 = (R - S\tilde{U})^T \cdot \tilde{\Sigma}^{-1}(R - S\tilde{U}) \;/\; (n - r) \tag{4.13}$$

where $\tilde{\Sigma} = \begin{bmatrix} \tilde{\Sigma}_\varepsilon & \\ & \tilde{\Sigma}_\gamma \end{bmatrix}$, $\quad \tilde{\Sigma}_\gamma = \tilde{\Sigma}_\gamma (1 + 2d + 2d^2)$

Thus the formulated above model (4.2) and solution (4.8)-(4.13) are considered to be the solution of the problem of phototheodolite survey with the use of control data.

CONSOLUSIONS

1.The analysis of photogrammery data and methods of adjustment permitted to make a scheme containing main characteristics of measured quantities series, extra data and geometrical conditions that appear in different photogrammetry tasks. This scheme may be used for the choice of the corresponding model of mathematical processing of photogrammetric survey.
2. Chapter 2 presents the mathematical model of joint adjustment of the measured quantities, their functions and control data. This model is generalized in accordance with the wide class of photogrammetrical tasks. Theoretical solution of this model covers the wide range of both, classical solution and new adjusting tasks of photogrammetry. It is proved that the notion of functions of loses (which is widely used in regressive analysis) makes it possible to process the series of measurements with error division different from Gauss.
3. The introduction of new geodetic and photogrammetrical methods and equipment (in particular – the use of GPS observations provides with control of photo stations and control points) require new approaches to the process of analytical processing of survey materials. The generalized mathematical model of phototheodolite survey (3.1) takes into accounts all new technologies of collecting and processing data if the joint adjustment of photogrammetrical measurements and control data becomes useful and possible. Traditional methods may be treated as partial cases of this model.
The authors present the theoretical solution of the adjustment task when control points and linear elements of exterior orientation are known with high and almost equal accuracy (as in case with the use of GPS observation).
The presented mathematical models (2.10) and (3.1) and their solutions may be used to create new algorithms, software, for extension of functional abilities of software for analytical photogrammetric equipment and digital stations.

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